International Negotiations
Games, Strategies and Negotiations
Game Theory, solution concepts: Dominance

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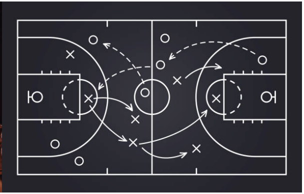


Solution notions of normal or strategic form games

- Given a strategic form game, what constitutes a solution?
- The object of game theory is to predict which outcomes are more likely to occur in a strategic situation
- There are various solution concepts for strategic form games
- We begin with dominance

Solutions of strategic form games: Dominant/dominated strategies

- In some games might there might be an obvious of play for one or more players: one strategy might be preferred by this player, *irrespective of what the other player plays*
- Such a strategy is called *dominant strategy* for this player

Dominant/dominated strategies

- The prisoner's dilemma "Two prisoners are interrogated *separately*. If both keep their "mouths shut" (that is if both *cooperate* with one another), they spend one period in jail. If one betrays (*defects*) the other by giving the authorities incriminated evidence against the other while the other cooperates, then the one who defects is freed and the one who cooperates is sentenced to imprisonment for 9 periods of time. If both defect, then both are incarcerated fo 6 periods of time. Their preferences depend only on the time spent in prison"
- Players: The two prisoners $N = \{1, 2\}$
- Strategies: The strategy sets for the two players are $S_i = \{\text{C, D}\}$ for each player $i \in \{1, 2\}$

Dominant/dominated strategies

- Let's see their payoffs in a table (strategic form):

	C	D
C	$-1, -1$	$-9, 0$
D	$0, -9$	$-6, -6$

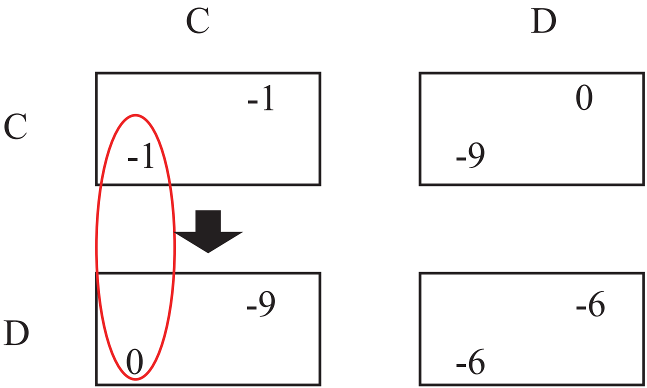
Dominant strategies in prisoner's dilemma

	C	D
C	-1 -1	0 -9
D	-9 0	-6 -6

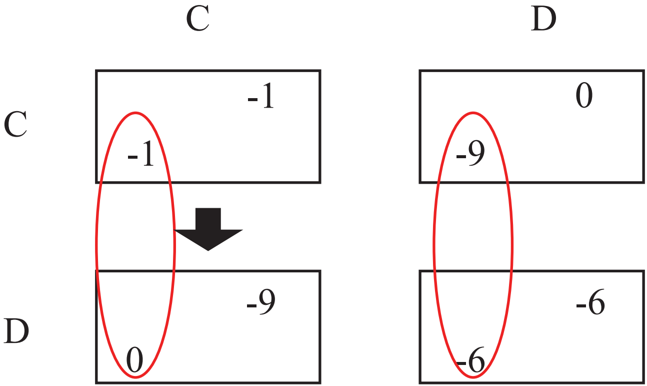
Dominant strategies in prisoner's dilemma

	C	D
C	-1 / -1	0 / -9
D	-9 / 0	-6 / -6

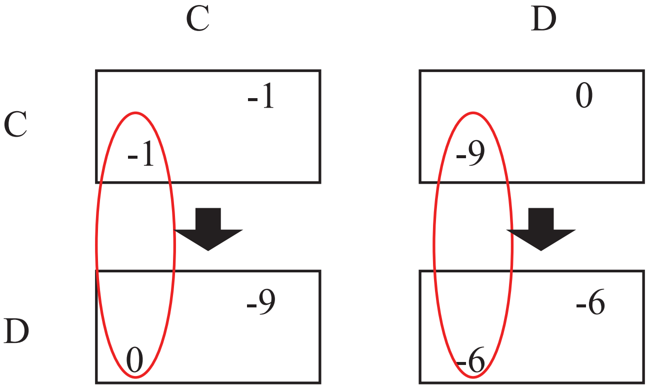
Dominant strategies in prisoner's dilemma



