REGRESSION TWO VARIABLES CASE

$$y_i = \beta_1 + \beta_2 X_i + \varepsilon_i \, , \quad \varepsilon_i \overset{iid}{\sim} (0, \sigma^2)$$

$$i = 1, 2, \ldots, n$$

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \beta_1 + \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \beta_2 + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix}$$

$$Y = X \underbrace{\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}}_{\beta} + \varepsilon \, , \quad X = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix}$$

$$b = (X'X)^{-1} X'Y$$

$$X'X = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \end{pmatrix} \begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{pmatrix} =$$

$$= \begin{pmatrix} n & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 \end{pmatrix}$$

$$(X'X)^{-1} = \frac{1}{n} \frac{1}{\sum (x_i - \bar{x})^2} \begin{pmatrix} \sum x_i^2 & -\sum x_i \\ -\sum x_i & n \end{pmatrix}$$

$$X'Y = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} y_i x_i \end{pmatrix}$$

$$\sum (x_i - \bar{x})^2 = \sum x_i^2 - n\bar{x}^2$$

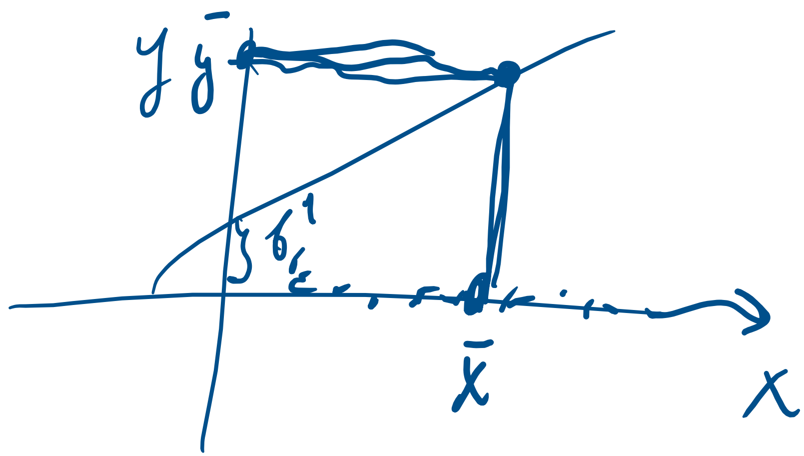
$$\sum (x_i - \bar{x})(y_i - \bar{y}) = \sum x y_i - n\bar{x}\bar{y}$$

$$\hat{b}_2 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$$\hat{b}_1 = \bar{y} - \bar{x}\,\hat{b}_2$$

$$\bar{x} = \frac{1}{n}\sum x_i$$

$$\bar{y} = \frac{1}{n}\sum y_i$$



$$X'\hat{\varepsilon} = 0 \qquad \int (0')?$$

$$X' \underbrace{M\varepsilon}_{\hat{\varepsilon}} = \underbrace{X'M}_{=\,0}\varepsilon = 0 \cdot \varepsilon = 0.$$

$$\hat{\beta} = (X'X)^{-1}X'y, \qquad \hat{\varepsilon} = y - \hat{y} = y - X\hat{\beta}$$

$$\hat{y}'\hat{\varepsilon} = (X\hat{\beta})'\hat{\varepsilon} = \hat{\beta}'X'\hat{\varepsilon} = \hat{\beta}' \cdot 0 = 0.$$

$$\hat{\varepsilon} = y - \hat{y} \Rightarrow y = \hat{y} + \hat{\varepsilon}$$

$$y'y = (\hat{y} + \hat{\varepsilon})'(\hat{y} + \hat{\varepsilon}) \Rightarrow$$

$$\Rightarrow y'y = \hat{y}'\hat{y} + \underbrace{\hat{\varepsilon}'\hat{y}}_{=0} + \underbrace{\cancel{\hat{y}'\hat{\varepsilon}}}_{=0} + \hat{\varepsilon}'\hat{\varepsilon}$$

