$$\underset{n\times1}{Y} = \underset{n\times k}{X}\,\underset{k\times1}{\beta} + \underset{n\times1}{\varepsilon} \qquad \varepsilon \sim (0, \sigma^2 \Omega)$$

ASSUME WRONGLY $\varepsilon \sim (0, \sigma^2 I_n)$

$$\hat{\beta} = (X'X)^{-1}X'Y = \beta + (X'X)^{-1}X'\varepsilon$$

$$E(\hat{\beta}) = \beta + (X'X)^{-1}X'E(\varepsilon) = \beta$$

$$V(\hat{\beta}) = E\left[(X'X)^{-1}X'\varepsilon\left[(X'X)^{-1}X'\varepsilon\right]'\right]$$

$$= (X'X)^{-1}X'\sigma^2\Omega X(X'X)^{-1} \neq \sigma^2(X'X)^{-1}$$

UNLESS $\Omega = I_n$

$$V(\hat{b}) = E\left[(\hat{b} - E(\hat{b}))(\hat{b} - E(\hat{b}))'\right]$$

$$= E\left[(\hat{b} - b)(\hat{b} - b)'\right] =$$

$$= E\left[(X'X)^{-1}X'\varepsilon\left[(X'X)^{-1}X'\varepsilon\right]'\right] =$$

$$= E\left[(X'X)^{-1}X'\varepsilon\varepsilon'X(X'X)^{-1}\right] =$$

$$= (X'X)^{-1}X'E(\varepsilon\varepsilon')X(X'X)^{-1} =$$

$$= \sigma^2(X'X)^{-1}X'\Omega X(X'X)^{-1}$$

ECONOMETRIC PACKAGES EVALUATE

$$\hat{V}(\hat{\beta}) = s^2 (X'X)^{-1} \leftarrow \text{ΛΑΘΟΣ}$$

$$s^2 = \frac{1}{n-k} \sum_{t=1}^{n} \hat{\varepsilon}_t^2$$

$$V(\hat{\beta}) = \sigma^2 (X'X)^{-1} X' \Omega X (X'X)^{-1}$$

$\Rightarrow$ standard errors of $\hat{\beta}_i$ $i = 1, 2, \ldots, K$

ARE WRONG $\Rightarrow$

$\Rightarrow$ WRONG INFERENCE.

HOMOSKEDASTICITY

$$V(\varepsilon_i) = \sigma^2 \quad \forall\, i = 1, 2, \ldots, n$$

$$\operatorname{cov}(\varepsilon_i, \varepsilon_j) = 0 \quad \forall\, i \neq j = 1, 2, \ldots, n$$

$$V(\varepsilon) = \sigma^2 I_n$$

$$\varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix}$$

$$C_t = \alpha + b\, y_t + \varepsilon_t \qquad t = 1 \to n \text{ Πολλήτος}$$

$$C_t = \alpha^* + b^* y_t + \varepsilon_t \qquad t = \text{Πολλήτος}$$

(HOW TEST (STRUCTURAL BREAK)

$$\underset{(n_1 \times 1)}{Y_1} = \underset{n_1 \times k}{X_1}\, \underset{k \times 1}{b_1} + \underset{n_1 \times 1}{\varepsilon_1} \Big\} \; n_1$$

$$\underset{n_2 \times 1}{Y_2} = \underset{n_2 \times k}{X_2}\, \underset{k \times 1}{b_2} + \underset{n_2 \times 1}{\varepsilon_2} \Big\} \; \underbrace{n_1 + 1 \; \cdots \; n}_{n_2}$$

$$Y = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} \begin{matrix} \to n_1 \\ \to n_2 \end{matrix} \qquad n_1 + n_2 = n$$

$$Y = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} b + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix}$$

$$Y = X\hat{b} + \hat{\varepsilon} \qquad n = n_1 + n_2$$

$$Y_1 = X_1\hat{b}_1 + \hat{\varepsilon}_1 \qquad n_1$$

$$Y_2 = X_2\hat{b}_2 + \hat{\varepsilon}_2 \qquad n_2$$

$$F = \frac{\left[\hat{\varepsilon}'\hat{\varepsilon} - \left(\hat{\varepsilon}_1'\hat{\varepsilon}_1 + \hat{\varepsilon}_2'\hat{\varepsilon}_2\right)\right] / k}{\dfrac{\hat{\varepsilon}_1'\hat{\varepsilon}_1 + \hat{\varepsilon}_2'\hat{\varepsilon}_2}{n-2k}} \overset{H_0}{\sim} F(k, n-2k)$$

$$H_0 : b_1 = b_2$$

RAMSEY (RESET) TEST $\quad Z = [\hat{Y}^2, \hat{Y}^3, \hat{Y}^4]$

$$\underset{n\times 1}{Y} = \underset{n\times k}{X}\,\underset{k\times 1}{b} + \underset{n\times 1}{\varepsilon} \longrightarrow \hat{Y} = X\hat{b}$$

$$\underset{n\times 1}{Y} = \underset{n\times k}{X}\,b + \underset{n\times v}{Z}\,\underset{v\times 1}{\alpha} + \underset{n\times 1}{\varepsilon^*}$$

