

$$y_i = \begin{cases} 1 & \pi_i \\ 0 & 1 - \pi_i \end{cases}$$

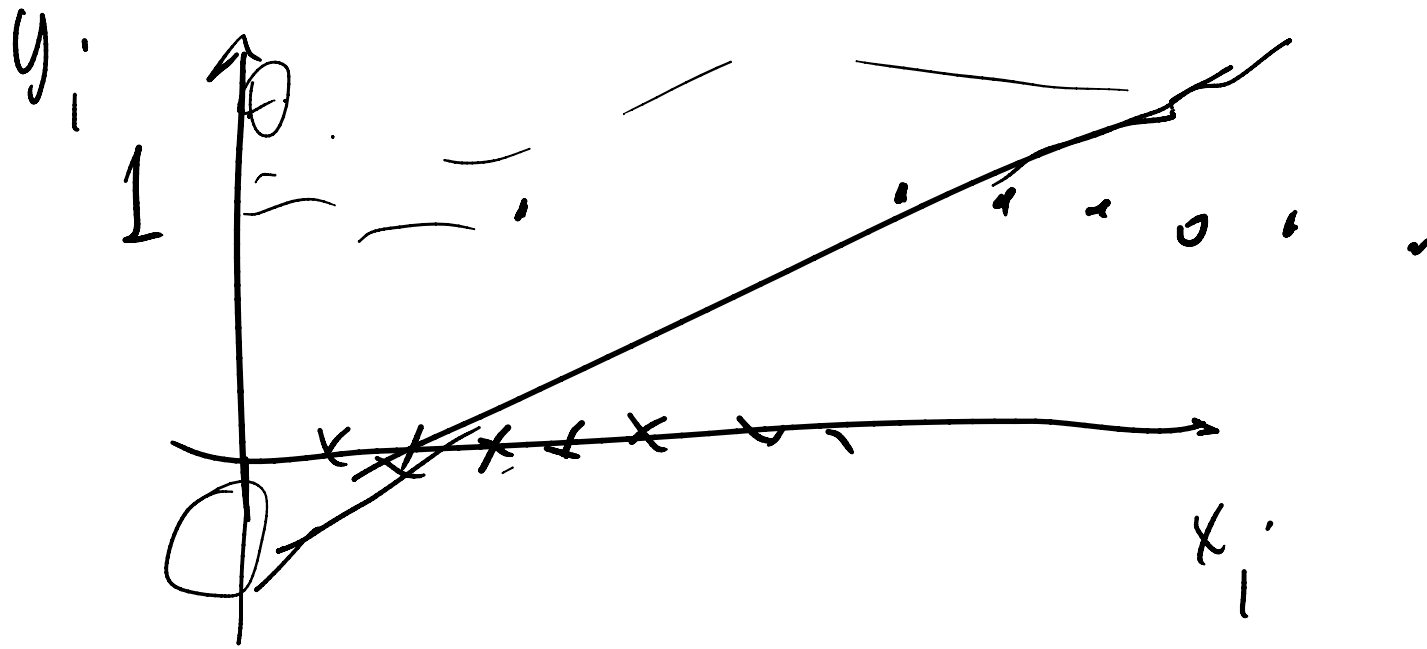
$$y_i = x_i' \beta + \varepsilon_i \rightarrow \varepsilon_i = y_i - x_i' \beta \begin{cases} \rightarrow 1 - x_i' \beta \pi_i \\ \rightarrow 0 - x_i' \beta (1 - \pi_i) \end{cases}$$

$$E(\varepsilon_i) = 0 \Rightarrow (1 - x_i' \beta) \pi_i + (-x_i' \beta) (1 - \pi_i) = 0$$

$$\Rightarrow \pi_i = x_i' \beta$$

$$V(\varepsilon_i) = \pi_i (1 - \pi_i) = x_i' \beta (1 - x_i' \beta) \Rightarrow \text{HETERO}$$

\Rightarrow ROBUST WHITE ESTIMATION



$\tilde{\pi}_i$ either > 1 or < 0 via x_i METHA
 or < 0 via x_i MIKPA

PROBIT

y_i^* latent variable (UNOBSERVED)

$$y_i^* = x_i' \beta + \varepsilon_i \quad \varepsilon_i \sim N(0, \sigma^2)$$

$$y_i = \begin{cases} 1 & y_i^* > 0 \\ 0 & y_i^* \leq 0 \end{cases}$$

$$P(y_i = 1) = P(y_i^* > 0) \Rightarrow P(x_i' \beta + \varepsilon_i > 0)$$
$$\Rightarrow P(\varepsilon_i > -x_i' \beta) = P\left(\frac{\varepsilon_i}{\sigma} > -x_i' \beta / \sigma\right)$$

$$\begin{aligned} P(y_i = 1) &= P\left(\frac{\varepsilon_i}{\sigma} > -x_i' \frac{\beta}{\sigma}\right) \quad (\sigma = 1) \\ &= P(\varepsilon_i > -x_i' \beta) = P(\varepsilon_i < x_i' \beta) \\ &= \Phi(x_i' \beta) \end{aligned}$$

$$P(y_i = 0) = P(y_i^* < 0) = 1 - \Phi(x_i' \beta)$$

$$l = \sum_{i=1}^n \left\{ y_i \ln [\Phi(x_i; \theta)] + \right.$$

$$\left. (1 - y_i) \ln [1 - \Phi(x_i; \theta)] \right\} \leq 0$$

$\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_k \end{pmatrix} \leftarrow \text{standard}$

$$LR = \left[\chi^2_{k-1} \right] \left[l(\theta_1, \theta_2, \dots, \theta_k) - l(\theta_1, 0, 0, \dots, 0) \right]$$

$$P(y_i = 1) = \Lambda(x_i' \beta) \stackrel{\text{LOGIT}}{=} \frac{e^{x_i' \beta}}{1 + e^{x_i' \beta}} \leftarrow$$

$$y_i = \Lambda(x_i' \beta) + \varepsilon_i \quad \varepsilon_i \sim \text{extreme value distribution}$$

$$E(y_i) = F(x_i' \beta) \quad \begin{cases} \rightarrow \Phi(x_i' \beta) \\ \rightarrow \Lambda(x_i' \beta) \end{cases}$$

$$\frac{\partial E(y_i)}{\partial x_v} = \phi(x_i' \beta) \beta_v$$

$$v = 1, 2, \dots, k$$

$$\frac{\partial E(y_i)}{\partial x_v} = \frac{e^{x_i' \beta}}{[1 + e^{x_i' \beta}]^2} \beta_v$$

LINEAR MODEL

$$\frac{\partial E(y_i)}{\partial x_v} = b_v \leftarrow$$

likelihood value
π₁'s π₂'s π₃'s

R² difficult to interpret.

$$y_i = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

MacFadden R² = R_M²

$$R_M^2 = 1 - \frac{\ell(\hat{\beta})}{\ell(\hat{\beta}_{1000})}$$

likelihood value
of the model