CONDITIONAL HETEROSK. MODELS

$$r_t = x_t' b + \varepsilon_t =$$
$$= x_t' b + \sigma_t z_t, \quad z_t \sim N(0,1)$$

1) GARCH(1,1)

$$\sigma_t^2 = w + \alpha \varepsilon_{t-1}^2 + \delta \sigma_{t-1}^2$$

POSIT. CONSTR. $w > 0, \quad \alpha \geq 0, \quad \delta \geq 0$

STATIONAR.

$$\alpha + \delta < 1$$

NO DYNAMIC ASYMMETRY (if negative LEVERAGE EFFECT)

2) GQARCH (1,1)

$$\sigma_t^2 = w + \alpha (\varepsilon_{t-1} + \gamma)^2 + \delta \sigma_{t-1}^2$$

LEVERAGE iff $\gamma < 0$

3) GJR (1, 1)

$$\sigma_t^2 = w + \alpha \varepsilon_{t-1}^2 + \gamma \varepsilon_{t-1}^2 \, \mathbb{I}(\varepsilon_{t-1} < 0) + \delta \sigma_{t-1}^2$$

LEVERAGE $\underline{\underline{\text{iff}}}$ $\gamma > 0$

4) EXPONENTIAL GARCH

EGARCH (1, 1)

$$\ln \sigma_t^2 = w + \gamma z_{t-1} + \theta(|z_{t-1}| - E|z_{t-1}|) + \delta \ln \sigma_{t-1}^2$$

$\sigma_t^2 = e^{\ln \sigma_t^2} > 0$ W.P. 1 $\forall\, w, \gamma, \theta, \delta \in \mathbb{R}$

NO POSITIVITY, STATION. $\delta < 1$, LEVER. iff $\gamma < 0$

Dependent Variable: EXR1
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Sample: 1960M01 2003M12
Included observations: 528
Convergence achieved after 29 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(7) + C(8)*ABS(RESID(-1)/@SQRT(GARCH(-1))) +
C(9)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
JANDUM	2.573751	0.294561	8.737571	0.0000
RMRF	0.895623	0.016175	55.36964	0.0000
RMJAN	-0.060973	0.042835	-1.423444	0.1546
HML	0.242499	0.025189	9.627083	0.0000
HMJAN	-0.229997	0.076218	-3.017620	0.0025
SMB	1.214897	0.020217	60.09151	0.0000
Variance Equation				
C(7)	-0.114345	0.028775	-3.973802	0.0001
C(8)	0.170104	0.040102	4.241809	0.0000
C(9)	0.977167	0.011749	83.16798	0.0000

R-squared	0.934963	Mean dependent var	0.791042
Adjusted R-squared	0.934340	S.D. dependent var	6.384388
S.E. of regression	1.635950	Akaike info criterion	3.687386
Sum squared resid	1397.046	Schwarz criterion	3.760155
Log likelihood	-964.4700	Hannan-Quinn criter.	3.715874
Durbin-Watson stat	1.927902		

$\delta = C(9)$

$H_0: \delta \geq 1$ v.s.

$H_1: \delta < 1$

$$z = \frac{\hat{\delta} - 1}{s.e.(\hat{\delta})} = \frac{0.9772 - 1}{0.0117} \approx -2.3$$

Reject H_0 iff $z < z_\alpha$, $\alpha = 5\%$

$z < -1.645$ ΕΠΕΙΔ $z = -2.3 < 1.645 \Rightarrow$

$\Rightarrow$ Reject $H_0 \Rightarrow \delta < 1$

ΣΥΓΚΡΙΣΗ GARCH ΜΕ EGARCH

$\sigma_t^2 = \ldots$ NON-NESTED.

$\ln \sigma_t^2 =$

ΔΙΑΛΕΞΕ ΤΟ ΜΟΝΤΕΛΟ ΠΟΥ ΕΧΕΙ ΤΑ ΜΙΚΡΟΤΕΡΑ INFORMATION CRITERIA

AKAIKE, Schwatz, Hannan - Quinne

ΣΥΓΚΡΙΣΗ GARCH(1,1) ΜΕ GQARCH(1,1)

γιά γ=0 ΤΟ GQARCH ≡ GARCH

=) GARCH NESTED IN GQARCH(1,1)

