International Negotiations
Games, Strategies and Negotiations
Game Theory: Introduction

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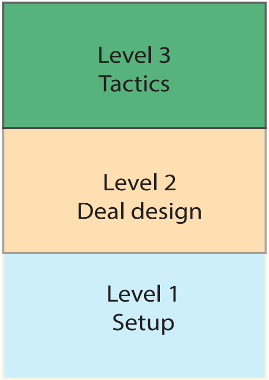
Negotiations. Strategic behaviour

Negotiations:

In this section we will discuss the basic concepts of Game Theory

- Basic theory: need of a formal modelling of strategic behaviour. Usually it involves unidimensional analysis. Think of voting on a single-dimensional strategy space
- Strategic interaction is what takes place at the higher level (level 3 of a negotiation)

Levels of a negotiation



What is a game?

- Game: A formal (mathematical) representation/analysis of behaviour when there are STRATEGIC INTERACTIONS
- The above definition is general
- We need certain components to describe a game

What does a game consist of?

To describe a game we need to know:

1. The players: Who participates? E.g. side A and side B in a game of nuclear armament. Or two or more parties in a bilateral or multilateral negotiation. Or black and white in chess
2. The rules: who moves and when? What knowledge does the mover have? What moves are permissible etc.? E.g. In chess, the white moves first and then the two players take turns. The white can move either their pawns in such and such ways or the knights etc...
3. The possible outcomes: white wins, black wins or draw
4. The payoffs: how much each player values each possible outcome. E.g. if we bet €10 on a head or tails toss: $U(\text{Heads}) =$€10, $U(\text{Tails}) =$€-10.

Kinds of Games: cooperative and non-cooperative games

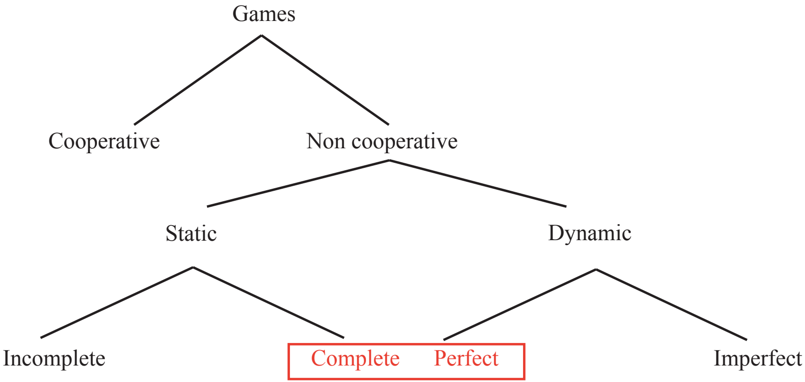
- Depending on whether players can sign *binding, enforceable* contracts on how to play the game or not, we distinguish between:
  1. Cooperative games (It is possible to agree in binding, enforceable contracts)
  2. Non-cooperative games (signing of binding contracts not possible)

Kinds of Games: static and dynamic games

- Depending on whether the players move simultaneously (once) or serially, the games we distinguish between:
  1. Simultaneous or static games (e.g. rock-paper-scissors)
  2. Dynamic games (e.g. games of threat and response)

Kinds of games: games of complete and incomplete information

- Depending on the kind of information players have, static games are distinguished into:
  1. Games of complete information: the players know everybody's payoffs
  2. Games of incomplete information: at least one players doesn't know his or another players' payoffs
- Depending on the kind of information players have, dynamic games are distinguished into:
  1. Games of perfect information: whoever moves knows which knot of the game he is on
  2. Games of imperfect information: at least one player doesn't know which knot he is moving from
- We will only look into complete and perfect information


Games
Cooperative
Non cooperative
Static
Dynamic
Incomplete
Complete
Perfect
Imperfect

Representing games: games in normal or extensive form

For static games of 2 players we juse a simple form of representation: a table of payoffs

- E.g. Player 1 (row) has three moves: U, M or D
  Player 2 (column) has two moves: L or R:
- Each cell of the table corresponds to one outcome. Inside each cell we write the players' payoffs (first for row and second for column, separated by commas)
- In static games each move is a strategy. We call this representation *strategic* or *normal form representation* of a game

Strategic or normal form representation of a game

- Simultaneous-move games with 2 players admit a very simple form of representation (strategic or normal form representation) as a table:

	L	R
U	10,10	0,3
M	3,0	2,2
D	21,–5	22,2

- Player 1 is row. Every row represents one possible action for player 1
- Every row is also a strategy (in static games actions and strategies coincide)

Strategic or normal form representation of a game

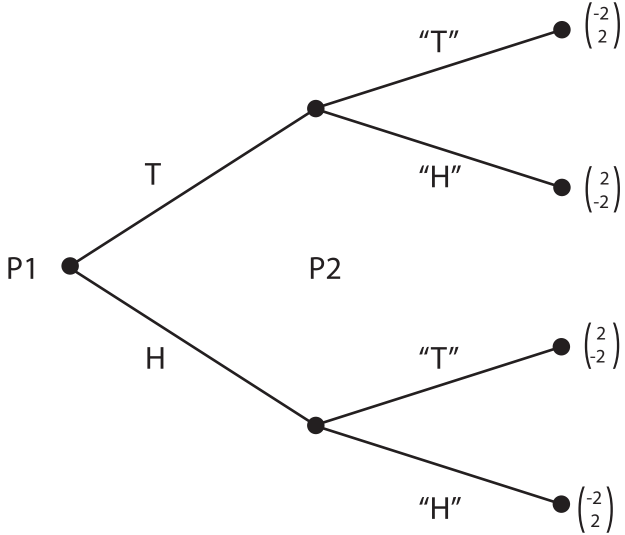
	L	R
U	10,10	0,3
M	3,0	2,2
D	21,–5	22,2

- Player 2 is column. Every column represents one possible action for player 2
- Every column is also a strategy
- Every cell in the table is a possible outcome of the game
- In every cell of the table we write first row's payoff from this outcome and then column's, separated by commas
- So that in the *strategy profile* where Row plays D and Column plays L, Row receives 21 units of utility (payoff) and Column receives –5 units of utility