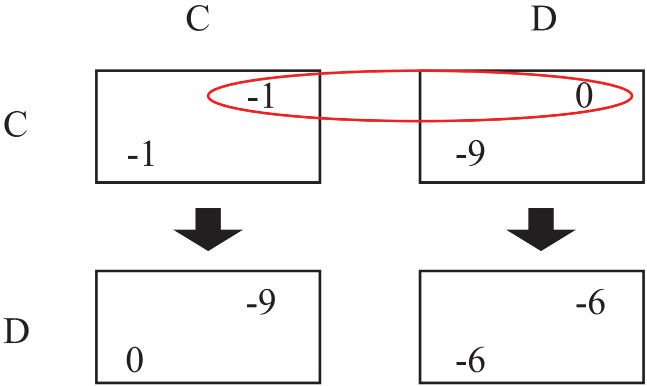
Dominant strategies in prisoner’s dilemma



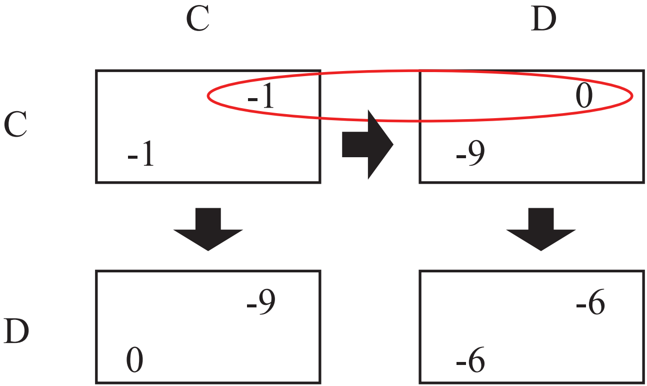
Dominant strategies in prisoner's dilemma



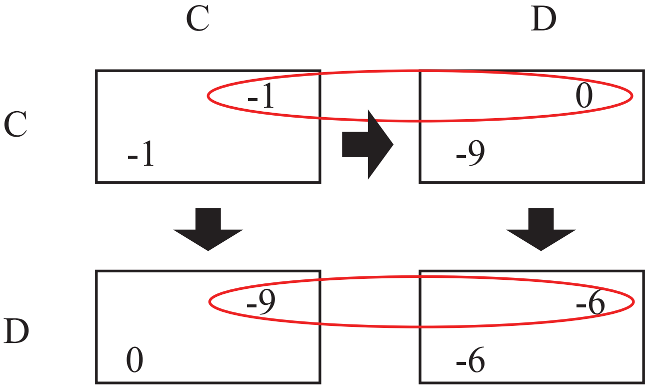
Dominant strategies in prisoner's dilemma



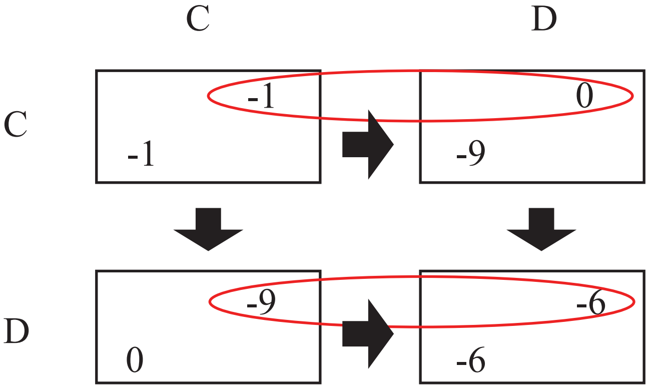
Dominant strategies in prisoner's dilemma



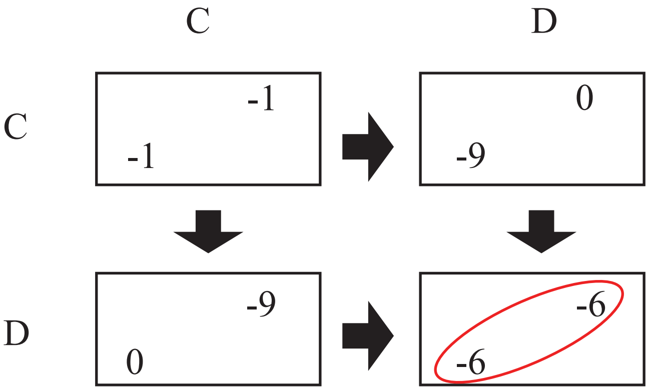
Dominant strategies in prisoner's dilemma