ΑΝ γ ΣΤΑΤΙΣΤΙΚΑ ΣΗΜΑΝΤΙΚΟ ΤΟΤΕ

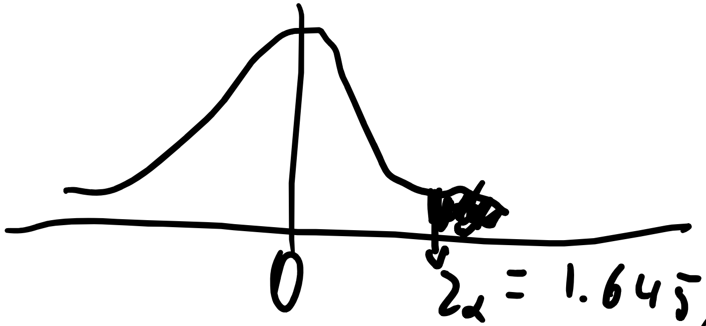
GQARCH ΚΑΛΥΤΕΡΟ ↑ P.V. < 5%

GARCH NESTED IN GJR (γ=0)

γ ΣΤΑΤΙΣΤΙΚΑ ΣΗΜΑΝΤΙΚ (P.V. < 5%) =) GJR ΚΑΛΥΤΕΡΟ

ΑΝ P.V. > 5% =) GARCH ΚΑΛΥΤΕΡΟ.

$H_0: \delta \leq 1$ v.s. $H_1: \delta > 1$ ← ~~WRONG~~



Reject H_0 iff $\dfrac{\hat{\delta} - 1}{s.e.(\hat{\delta})} > 1.645$

$$\frac{\hat{\delta} - 1}{s.e.(\hat{\delta})} = \frac{0.977 - 1}{0.011} < 0$$

AN $\hat{\delta} = 0.99$ $\quad \dfrac{0.99 - 1}{0.011} \approx -1$

$$y_t = \alpha y_{t-1} + \varepsilon_t \qquad \varepsilon_t \overset{iid}{\sim} (0, \sigma^2)$$

AR(1) STATIONARY iff $|\alpha| < 1$

$$E(y_t) = 0, \quad V(y_t) = \frac{\sigma^2}{1-\alpha^2}$$

$$\rho_k = \alpha^k \longrightarrow 0 \text{ for } k \to \infty$$

$\alpha = 1$

$$y_t = \varepsilon_t + \alpha \varepsilon_{t-1} + \alpha^2 \varepsilon_{t-2} + \alpha^3 \varepsilon_{t-3} + \dots$$

$$E(y_t) = E(\varepsilon_t) + \alpha E(\varepsilon_{t-1}) + \alpha^2 E(\varepsilon_{t-2}) + \dots$$

$$= 0 + 0 + 0 = 0 \ \forall \alpha$$

$\underline{V(y_t)} = \sigma^2 + \alpha^2 \sigma^2 + \alpha^4 \sigma^2 + \alpha^6 \sigma^2 + \dots$

$$= \sigma^2 (1 + \alpha^2 + \alpha^4 + \alpha^6 + \dots) \overset{\alpha=1}{=} \sigma^2 (1 + 1 + 1 + 1 + \dots) \to \infty$$

για $\alpha=1$ $V(y_t)$ ΔΕΝ ΥΠΑΡΧΕΙ

$\Rightarrow$ y_t ΔΕΝ ΕΙΝΑΙ 2^{nd} ORDER STATIONARY

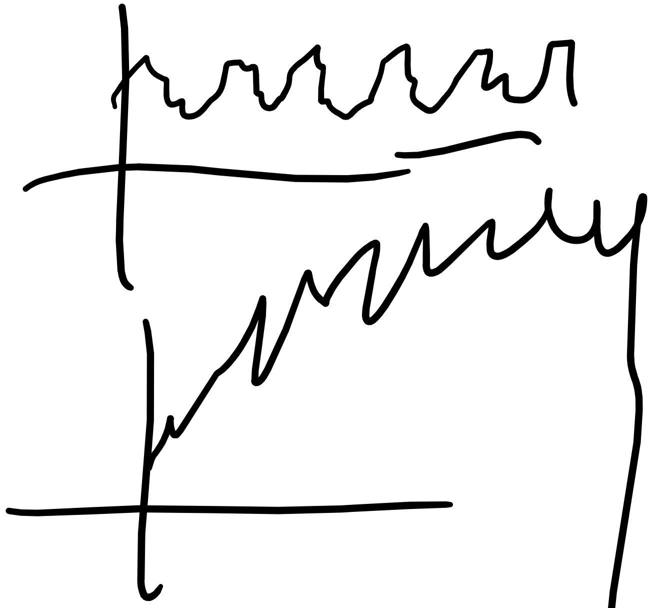
$$\rho_k = \alpha^k \xrightarrow{\alpha=1} 1^k = 1 \quad \forall k$$