$$\Rightarrow y'y = \hat{y}'\hat{y} + \hat{\varepsilon}'\hat{\varepsilon}$$

$\bar{y} = \bar{\hat{y}}$ διότι

$X'\hat{\varepsilon} = 0$ ΑΝ ΥΠΑΡΧΕΙ ΣΤΑΘΕΡΑ ΣΤΗΝ ΠΑΛΙΝΔΡΟΜΗΣΗ

$$X = \begin{pmatrix} 1 & & \\ 1 & 1 & 1 \\ \vdots & & \\ 1 & 1 & 1 \end{pmatrix}$$

$$X'\hat{\varepsilon} = 0 \Rightarrow \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ & & \dots & & \end{pmatrix} \begin{pmatrix} \hat{\varepsilon}_1 \\ \hat{\varepsilon}_2 \\ \vdots \\ \hat{\varepsilon}_n \end{pmatrix} = 0 \Rightarrow$$

$$\sum_{i=1}^{n} \hat{\varepsilon}_i = 0$$

$$y_i = \hat{y}_i + \hat{\varepsilon}_i \quad \forall i \Rightarrow \frac{1}{n}\sum y_i = \frac{1}{n}\sum \hat{y}_i + \frac{1}{n}\sum \hat{\varepsilon}_i$$

$$\Rightarrow \bar{y} = \bar{\hat{y}}$$

RSS = RESIDUALS SUM OF SQUARES

$$\underbrace{y'y}_{\sum_{i=1}^{n} y_i^2} = \hat{y}'\hat{y} + \hat{\varepsilon}'\hat{\varepsilon} \qquad n\bar{y}^2$$

$$\bar{\hat{y}}^2$$

$$\Rightarrow y'y - n\bar{y}^2 = \hat{y}'\hat{y} - n\bar{y}^2 + \hat{\varepsilon}'\hat{\varepsilon}$$

$$\Rightarrow \underbrace{\sum_{i=1}^{n} (y_i - \bar{y})^2}_{TSS} = \underbrace{\sum (\hat{y}_i - \bar{y})^2}_{ESS} + \underbrace{\sum \hat{\varepsilon}_i^2}_{RSS}$$

TSS = TOTAL SUM OF SQUARES (ΑΠΟΚΛΙΣΕΩΝ ΑΠΟ ΤΟΝ ΜΕΣΟ)

ESS = EXPLAINED " " " "

$$R^2 = \frac{ESS}{TSS}$$ = ΠΟΣΟΣΤΟ ΤΟΥ VARIABILITY ΤΟΥ Y ΠΟΥ ΕΞΗΓΕΙΤΑΙ ΑΠΟ ΤΟ VARIABILITY ΤΩΝ X

$$\Rightarrow R^2 = \frac{TSS - RSS}{TSS} \Rightarrow R^2 = 1 - \frac{RSS}{TSS}$$

$\bar{R}^2$ (ADJUSTED R^2) $= 1 - \dfrac{RSS/n-k}{TSS/n-1}$ ← NO OF X's VARIABLES

$H_0: \underset{v \times k}{R}\beta = q$ v.s. $H_1: R\beta \neq q$

K = NO. EXPLAN. VAR.

V = NO. RESTRICTIONS

$$F = \frac{(R\hat{\beta} - q)' \left[R (X'X)^{-1} R' \right]^{-1} (R\hat{\beta} - q) / v}{\hat{\varepsilon}' \hat{\varepsilon} \,/\, (n-k)} \overset{H_0}{\sim} F(v, n-k)$$

AN $\varepsilon \sim N(0, \sigma^2 I_n)$

$$Y = X\beta + \varepsilon \Rightarrow \hat{\beta} = (X'X)^{-1}X'Y \Rightarrow \hat{\varepsilon} = Y - X\hat{\beta}$$