Dominant strategies in prisoner's dilemma



Dominant strategies in prisoner's dilemma



Dominant strategies in Prisoner's dilemma

- D is a strictly dominant strategy for both players C is a strictly dominated strategy
- That is, D is preferred to C irrespective of the opponent's move
- More formally: s_1 is **a strictly dominant strategy** for player 1 iff

$$u(s_1, s_{-1}) > u(s'_1, s_{-1}) \tag{1}$$

  for every other strategy s'_1 player's 1 strategy set and for every strategy s_{-1} of the other player/players
- s_1 is **weakly** dominant strategy for player 1 if:

$$u(s_1, s_{-1}) \geq u(s'_1, s_{-1}) \tag{2}$$

Cooperation in an environment that rewards betrayal

- Why does the prisoner’s dilemma enjoy such recognition? Because it highlights in a clear way situations in which both parties would benefit from cooperation, however both parties have a strong incentive to “cheat”
- Arms race: think of variation of prisoner’s dilemma: Two neighbouring countries with tense diplomatic relations, decide on whether to invest in armament. If neither invests in weapons, both countries can divert resources in other sectors they prefer to invest in. However, irrespective of what the opponent does, each country individually prefers to invest: if only I invest, I will dominate strategically. If you invest, I wish to invest too, to avoid being dominated

Arms races as a prisoner's dilemma

		United States	
		Disarm	Arm
China	Disarm	3, 3	1, 4
	Arm	4, 1	2, 2

- We observe that whereas a deal for both to disarm improves both countries' welfare, both countries have a strategic motive to renege and increase (individually) their payoff, IRRESPECTIVE OF WHAT THE OTHER DOES
- Could they move towards a mutually beneficial deal?

Lessons to take away from Prisoner's dilemma

- Economic Theory had for a very long time regarded pursue of personal interest as a way to promote social welfare:

  > *The natural effort of every individual to better his own condition, when suffered to exert itself with freedom and security, is so powerful a principle, that it is alone, and without any assistance, not only capable of carrying on the society to wealth and prosperity, but of surmounting a hundred impertinent obstructions with which the folly of human laws too often encumbers its operations*

  Adam Smith, The Wealth of Nations, Book IV, Chapter V, Digression on the Corn Trade
- Game Theory can provide many examples where rational pursue of own interest can have detrimental welfare effects for individual and society

Lessons to take away from Prisoner's dilemma

- The P. D. provides a good depiction of the countervailing forces between cooperating and deviating
- Despite obvious benefits from cooperation, in a STATIC environment, when that is the game is only played once, there is a strong incentive to deviate and compete against one another
- We need to revisit the P. D. under the light of a stable long-term cooperation. Time adds another perspective into the strategic environment and can change a lot of the conclusions of static analysis

Dominated strategies

- A dominant strategy is a very powerful notion (and an obvious way to play), however it is not encountered usually in games. Because a strategy needs to be preferred to all my other strategies, whatever the other players might play
- A somewhat weaker notion is that of a dominated strategy: a strategy that is inferior to another strategy (not to all other strategies), whatever the other players might play
- Let's see an example

Iterated Deletion of Strictly Dominated Strategies

	L	C	R
U	5 -2	4 4	3 6
M	-1 0	0 3	-1 6
D	2 -1	8 5	5 5

Iterated Deletion of Strictly Dominated Strategies

	L	C	R
U	-2, 5	4, 4	6, 3
M	0, -1	3, 0	6, -1
D	-1, 2	5, 8	5, 5

Here, column R is strictly dominated for player column by column C

Iterated Deletion of Strictly Dominated Strategies

	L	C	R
U	-2, 5	4, 4	6, 3
M	0, -1	3, 0	6, -1
D	-1, 2	5, 8	5, 5

Here, column R is strictly dominated for player column by column C

Strictly dominated strategies

- A strategy s_i for player i is strictly dominated for$i$, if player i has another strategy s_i' which he always prefer to strategy s_i More formally: if $\exists\ s_i' \in S_i$, such that $\forall s_{-i} \in S_{-i}$:

$$u(s_i, s_{-i}) < u(s_i', s_{-i}) \tag{3}$$

- Here we not that s_i is dominated for i only by strategy s_i', and not necessarily by all other strategies of player i.
- Weak dominance is defined with $\leq$ instead of $<$