ΠΕΡΙΜΕΝΩ ρ_k να είναι ΚΟΝΤΑ ΣΤΟ 1 ~~α~~ κόμα και μεγάλο k

$$\hat{\rho}_k = 1 - \frac{k}{T}$$

T ← NO. OF OBSERVATIONS

AR(1) STATIONARY



AR(1) NON-STATION.

$$y_t = \alpha y_{t-1} + \varepsilon_t$$

LAG OPERATOR (ΣYMB. L)

$$L^K y_t = y_{t-K}, \quad L^2 y_{t-2} = y_{t-2-2}, \quad L^3 y_t = y_{t-3}$$

$$y_t = \alpha L y_t + \varepsilon_t \Rightarrow y_t(1 - L\alpha) = \varepsilon_t$$

CHARECTERISTIC POLYNOMIAL

$1-\alpha L=0 \Rightarrow L=\frac{1}{\alpha}>0$ για $\alpha<0$

$\alpha=1 \Rightarrow 1-L=0 \Rightarrow \boxed{L=1}$

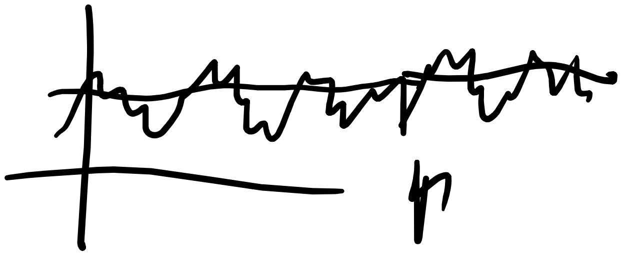
OTAN $\alpha=1$

$$\boxed{y_t = y_{t-1}+\varepsilon_t \quad \text{UNIT ROOT} \quad \text{NON-STATIONARY}}$$

$$y_t=\varepsilon_t+\varepsilon_{t-1}+\varepsilon_{t-2}+\varepsilon_{t-3}+\varepsilon_{t-4}+\dots+\varepsilon_{t-k}+\dots$$

~~IF~~ STATIONARY

$$y_t=\varepsilon_t+\alpha\varepsilon_{t-1}+\alpha^2\varepsilon_{t-2}+\alpha^3\varepsilon_{t-3}+\alpha^4\varepsilon_{t-4}+\dots+\underline{\alpha^k\varepsilon_{t-k}}+\dots$$



STATIONARY AR(1)

MEAN_REVERTING

NON STATIONA AR(1)

NON-MEAN_REVERTING

UNIT ROOT TESTING
STATIONARITY >>

$$y_t = \alpha y_{t-1} + \varepsilon_t \Rightarrow$$

$$y_t - y_{t-1} = \alpha y_{t-1} - y_{t-1} + \varepsilon_t \Rightarrow$$

$$\Delta y_t = \underbrace{(\alpha - 1)}_{\gamma} y_{t-1} + \varepsilon_t \Rightarrow \Delta y_t = \gamma y_{t-1} + \varepsilon_t$$

$$H_0 : \alpha = 1 \text{ v.s. } H_1 : \alpha < 1 \equiv H_0 : \underline{\gamma = 0} \text{ v.s. } H_1 : \gamma < 0$$

$$\text{"}z\text{"} = \cancel{\frac{\hat{\gamma} - 0}{s.e.(\hat{\gamma})}} \not\to N$$

$$\underline{\Delta y_t} = \hat{\gamma}\, y_{t-1} + \varepsilon_t \leftarrow$$

$H_0: \gamma = 0$ v.s. $H_1: \gamma < 0$

$$\text{"}z\text{"} = \frac{\hat{\gamma} - 0}{s.e.(\hat{\gamma})} \sim \text{DICKEY-FULLER}$$

Reject H_0 iff "z" < Critical Value

TON ΠΙΝΑΚΑ D-F.

$$\Delta y_t = \gamma y_{t-1} + \alpha_1 \Delta y_{t-1} + \alpha_2 \Delta y_{t-2} + \dots + \alpha_k \Delta y_{t-k} + \varepsilon_t$$

K είναι τέτοιο ώστε ε_t να είναι iid

AUGMENTED D-F.

$H_0: \gamma = 0$ (NON-STATION.) v.s. $H_1: \gamma < 0$

"z" $= \dfrac{\hat{\gamma}}{s.e.(\hat{\gamma})}$ Reject iff "z" < D.F.