ΕΣΤΩ $\tilde{\beta}$ Η ΕΚΤΙΜΗΤΡΙΑ ΤΟΥ β

ΑΦΟΥ ΕΧΩ ΕΝΣΩΜΑΤΩΣΕΙ ΤΟΥΣ

ΠΕΡΙΟΡΙΣΜΟΥΣ $\Rightarrow \tilde{\varepsilon} = Y - X\tilde{\beta}$

$$F = \frac{(\tilde{\varepsilon}'\tilde{\varepsilon} - \hat{\varepsilon}'\hat{\varepsilon})/v}{\hat{\varepsilon}'\hat{\varepsilon}/(n-k)} \overset{H_0}{\sim} F(v, n-k)$$

$$\frac{(RRSS - URSS)/v}{URSS/n-k} \overset{H_0}{\sim} F(v, n-k)$$

$RRSS = \tilde{\varepsilon}'\tilde{\varepsilon}$ = RESTRICTED RESID. SUM. SQUA.

$URSS = \hat{\varepsilon}'\hat{\varepsilon}$ = UNRESTRICTED " " "

$$y_i = \beta_1 + \beta_2 x_{i2} + \beta_3 x_{i3} + \cdots + \beta_k x_{ik} + \varepsilon_i$$

$i = 1, 2, \ldots, n$

$H_0: \beta_2 = \beta_3 = \cdots = \beta_k = 0$ V.S. H_1: AT LEAST ONE $\neq 0$

RESTRICTED MODEL

$$y_i = \beta_1 + \varepsilon_i \Rightarrow \hat{\beta}_1 = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \beta_1 + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix}$$

$$Y = X\beta_1 + \varepsilon$$

$$\hat{\beta}_1 = (X'X)^{-1} X'Y = \left((1\ 1\ 1 \ldots 1) \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \right)^{-1} (1\ 1 \ldots 1) \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

$$= n^{-1} \sum_{i=1}^{n} y_i = \frac{1}{n} \sum y_i = \bar{y}$$

$$F = \frac{\left(\tilde{\varepsilon}'\tilde{\varepsilon} - \hat{\varepsilon}'\hat{\varepsilon}\right)/k-1}{\dfrac{\hat{\varepsilon}'\hat{\varepsilon}}{n-k}} \sim F(k-1,\ n-k)$$

ΤΥΠΩΝΕΤΑΙ ΑΠΟ ΟΛΑ ΤΑ ΟΙΚΟΝΟΜ. ΠΑΚΕΤΑ

ΕΙΝΑΙ F STATISTIC ΓΙΑ ΤΗΝ ΥΠΟΘΕΣΗ ΟΤΙ ΟΛΕΣ ΟΙ ΜΕΤΑΒΛΗΤΕΣ ΠΛΗΝ ΤΗΣ ΣΤΑΘΕΡΑΣ ΕΙΝΑΙ ΜΗΔΕΝ. Δηλαδή ΕΛΕΓΧΩ ΤΗΝ ΣΤΑΤΙΣΤΙΚΗ ΣΥΜΑΝΤΙΚΟΤΗΤΑ ΟΛΩΝ ΤΩΝ ΜΕΤΑΒΛΗΤΩΝ (πλην ΣΤΑΘΕΡΑΣ) ΑΠΟ ΚΟΙΝΟΥ.

$H_0: b_1 = b_5 = 0$ vs. H_1: At least one $\neq 0$

$$F = \frac{(RRSS - URSS)/2}{\frac{URSS}{n-k}} =$$

$$= \frac{(1655.775 - 1655.006)/2}{\frac{1655.006}{528-5}} \overset{H_0}{\sim} F(2, 523)$$

Reject H_0 iff $F > F_{5\%}(2, 523)$

INFO. CRITERIA

n = NO. OF OBS.
k = NO. OF EXPLAN. VAR.