$H_0: \alpha = 0$

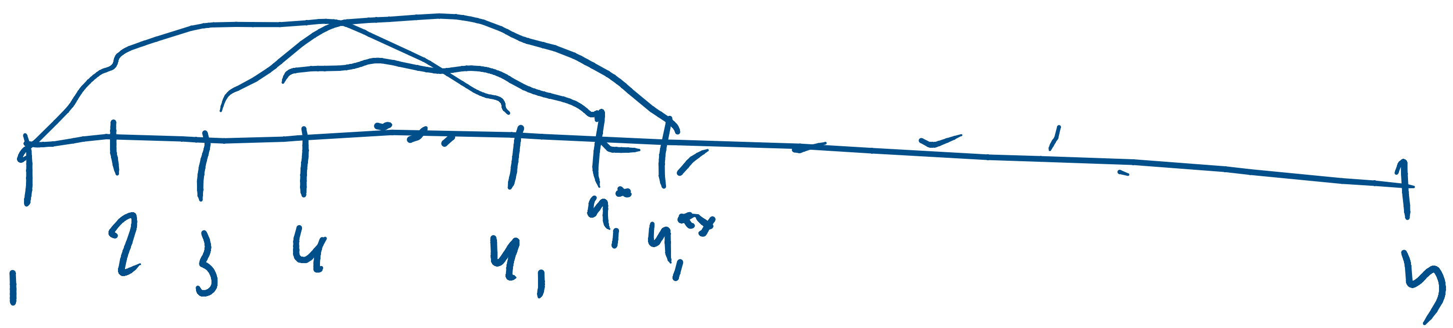
$(\alpha_1 = \alpha_2 = \dots = \alpha_v = 0)$

$$F = \frac{(\hat{\varepsilon}'\hat{\varepsilon} - \hat{\varepsilon}^{*\prime}\hat{\varepsilon}^*)/v}{\hat{\varepsilon}^{*\prime}\hat{\varepsilon}^* / (n-k-v)} \overset{H_0}{\sim} F(v, n-k-v)$$

ANDREWS-QUANDT

F-statistics

για $n_1 = n_{\alpha\rho\chi\iota}$ $\longrightarrow$ $n_1^* = n_{\tau\epsilon\rho\mu\alpha\tau\omicron}$



$$\text{MAX } F = \max_{n_1 \le \tau \le n_1^*} F(\tau) \sim F(\cdot,\cdot)$$

Από SIMULATIONS

$$r_t = \alpha + b\, r_{M_t} + \varepsilon_t$$

$$D_t = \begin{cases} 0 & \forall \text{ month except january} \\ 1 & \text{for JANUARY} \end{cases}$$

$$r_t = \alpha + \gamma D_t + b\, r_{M_t} + \varepsilon$$

$\Rightarrow$ FOR NON-JANUARY

$$r_t = \alpha + \gamma \cdot 0 + b\, r_{M_t} + \varepsilon_t = \alpha + b\, r_{M_t} + \varepsilon_t$$

FOR JANUARY

$$r_t = \alpha + \gamma \cdot 1 + b\, r_{M_t} + \varepsilon_t = \alpha + \gamma + b\, r_{M_t} + \varepsilon$$

$$r_t = \alpha + \gamma D_t + \beta r_{M_t} + \delta D_t \cdot r_{M_t} + \xi_t$$

NON-JANUARY

$$r_t = \alpha + \gamma \cdot 0 + \beta r_{M_t} + \delta \cdot 0 \cdot r_{M_t} + \xi_t$$
$$= \alpha + \beta r_{M_t} + \xi_t$$

JANUARY

$$r_t = \alpha + \gamma \cdot 1 + \beta r_{M_t} + \delta \cdot 1 \cdot r_{M_t} + \xi_t$$
$$= \alpha + \gamma + (\beta + \delta) r_{M_t} + \xi_t$$

$r_{mt} = 10\%$, $HML = 5\%$, $SMB = -5\%$

$\hat{r_1 - r_f} = ?$

1) NON-JANUARY

$\hat{r}_1 - r_f = -0.19 + 0.89 \cdot 10\% + 0.24 \cdot 5\%$
$+ 1.21 \cdot (-5\%) = \ldots$

2) JANUARY

$\hat{r}_1 - r_f = -0.19 + 3.12 + 0.89 \cdot 10\% + 0.24 \cdot 5\%$
$- 0.25 \cdot 5\% + 1.21(-5\%) = \ldots$

$Y = X\beta + \varepsilon, \quad \varepsilon_i \sim (0, \sigma_i^2) \quad \forall i = 1, 2, \dots$

$Cov(\varepsilon_i, \varepsilon_j) = 0 \quad \forall i \neq j = 1, 2, \dots, n$

$$V(\varepsilon) = \begin{pmatrix} \sigma_1^2 & & & 0 \\ & \sigma_2^2 & & \\ & & \sigma_3^2 & \\ 0 & & & \ddots \; \sigma_n^2 \end{pmatrix}$$

$\hat{\beta} = (X'X)^{-1} X'Y \rightarrow$ UNBIASED BUT INEFFICIENT.

$$V(\hat{\beta}) = (X'X)^{-1} X' \underset{\uparrow}{\underline{\Omega}} X (X'X)^{-1}$$