$H_0: \gamma = 0$ v.s. $H_1: \gamma < 0$

A) $\Delta y_t = \underline{\underline{c}} + \gamma y_{t-1} + \alpha_1 \Delta y_{t-1} + \dots + \alpha_k \Delta y_{t-k} + \varepsilon_t$

$$\text{"Z"} = \frac{\hat{\gamma} - 0}{s.e.(\hat{\gamma})}$$ Reject H_0 iff

"Z" < APPROPRIATE D.F.

B) $\Delta y_t = c + bt + \underline{\underline{\gamma}} y_{t-1} + \alpha_1 \Delta y_{t-1} + \dots + \alpha_k \Delta y_{t-k} + \varepsilon_t$

ΑΝ ΣΤΗΝ Α) ή Β) ADF ΤΟ

C ή/ΚΑΙ ΤΟ δ είναι στατιστικά σημαντικό ΤΟΤΕ

$$Z = \frac{\hat{\gamma}-0}{s.e(\hat{\gamma})} \xrightarrow[A]{} N(0,1)$$

ΓΙΑ ΜΕΓΑΛΟ ΔΕΙΓΜΑ.

Reject H_0 iff $Z < -1.645$

$y_t = \alpha y_{t-1} + \varepsilon_t \Rightarrow \quad y_t - y_{t-1} = \underline{\underline{\Delta y_t}} = \underline{\underline{\varepsilon_t}}$

$\alpha = 1$

If x_t is stationary $\Rightarrow X_t \sim I(0)$

INTEGRATED of ORDER 0

$\alpha = 1 \Rightarrow \quad y_t \sim I(1) \Leftrightarrow \Delta y_t \sim I(0)$

$x_t \sim I(0)$ (STAT.)
$y_t \sim I(0)$ >>

$\Rightarrow \alpha x_t + b y_t \sim I(0)$ STATION.

$x_t \sim I(1)$
$y_t \sim I(0)$

$\Rightarrow \alpha x_t + b y_t \sim I(1)$

$x_t \sim I(0)$
$y_t \sim I(1)$

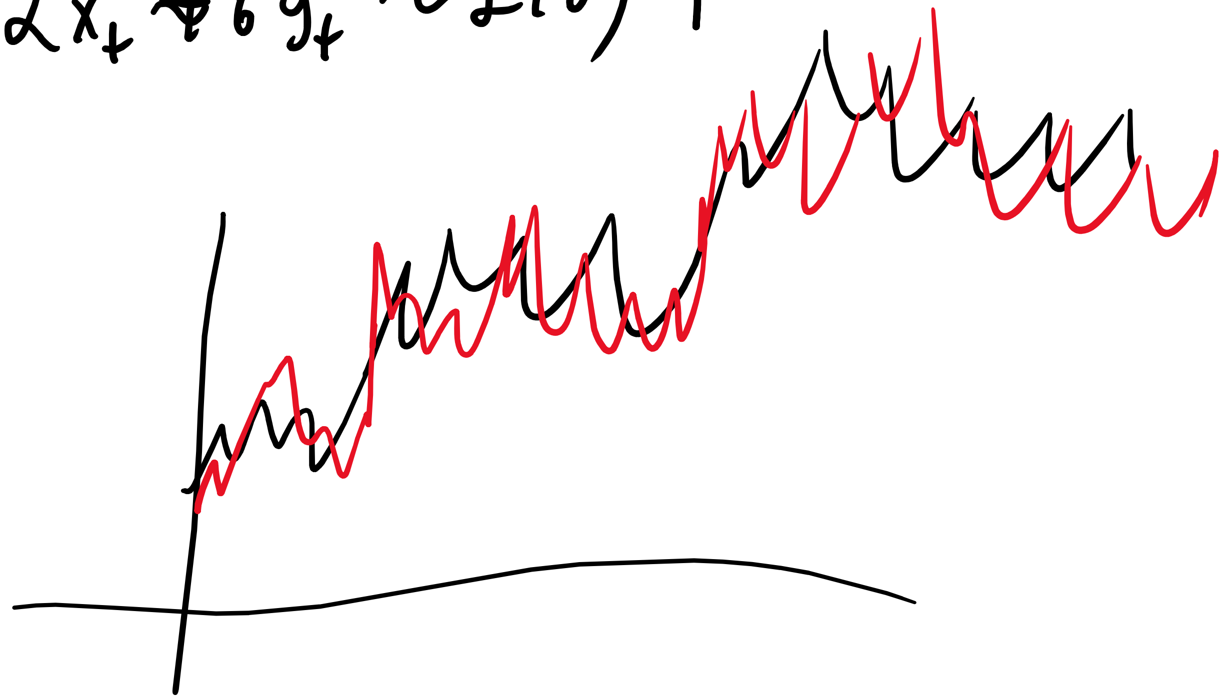
$\Rightarrow \alpha x_t + b y_t \sim I(1)$