AKAIKE INFO. = $AIC = \ln \dfrac{\hat{\varepsilon}'\hat{\varepsilon}}{n} + \dfrac{2k}{n}$

SCHWARZ INFO. = $SC = \ln \dfrac{\hat{\varepsilon}'\hat{\varepsilon}}{n} + \dfrac{k}{n} \ln n$

Hannan-Quinn = $HQ = \ln \dfrac{\hat{\varepsilon}'\hat{\varepsilon}}{n} + \dfrac{k}{n} \ln(\ln n)$

ΣΥΓΚΡΙΝΟΝΤΑΣ ΜΟΝΤΕΛΑ (ΠΑΛΙΝΔΡΟΜΗΣΕΙΣ) ΔΙΑΛΕΓΟΥΜΕ ΤΟ ΜΟΝΤΕΛΟ ΜΕ ΤΑ ΜΙΚΡΟΤΕΡΑ INFO. CRIT.

ΠΡΟΣΟΧΗ: ΤΑ INFO CRITERIA ΧΡΗΣΙΜΟΠΟΙΟΥΝΤΑΙ ΑΝ ΤΑ ΜΟΝΤΕΛΑ ΕΙΝΑΙ NON-NESTED

(A) $y_i = \beta_1 + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \varepsilon_i$

(B) $y_i = \delta_1 + \delta_2 x_{i2} + \delta_3 x_{i3} + n_i$

ΑΝ ΣΤΟ (A) ΘΕΣΩ $\beta_4 = 0 \Longrightarrow$ (B)

ΑΡΑ (B) IS NESTED INTO (A)
(ΕΜΦΩΛΕΥΕΙ)

ΓΕΝΙΚΩΣ

ΕΝΑ ΜΟΝΤΕΛΟ (A) IS NESTED IN ΤΟ (B) ΑΝΝ ΜΕ ΜΗΔΕΝΙΚΟΥΣ ΠΕΡΙΟΡΙΣΜΟΥΣ ΤΩΝ ΣΥΝΤΕΛΕΣΤΩΝ ΤΟΥ (B) ΕΧΩ ΤΟ (A)

ΣΤΑ NESTED MODELS ΚΑΝΩ ΧΡΗΣΗ ΤΟΥ F (ΑΝ ΕΧΩ 2 Η ΠΕΡΙΣΣΟΤΕΡΟΥΣ ΠΕΡΙΟΡΙΣΜΟΥΣ) Ή ΤΟΥ t (ΑΝ ΕΧΩ 1 ΠΕΡΙΟΡΙΣΜΟ).

ΑΝ ΤΑ MODELS ARE NON-NESTED
ΧΡΗΣΗ ΤΩΝ INFO. CRIT.

ΔΗΛΑΔΗ ΔΙΑΛΕΓΩ ΤΟ ΜΟΝΤΕΛΟ
ΜΕ ΤΑ ΜΙΚΡΟΤΕΡΑ INFO. CRITER.

(A) $y_i = \beta_1 + \beta_2 x_{i2} + \beta_3 x_{i3} + \varepsilon_i$

(B) $y_i = \delta_1 + \delta_2 x_{i2} + \delta_3 z_i + u_i$

(A), (B) NON-NESTED. (ΤΟΤΕ INFO. CRITER.)

$$\underset{n\times 1}{Y} = \underset{n\times k}{X}\,\underset{k\times 1}{b} + \underset{n\times 1}{\varepsilon} \longrightarrow \hat{b} = (X'X)^{-1}X'y$$

$\hat{b}$ MINIMIZES $\varepsilon'\varepsilon = \min_{b} (y - Xb)'(y - Xb)$

$\hat{b}_{OLS}$

$x_i \overset{iid}{\sim} N(\mu, \sigma^2) \quad i = 1, 2, \ldots, n$

(JOINT DENSITY)

$$f(x_1, x_2, \ldots, x_n) \overset{\text{INDEP.}}{=} f(x_1)\, f(x_2) \cdots f(x_n)$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-\mu)^2}{2\sigma^2}} \cdots \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_n-\mu)^2}{2\sigma^2}}$$

$$= f(x_1, x_2, \ldots, x_n; \underbrace{\mu, \sigma^2})$$

$$= L(\mu, \sigma^2 \mid x_1, x_2, \ldots, x_n)$$

$\uparrow$

LIKELIHOOD.