Iterated Deletion of Strictly Dominated Strategies

- When i has a strictly dominated strategy s_i, it is reasonable to delete it (since he would never play s_i if he is rational)
- If all players are rational, they would never play strictly dominated strategies and they would delete them. They would also know that other players would never play dominated strategies and have deleted them and so on
- For as long as strictly dominated strategies exist, players will keep deleting them until there are now strictly dominated strategies left
- The strategies we are left with after Iterated Deletion of Strictly Dominated Strategies (IDSDS) are calledrationalizable strategies

Iterated Deletion of Strictly Dominated Strategies

- Important notes:
- IDSDS doesn't mean that the players move one after the other: the game is not dynamic, it is static
- This whole procedure takes place in players' mind: they analyse the game and their strategies before playing
- If I know that you have a strictly dominated strategy and hence that you would never play it, I can check for my dominated strategies in the reduced game, after deletion and delete possible strictly dominated strategies in it
- We can delete only strictly dominated strategies, not weakly dominated ones. Some authors proceed with deletion of weakly dominated strategies as well, but this as we shall see, can lead to eliminating ways of playing the game that are the very likely to be played

Iterated Deletion of Strictly Dominated Strategies

	L	C	R
U	5 -2	4 4	3 6
M	-1 0	0 3	-1 6
D	2 -1	8 5	5 5

Iterated Deletion of Strictly Dominated Strategies

	L	C	R
U	-2, 5	4, 4	6, 3
M	0, -1	3, 0	6, -1
D	-1, 2	5, 8	5, 5

Iterated Deletion of Strictly Dominated Strategies

	L	C	R
U	-2, 5	4, 4	6, 3
M	0, -1	3, 0	6, -1
D	-1, 2	5, 8	5, 5

Iterated Deletion of Strictly Dominated Strategies

	L	C	R
U	-2, 5	4, 4	6, 3
M	0, -1	3, 0	6, -1
D	-1, 2	5, 8	5, 5

Iterated Deletion of Strictly Dominated Strategies

	L	C	R
U	5 -2	4 4	3 6
M	-1 0	0 3	-1 6
D	2 -1	8 5	5 5

Iterated Deletion of Strictly Dominated Strategies

	L	C	R
U	-2, 5	4, 4	6, 3
M	0, -1	3, 0	6, -1
D	-1, 2	5, 8	5, 5

Iterated Deletion of Strictly Dominated Strategies

	L	C	R
U	-2, 5	4, 4	6, 3
M	0, -1	3, 0	6, -1
D	-1, 2	5, 8	5, 5

Rationalizable strategies

	L	C	R
U	7,2	0,1	-2, 7
M	3 ,4	1,2	0,0
D	-2 ,0	2, -1	-3,4

Column C is strictly dominated (by column L)

Rationalizable strategies

	L	C	R
U	7,2	0,1	-2, 7
M	3 ,4	1, 2	0,0
D	-2 ,0	2, -1	-3,4

Column C is strictly dominated (by column L)
We can delete it

Rationalizable strategies

	L	R
U	7,2	-2, 7
M	3 ,4	0,0
D	-2 ,0	-3,4

In the new game that ensues, row D is strictly dominated (by say row U)

Rationalizable strategies

	L	R
U	7,2	-2, 7
M	3 ,4	0,0
D	-2,0	-3, 4

In the new game that ensues, row D is strictly dominated (by say row U)
We delete it

Rationalizable strategies

	L	R
U	7,2	-2, 7
M	3 ,4	0,0

We can no longer delete another row or column. The remaining strategies are rationalizable strategies

Note that whereas sometimes it leads to a unique solutions, in other cases, the Iterated Delection of Strictly Dominated Strategies leads to a subset of the initial game without determining a unique way to play it.

IDSDS, some key points: ATTENTION

Three key points when practicing iterated deletion:

- IDSDS is a thought process by the players when analysing the game. Deletion of strategies is not an actual move in the game. That is why we apply it to static games. While analysing how the game might be played, the players *exclued* from play strategies that cannot be played by rational agents, they don't actually move to delete a strategy
- As we mentioned, in order to delete a player's strategy, it needs to be strictly dominated. Deleting a weakly dominated strategy can lead to a wrong prediction of how the game will be played
- When more than one strategies are strictly dominated, the order of deletion has no effect on the final outcome. We can start either by row or column and delete any dominated strategy they might have in arbitrary order

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