$x_t \sim I(1)$

$y_t \sim I(1)$

$\alpha x_t + \beta y_t \sim I(0)$

$\Rightarrow$ x_t, y_t COINTEGRATED



$x_t, y_t \sim CI(1,1)$

COINTEGRATION TEST

1) TEST UNIT ROOT ΣΤΟ x_t (YES UNIT. ROOT)
   D.F. TEST

2) TEST UNIT ROOT ΣΤΟ y_t (YES UNIT ROOT)
   D.F. TEST

3) $x_t = \hat{b} y_t + \tilde{\varepsilon}_t \Rightarrow \tilde{\varepsilon}_t = x_t - \tilde{b} y_t$

TEST UNIT ROOT ΣΤΟ $\tilde{\varepsilon}_t$ (NO UNIT ROOT)

$\Rightarrow$ x_t, y_t COINTEGRATED.

$x_t = b y_t + \varepsilon_t$ ΜΑΚΡΟΧΡΟΝΙΑ ΣΧΕΣΗ
LONG RUN RELATIONSHIP.

ΑΝ $x_t, y_t \sim CI \Longleftrightarrow$

ΥΠΑΡΧΕΙ ΜΗΧΑΝΙΣΜΟΣ ΔΙΟΡΘΩΣΗΣ

ERROR CORRECTION MECHANISM.

$$\underbrace{\Delta x_t}_{\sim I(0)} = \alpha \underbrace{\Delta y_t}_{I(0)} - q \underbrace{\varepsilon_{t-1}}_{I(0)} + \underbrace{\eta_t}_{\sim I(0)}$$

$$x_t = b y_t + \varepsilon_t \qquad x_{t-1} = b y_{t-1} + \varepsilon_{t-1}$$

$$x_t = x_{t-1} + \alpha \Delta y_t - q \varepsilon_{t-1} + \eta_t$$

$y_t \sim I(1) \Rightarrow \Delta y_t \sim I(0)$

$y_t \sim I(2) \Rightarrow \Delta^2 y_t \sim I(0)$

$\downarrow$ $\quad\quad \Delta \Delta y_t$

$\Delta y_t \sim I(1) \longrightarrow \Delta \Delta y_t \sim I(0) \Rightarrow \Delta^2 y_t \sim I(0)$

$\Delta^2 y_t = \Delta(\Delta y_t) = \Delta y_t - \Delta y_{t-1} =$

$= y_t - y_{t-1} - y_{t-1} + y_{t-2} = y_t - 2y_{t-1} + y_{t-2}$

$r_t \quad \rho_k = \rho(r_t, r_{t-k}) \approx 0$

M.X.

$\hat{\rho}(r_t, r_{t-1}) \approx -0.08$ Dow Jones

Prices of STOCKS

$$P_t \sim I(1) \Rightarrow \Delta P_t \sim I(0)$$

R_t

r_t ΔΕΙΚΤΗ ή ΧΑΡΤΟΦΥΛΑΚΙΟ

$E(r_t) =$ RISK PREMIUM

$V(r_t) =$ MEASURE OF RISK.

MERTON + MARKOWICH

$\underline{E(r_t)} = f(\text{RISK})$ $\qquad \dfrac{\partial f}{\partial \text{RISK}} > 0$

$f(\underline{0}) = 0$

MEAN

GARCH(1,1)-M

IN MEAN

$$\varepsilon_t | I_{t-1} \sim N(0, \sigma_t^2)$$

$$\boxed{\underline{r_t} = c + \delta \underline{\sigma_t}^2 + \varepsilon_t}$$

$$\boxed{\underline{\sigma_t}^2 = w + \alpha \varepsilon_{t-1}^2 + b \sigma_{t-1}^2}$$

COND. VAR.

$$E(r_t) = c + \delta E(\sigma_t^2) + 0$$

$$\boxed{E(\underline{r_t | I_{t-1}}) = c + \delta \sigma_t^2}$$

PRICE OF RISK

$$\frac{\partial E(r_t | I_{t-1})}{\partial \sigma_t^2} > 0$$

$$\delta > 0$$

$$\sigma_t^2 = 0 \Rightarrow E(r_t | I_{t-1}) = 0 \Rightarrow c = 0$$

$$r_t = \delta_1 \sigma_t^2 + \delta_2 \sigma_t + \varepsilon_t$$

$$\sigma_t^2 = \dots$$

$$r_t = \underline{\underline{\delta_3}} \ln \sigma_t^2 + \varepsilon_t$$