JOINT DENSITY $\overset{\text{IND.}}{=}$ PRODUCT MARGINAL

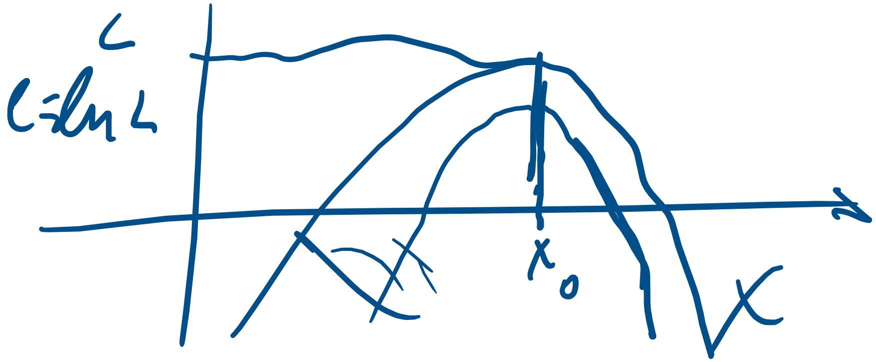
$$f(x_1, x_2, \ldots, x_n) \overset{\text{IND}}{=} f(x_1) f(x_2) \cdots f(x_n)$$

ΕΚΤΙΜΗΤΡΙΕΣ ΜΕΓΙΣΤΗΣ ΠΙΘΑΝΟΦΑΝΕΙΑΣ

= ΟΙ ΕΚΤΙΜΗΤΡΙΕΣ ΜΕΓΙΣΤΟΠΟΙΟΥΝ ΤΗΝ ΑΠΟ-ΚΟΙΝΟΥ ΣΥΝΑΡΤΗΣΗ ΚΑΤΑΝΟΜΗΣ ΤΟΥ ΔΕΙΓΜΑΤΟΣ ΔΕΔΟΜΕΝΩΝ ΤΩΝ ΠΑΡΑΤΗΡΗΣΕΩΝ.

$$L(\mu,\sigma^2 | \dots) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2}$$

$$\ell(\mu,\sigma^2|x_1,\dots x_n) = \ln L(\mu,\sigma^2|x_1,x_2,\dots x_n)$$



$$\ell(\vartheta) = \ln L(\vartheta)$$

$$\underset{0}{\underset{\shortparallel}{\frac{\partial \ell}{\partial \vartheta}}} = \frac{1}{L(\vartheta)} \underset{0}{\underset{\shortparallel}{\frac{\partial L}{\partial \vartheta}}}$$

$$\max_{\mu,\sigma^2} \ell(\mu,\sigma^2 \mid x_1, x_2, \dots, x_n)$$

$$\ell(\mu,\sigma^2) = -\frac{n}{2}\ln 2\pi - \frac{n}{2}\ln\sigma^2 - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$\frac{\partial\ell}{\partial\mu} = 0 \Rightarrow -\frac{1}{2\sigma^2}\sum_{i=1}^{n} -2(x_i-\hat{\mu}) = 0 \Rightarrow$$

$$\sum(x_i-\hat{\mu}) = 0 \Rightarrow \sum x_i - \sum\hat{\mu} = 0 \Rightarrow$$

$$\Rightarrow \sum x_i = n\hat{\mu} \Rightarrow \hat{\mu} = \frac{1}{n}\sum x_i = \bar{x}$$

