# International Negotiations
# Games, Strategies and Negotiations
## Game-theoretic models of bargaining

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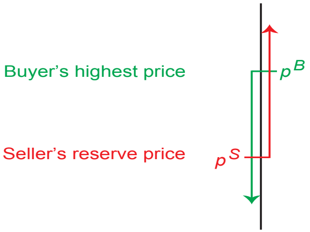
# Bargaining models: static analysis

- Two players are to divide €1 between them
- They simultaneously suggest an amount they wish to receive. If the sum of the amounts is less or equal than €1, then each receives his/her suggestion. Else, they get 0
- Can you think of the Nash equilibria of this game?
- Any suggestion $x$ for P1 and $1 - x$ for P2 is a NE!
- NE cannot make any prediction about such a game $\longrightarrow$ indeterminancy

# Bargaining models: static analysis

- Most models of bargaining: splitting some surplus
- $p^B > p^S$
- What can Game theory tell us about such splits?
- alternative/complementary approaches to game theoretic/rational analysis?

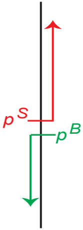
# Surplus and bargaining




- The buyer is willing to pay at most $p_A$
- The seller sells for a price no less than $p_S$

- The two parties bargain for the split of surplus $p_B - p^S$
- This is one of the most basic assumptions of most bargaining models

# Surplus and bargaining




- If $p_A < p_S$ the negotiation cannot lead to a mutually acceptable split
- We will revisit this case later. For now, let's assume that $p_A > p_S$

# Bargaining and indeterminance

- Bargaining over a €1 split
- Game:
  - The two players (1 and 2) simultaneously and independently suggest a split *v*
  - If the proposals are compatible (e.g. 0.40 and 0.50), then they are realised and the game ends
  - If not, each player, knowing the other player's proposal, decides whether to accept or not
  - If both insist on their initial proposal, they get nothing and the game ends
  - If one player (say P1) retracts and the other (e.g. P2) doesn't, the suggestion of the player who didn't retract (p2) is realised
  - If both retract, each take their share in the other's proposal (possibly leaving money at the table)

# Bargaining and indeterminance

- We created a game with well defined rules instead of a general “notion” of negotiation
- Nevertheless, we cannot predict how the game is to be played
- Any proposal of $n$ cents to P1 and $100 - n$ cents for P2 is a Nash equilibrium $\longrightarrow$ indetermiance
- N.E. does not suffice to determine the solution of the game

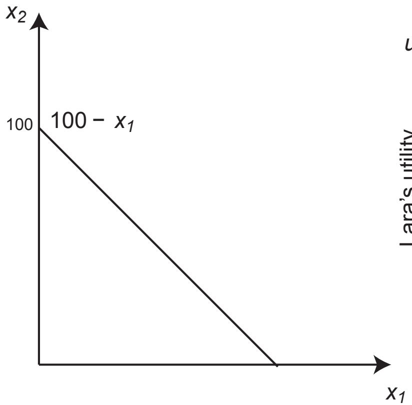
## Possible solutions to the “split the euro” game: focal equilibria

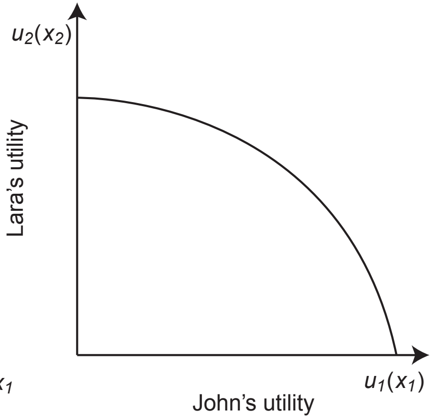
- Sometimes some equilibria are by nature focal
- E.g. often players play 50%-50%
- Or if say it is customary for the man to take 60% of the inheritance, then it is possible that they follow that split
- Experiments confirm that if for any reason players focus on a particular split before the game then they tend to play that split

# Nash bargaining solution

- Think of of the problem of John and Lara splitting €100
- John takes $x_1$ and Lara will be left with $x_2 = 100 - x_1$
- Any such split is a NE (indeterminancy)
- How can we depict possible allocations?
- Two ways:
  1. amount each gets in each split
  2. (psychological) utility each gets from the split

# Nash bargaining solution



# Nash bargaining solution

- Nash bargaining solution comes to answer which split (and corresponding utilities) should be chosen
- Some times negotiating parties cannot find a compromise
- If an arbitrator were to choose the optimal split of the €100 between John and Lara, what split would she choose?
- how would the arbitrator pick the fairest agreement among all possible points of the two lines?