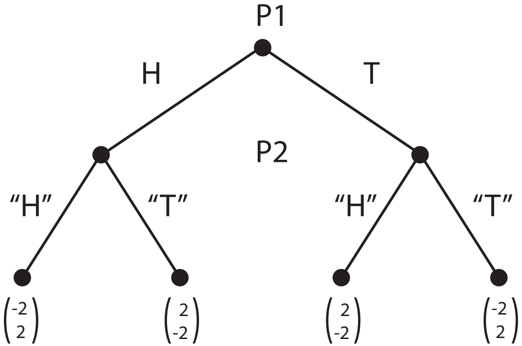
Extensive form representation of games

A more general form of representation is extensive form representation of a game. It can depict dynamic games

- Matching pennies: Player 1 places a coin on a table. She has two possible moves: place the coin with either heads (H) or tails (T) facing up
  Player 2 also has two moves: to call either heads ("H") or tails ("T")
- Payoffs: if player 2 matches the the side of player's 1 coin facing up, she wins €2. Else she loses €2.
- How can we depict this game?



P1
T
H
P2
"T"
"H"
"T"
"H"
-2
2
2
-2
2
-2
-2
2


P1
H
T
P2
"H"
"T"
"H"
"T"
-2
2
2
-2
2
-2
-2
2

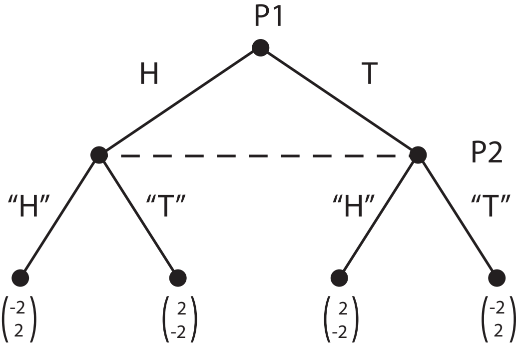
Extensive form representation of games

The extensive form representation of a game consists of:

- the players P1, P2,...
- the nodes: either final or nodes indicating that a player moves
- the moves of the player in each node
- the outcomes (all the final nodes)
- the payoffs for each player in each outcome

Extensive form representation of games

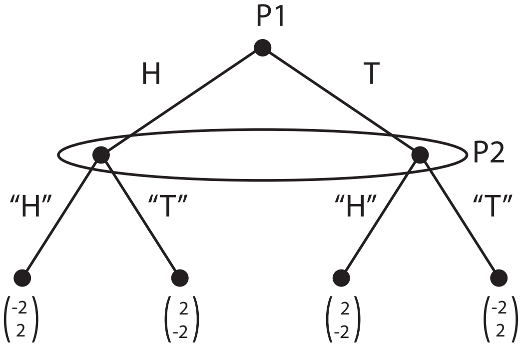
- Matching pennies described above is a dynamic game (first P1 moves and then P2), of perfect information (when P2 moves she knows which node P1 brought her to)
- Let's examine a variant of the game in which P1 places the coin on the table but covers it with his hand. P2 says either "H" or "T" without knowing which side P1 placed facing up
- How do we represent this game?



P1
H
T
P2
“H”
“T”
“H”
“T”
-2 2
2 -2
2 -2
-2 2

Extensive form representation of games

- If a player doesn't know which node she is brought at by a preceding player's move, we represent this "lack of knowledge" or incomplete information by either a dotted line connecting the set of nodes that the player is at (without knowing which precise node she has been brought to) or by ellipse encircling this set of nodes
- The set of nodes the player who doesn't know where she is, is called "information set"


P1
H
T
P2
"H"
"T"
"H"
"T"
-2
2
2
-2
2
-2
-2
2

Example: Rock-paper-scissors

Let's represent the rock-paper-scissors-game (I assume everyone is familiar with it) in strategic form representation:

		P2		
		R	P	S
	R	0, 0	-1, 1	1, -1
P1	P	1, -1	0,0	-1, 1
	S	-1, 1	1, -1	0, 0

Table: Rock-paper-scissors in strategic form representation

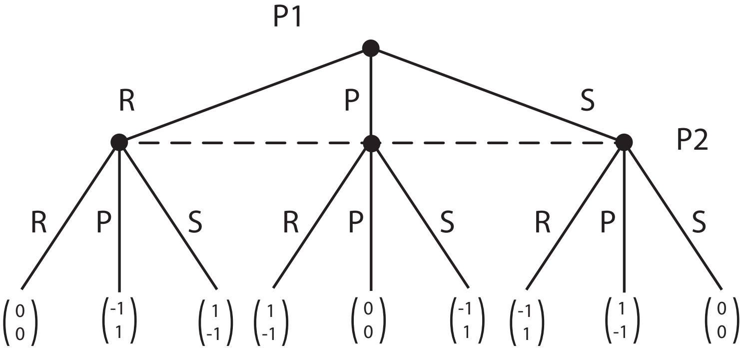


Figure: Rock-paper-scissors in extensive form representation

References

1. Dutta, Prajit K. (1999), ”Strategies and Games”, MIT Press, Cambridge, Massachusetts. Chapters: 1, 2, 3.1, 3.2.
2. Osborne, Martin. (2004), ”An Introduction to Game Theory”, Oxford University Press, Oxford. Chapters 1, 2.1-2.5.
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