$$\frac{\partial l}{\partial \sigma^2} = 0 \Rightarrow -\frac{n}{2}\frac{1}{\hat{\sigma}^2} + \frac{1}{2\hat{\sigma}^4}\sum (x_i - \hat{\mu})^2 = 0$$

$$\Rightarrow \hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{\mu})^2 =$$

$$= \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 \neq S^2 = \frac{1}{n-1}\sum (x_i - \bar{x})^2$$

$$y_i = \beta_1 + \beta_2 x_{i2} + \beta_3 x_{i3} + \dots + \beta_k x_{ik} + \varepsilon_i$$

$$\varepsilon_i \overset{iid}{\sim} N(0, \sigma^2)$$

$$E(y_i) = \beta_1 + \beta_2 x_{i2} + \dots + \beta_k x_{ik} + \underbrace{E(\varepsilon_i)}_{0}$$

$$= \beta_1 + \beta_2 x_{i2} + \dots + \beta_k x_{ik}$$

$$V(y_i) = E\left[(y_i - E(y_i))^2\right] = E(\varepsilon_i^2) = \sigma^2$$

$$y_i = f(\varepsilon_i) \sim N$$

$$y_i \overset{NIND}{\sim} N(\beta_1 + \beta_2 x_{i2} + \dots + \beta_k x_{ik}, \sigma^2)$$

$$f(y_i) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y_i - \beta_1 - \beta_2 x_{i2} - \dots - \beta_k x_{ik})^2}{2\sigma^2}}$$

$$f(y_1, \dots, y_n) \overset{IND.}{=} f(y_1) \cdots f(y_n)$$

$$\ell(\beta_1, \beta_2, \dots \beta_k, \sigma^2 / y_1, y_2, \dots, y_n) =$$

$$= -\frac{n}{2} \ln 2\pi - \frac{n}{2} \ln \sigma^2 - \frac{1}{2\sigma^2} \sum_{i=1}^{n} (y_i - \beta_1 - \beta_2 x_{i2} - \dots - \beta_k x_{ik})^2$$

$$= \underbrace{-\frac{n}{2}\ln 2\pi}\ \underbrace{-\frac{n}{2}\ln\sigma^2} - \frac{1}{2\sigma^2}(Y-X\beta)'(Y-X\beta)$$

$$\max_{\beta} \ell = \max_{\beta} \qquad > 0 \qquad =$$

$$= \min_{\beta} (Y-X\beta)'(Y-X\beta) \longrightarrow$$

$$\Rightarrow \hat{\beta}_{OLS} = (X'X)^{-1}X'Y \equiv \hat{\beta}_{MLE}$$

$$\frac{\partial \ell}{\partial \sigma^2} = 0 \Rightarrow -\frac{n}{2}\frac{1}{\hat{\sigma}^2} + \frac{1}{2\hat{\sigma}^4}(Y - X\vec{\beta})'(Y - X\hat{\beta}) = 0$$

$$\Rightarrow \hat{\sigma}^2_{MLS} = \frac{1}{n} \quad (Y - X\hat{\beta})'(Y - X\hat{\beta})$$

$$= \frac{1}{n} \hat{\varepsilon}' \hat{\varepsilon} = \frac{1}{n} \sum_{i=1}^{n} \hat{\varepsilon}_i^2$$

$$S^2_{OLS} = \frac{1}{n-k} \sum_{i=1}^{n} \hat{\varepsilon}_i^2$$

$$\hat{\beta}_{MLE} = \hat{\beta}_{OLS} = (X'X)^{-1}X'y$$

$$\hat{\sigma}^2_{MLE} = \frac{1}{n}\hat{\varepsilon}'\hat{\varepsilon} \qquad \hat{\varepsilon} = y - X\hat{\beta}$$

$$s^2_{OLS} = \frac{1}{n-k}\hat{\varepsilon}'\hat{\varepsilon}$$

$$Y = X\beta + \varepsilon \qquad \varepsilon \sim N(0, \sigma^2 \Omega)$$

Ω ΘΕΤΙΚΑ ΟΡΙΣΜΕΝΗ ΣΥΜΜΕΤΡΙΚΗ ΜΗΤΡΑ

$$V(\varepsilon_i) = \sigma^2 w_{ii}, \quad Cov(\varepsilon_i, \varepsilon_j) = \sigma^2 w_{ij}$$