# Nash bargaining solution

What properties should a solution to the bargaining problem have?

- Symmetry: symmetric players/situation should end up with symmetric outcomes: under the same circumstances I shouldn't get less than you if we face symmetric possibilities/reservation prices
- Efficiency: no way to make both parties better off
- Solution should be independent of the units of measurement: doesn't matter if we change scale of utility
- Independence of irrelevant alternatives (technical): If the best outcome of a large set of possibilities belongs to a smaller set (subset of the large set), then this should be the best outcome of the small (sub)set as well

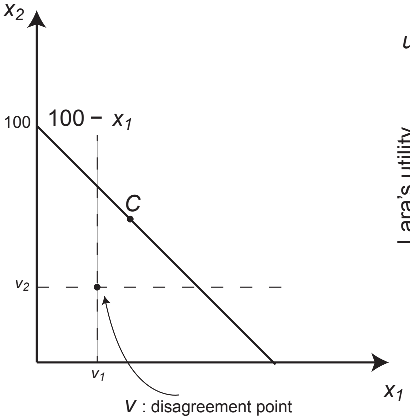
# Nash bargaining solution

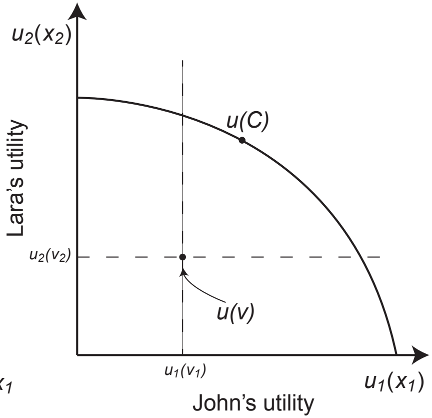
### Theorem (Nash bargaining solution)

*In the bargaining game presented in the graphs above, there exist a unique Nash equilibrium that satisfies symmetry, efficiency, independence of the units of measurement and independence of irrelevant alternatives*

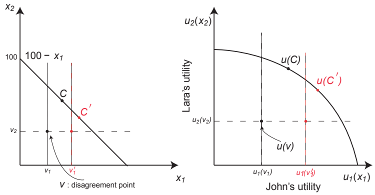
the unique NE, denoted $C$ should give both players more utility than the no-agreement point ($v$)

# Nash bargaining solution (N.B.S)



# Nash bargaining solution (N.B.S)




- With disagreement point $v$, Nash bargaining solution is $C$
- If John has higher reservation point $v_1'$, new NBS is $C'$ (more for John, less for Lara)

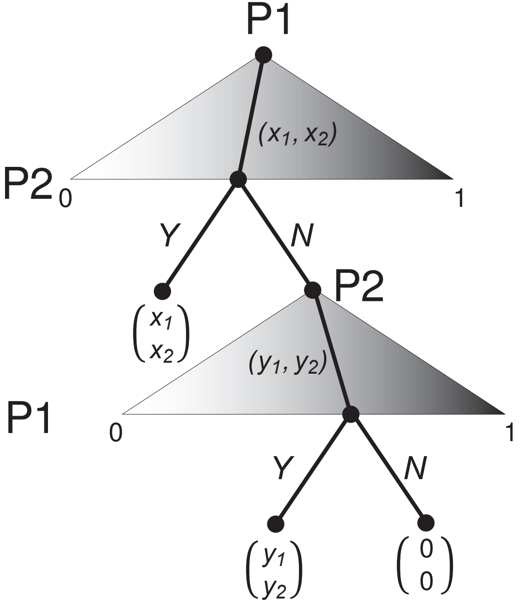
# Nash bargaining solution (N.B.S)