$V(\underline{\underline{\varepsilon_i}}) = \underline{\underline{\sigma^2}} \ \forall i$ ΔΕΝ ΙΣΧΥΕΙ

$Cov(\varepsilon_i, \varepsilon_j) = 0 \ \forall i \neq j$ ΔΕΝ ΙΣΧΥΕΙ

$$E(Y) = X\beta + \underset{0}{E(\varepsilon)} = X\beta$$

$$V(Y) = E\left[(Y - E(Y))(Y - E(Y))' \right] =$$

$$= E(\varepsilon \varepsilon') = \sigma^2 \Omega$$

$$\boxed{Y \sim N(X\beta, \sigma^2 \Omega)}$$

$$\ell(\beta, \sigma^2, \theta \mid Y) = -\frac{n}{2} \ln 2\pi - \frac{n}{2} \ln \sigma^2$$

$$- \frac{1}{2} \ln |\Omega| - \frac{1}{2\sigma^2} (Y - X\beta)' \Omega^{-1} (Y - X\beta) \leftarrow$$

$$\frac{\partial \ell}{\partial \beta} = 0 \Rightarrow \frac{1}{\sigma^2}\left(X'\Omega^{-1}y - X'\Omega^{-1}X\hat{\beta}\right) = 0$$

$$\Rightarrow \hat{\beta}_{MLE} = \left(X'\Omega^{-1}X\right)^{-1} X'\Omega^{-1} y$$

$$\frac{\partial \ell}{\partial \sigma^2} = 0 \Rightarrow \hat{\sigma}^2_{MLE} = \frac{1}{n}\left(y - X\hat{\beta}\right)'\left(y - X\hat{\beta}\right)$$

$\underline{\Omega}$ ΘΕΤΙΚΑ ΟΡΙΣΜΕΝΗ ΜΗΤΡΑ $\Rightarrow$

$\underline{\Omega}^{-1}$ » » » ? $\Rightarrow$

$\exists\, P : |P| \neq 0 : \underline{\Omega}^{-1} = P'P$

$$\hat{\beta}_{MLE} = (X'\underline{\Omega}^{-1}X)^{-1}X'\underline{\Omega}^{-1}y =$$

$$= (X'P'PX)^{-1}X'P'Py =$$

$$= [(PX)'PX]^{-1}(PX)'Py =$$

$= \hat{b}_{OLS}$ THE ΠΑΛΙΝΔΡΟΜΗΣΗΣ

PY ON PX

$PY = PX b + P\varepsilon$

$$\hat{b}_{OLS} = \left((PX)'PX\right)^{-1} \cdot (PX)'PY$$

$Y^* = X^* b + \varepsilon^*$

$\hat{b}_{OLS} = (X^{*\prime} X^*)^{-1} (X^*)' Y^* \rightarrow$ BLUE

iff $V(\varepsilon^*) = \sigma^2 I_n$

$$V(\varepsilon^*) = V(P\varepsilon) = P\,V(\varepsilon)\,P' =$$

$$= P\,\sigma^2 \Omega\, P' = \sigma^2 P (P'P)^{-1} P' =$$

$$= \sigma^2 \underbrace{P P^{-1}}_{I} \underbrace{(P')^{-1} P'}_{I} = \sigma^2 I_n .$$

$$V(\varepsilon) = \sigma^2 \Omega \neq \sigma^2 I_n$$

$$\hat{\beta}_{MLE} = (X'\Omega^{-1}X)^{-1} X'\Omega^{-1} Y = \hat{\beta}_{GLS}$$

GLS = GENERALISED LEAST SQUARES