- Note that the NBS does not predict how a specific bargain will end up
- rather it suggests the unique outcome an arbitrator would choose if he/she wanted the solution to satisfy a number of criteria of fairness and efficiency

# The Rubinstein alternating offers bargaining game

- Ariel Rubinstein (1982) proposed an extenstion to the ultimatum game:
- To split a €1, the two players proceed with alternating offers for a number of rounds. If a player accepts the split is realised and the game ends
- for as long as no player accepts the other player's offer, the game continues for $n$ rounds and the player who turned down the offer, makes a counter offer
- if the $n$th round is reached without agreement, they go away with zero payoffs
- let's examine the alternating offers game with 2 rounds:

# The alternating offers game with 2 rounds



# The alternating offers game with 2 rounds

- Can you find the SPNE of this game?
- In this game, P1 has a disadvantage: P2 has all the power of the ultimatum game
- P2 will always refuse any offer by P1
- starting at the subgame after P2's refusal, we have a classic ultimatum game
- in that, P2 suggests $\epsilon$ for P1 and she keeps $1 - \epsilon$ for herself
- NE of the 2 period alternating offers bargaining game:
  (
  P1: any $(x, 1 - x)$,
  Y if P2 rejects and counters $(y, 1 - y), y > 0$,
  P2: N, $(\epsilon, 1 - \epsilon)$
  )

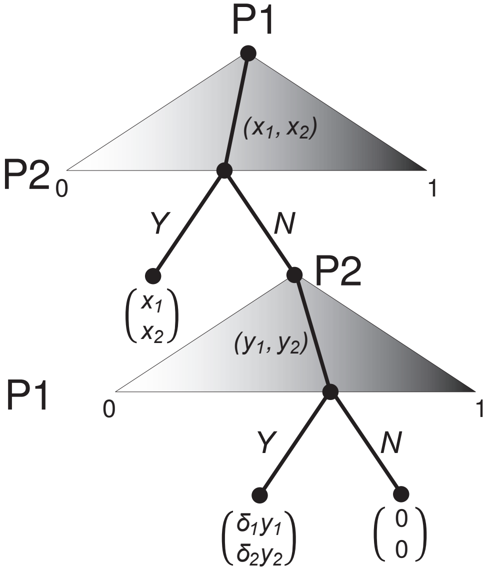
# The alternating offers game with $n$ rounds

- Can you find the SPNE of the alternating game with $n$ rounds?
- Who has the advantage (takes all the pie)?

# The alternating offers game with 2 rounds and impatient players

- Let's consider a variant of the 2-period alternating offers game where players are impatient: they discount the future with discount factors: $\delta_1, \delta_2$
- Then the game in extensive form is given below:

# The alternating offers game with 2 rounds and impatient players



# The alternating offers game with 2 rounds and impatient players

- If players are impatient, player 1 is not so powerless: she can exploit P2's impatience to extract some rents:
- If P2 rejects P1's offer and counters $(y_1, y_2$ in the second round, the present value of her share will be $\delta_2 y_2$
- At most P2 can achieve a payoff of $\delta_2$ if she keeps the whole amount for herself
- Hence in round 1, P1 could keep $1 - \delta_2$ and hand $\delta_2$ to P2
- SPNE of the game:
  (
  P1: $(1 - \delta_2, \delta_2)$, accept any offer by P2 in round 2
  P2: accept any offer with $x_2 \geq \delta_2$ in round 1, and offer $(\epsilon, 1 - \epsilon)$ in round 2
  )

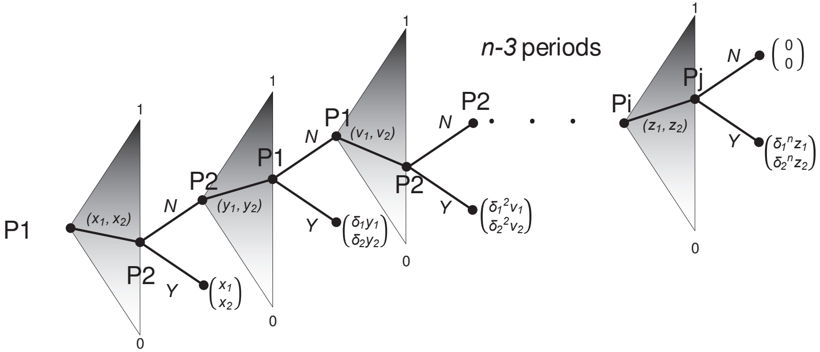
# The alternating offers game with 2 rounds and impatient players

- SPNE of the game:
  (
  P1: $(1-\delta_2, \delta_2)$, accept any offer by P2 in round 2
  P2: accept any offer with $x_2 \geq \delta_2$ in round 1, and offer $(\epsilon, 1-\epsilon)$ in round 2
  )
- Note that the more patient P2 is (the higher $\delta_2$), the larger the share of the pie she ends up with
- This is a standard result of the alternating offers bargaining game: more patient players end up with higher share of the pie!!!
- in negotiations it doesn't pay to be impatient!

# The alternating offers game with $n$ rounds and impatient players

- Suppose that the players keep alternating offers for $n$ rounds
- the game would then be represented in extensive form:

# The alternating offers game with $n$ rounds and impatient players



# The alternating offers game with 3 rounds and impatient players

- Can you find the SPNE of the alternating offers game with 3 rounds?
- Start at the final (third) node: who is playing there? (remember, at the first node P1 moves. At round 2, it's P2's turn and so on...)
- Think what the final offer would be and what that would mean for the penultimate node and so on...
- it is kind of difficult, don't worry if you can't solve it! but give it a try anyway!

# The alternating offers game with infinite periods

- What is the SPNE of the infinitely repeated alternating offers bargaining game?
- Suppose the two players discount the future with discount factors $\delta_1, \delta_2$

## Proposition

*The bargaining game with alternating offers and infinite horizon has a unique SPNE characterised by the strategies:*

- *player 1 always proposes $x$ and accepts a proposal $y$ if and only if $y_1 \geq y_1^*$*
- *player 2 always proposes $y$ and accepts a proposal $x$ if and only if $x_2 \geq x_2^*$, where*

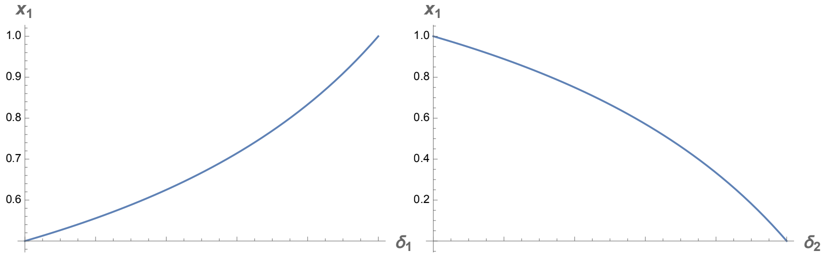
$$x^* = \left(\frac{1-\delta_2}{1-\delta_1\delta_2}, \frac{\delta_2(1-\delta_1)}{1-\delta_1\delta_2}\right)$$

$$y^* = \left(\frac{\delta_1(1-\delta_2)}{1-\delta_1\delta_2}, \frac{1-\delta_1}{1-\delta_1\delta_2}\right)$$

# Properties of the SPNE

- Efficient: offer is accepted in first round. No waste in welfare from waiting
- Patience pays: a players share is increasing in his patience and decreasing in opponent's patience (see plot next slide)
- First mover's advantage: with common $\delta$, first mover has an advantage which disappears as $\delta \to 1$
- if players have different costs each round without deal, the player with the smaller cost will take higher share
- If players require different amounts of time before making an offer, the player who takes more time, will take a higher share (why? Imagine a player taking for ever. What does it mean for the other player?)

# Patience in the alternating offers game




- a player's share rises with his/her patience
- a player's share decreases as his/her opponent becomes more patient