Χρηματοοικονομική Οικονομετρία.
Παρακαλώ απαντήστε σε όλες τις ερωτήσεις. Χρόνος 2 ώρες

Consider the following AR(1) process:

$$r_t = \alpha r_{t-1} + z_t$$

where z_t is an iid process with 0 mean and unit variance

1. **Assuming that the process is second order stationary, derive the autocorrelation function and the partial autocorrelation function.**

Notes or Book section for AR(1).

2. **How do you explain that the second order autocorrelation is nonzero and the second order partial autocorrelation is zero?**

This means that the second order autocorrelation is due to the fact that observations which are two periods apart are related due to their connection with observations that are 1 period apart, i.e. r_t is correlated with r_{t-2} due to the fact that r_t is correlated with r_{t-1}, and r_{t-1} is correlated to r_{t-2} and consequently r_t is correlated with r_{t-2}. In other words the correlation between r_t and r_{t-2} is coming through their connection with r_{t-1} and there is no direct connection between r_t and r_{t-2} (second order partial correlation zero).

Consider the FTSE 100 weekly excess returns, a sample size of 1500 observations. Table 1 presents the correlogram and the Q- tests (up to ten lags) for the excess returns and Table 2 the correlogram and the Q- tests for their squares.

Table 1 (Excess Returns - Correlogram)

	AC	PAC	Q-Stat	Prob
1	-0.017	-0.017	0.4161	0.519
2	0.062	0.062	6.2793	0.053
3	0.002	0.004	6.2838	0.099
4	-0.013	-0.017	6.5413	0.162
5	-0.005	-0.006	6.5814	0.254
6	-0.034	-0.033	8.3424	0.214
7	-0.054	-0.054	12.717	0.079
8	-0.018	-0.016	13.229	0.104
9	-0.023	-0.017	14.035	0.121
10	-0.003	-0.002	14.046	0.171

Table 2 (Squared Excess Returns - Correlogram)

	AC	PAC	Q-Stat	Prob
1	0.167	0.167	42.124	0.000
2	0.189	0.166	96.047	0.000
3	0.130	0.081	121.60	0.000
4	0.087	0.030	133.10	0.000
5	0.039	-0.012	135.37	0.000
6	0.040	0.008	137.77	0.000
7	0.069	0.052	145.03	0.000

8	0.055	0.031	149.58	0.000
9	0.017	-0.017	150.01	0.000
10	0.057	0.034	154.98	0.000

3. **According to the results presented in Table 1, are the excess returns autocorrelated of up order 2 at 10% level? At 5% level?**

At 10% they are (p-value=5.3%<10%). At 5% they are not autocorrelated (p-value=5.3%>5%).

4. **According to the results presented in Table 2, are the squared excess returns autocorrelated of order 1 at 5% level**

Yes they are (p-value=0.0%<5%)

GARCH (1,1) Model

$$r_t=\gamma+\varepsilon_t$$
$$\sigma^2_t=C+\alpha\ \varepsilon^2_{t-1}+\beta\ \sigma^2_{t-1}$$

Table 3 (GARCH (1,1) Estimation for the FTSE 100)

Dependent Variable: FTSE
Method: ML – ARCH
Sample: 1 1500
Included observations: 1500
Convergence achieved after 18 iterations
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
Γ	0.121721	0.043911	2.771969	0.0056
Variance Equation				
C	0.122750	0.030675	4.001607	0.0001
(α) RESID(-1)^2	0.098968	0.013245	7.471926	0.0000
(β) GARCH(-1)	0.871957	0.015888	54.88049	0.0000

R-squared	-0.000497	Mean dependent var	0.079125
Adjusted R-squared	-0.002503	S.D. dependent var	1.912061
S.E. of regression	1.914453	Akaike info criterion	4.035540
Sum squared resid	5483.033	Schwarz criterion	4.049709
Log likelihood	-3022.655	Durbin-Watson stat	2.028016

5. **What is meant by "positivity restrictions"? Do the estimated coefficients (Table 3) comply with these restrictions?**

Positivity restrictions are the restrictions on the conditional variance parameters so that $P(\sigma^2_t>0)=1$. For the GARCH(1,1) the positivity restrictions are: $\mathbf{C>0,\ \alpha\geq 0,\ \beta\geq 0}$. Yes they comply with the positivity restrictions as all the estimated coefficients are positive and p-value/2<5%.

Table 4: Variance-Covariance Matrix of the GARCH(1,1) estimation

	γ	C	resid(-1)^2	garch(-1)
γ	0.0019280	0.0000241	-0.0000365	0.0000372
C	0.0000241	0.0009410	0.0001535	-0.0003890
resid(-1)^2	-0.0000365	0.0001535	0.0001754	-0.0001760

garch(-1)	0.0000372	--0.0003890	-0.0001760	0.0002524

6. **Is the GARCH(1,1) stationary?**

For stationarity we need that **α+β<1**. Let H_0: **α+β≥1** versus H_1: **α+β<1**. The t statistic is given by $t = \frac{a+b-1}{s.e.(a+b)} = \frac{a+b-1}{\sqrt{Var(a+b)}} = \frac{0.09897+0.87196-1}{\sqrt{Var(a)+Var(b)+2Covr(a,b)}}$

$t = \frac{0.09897+0.87196-1}{\sqrt{0.0001754+0.0002524-2*0.000176}} = -3.3389$ where a and b are the estimators of α and β.

Hence H_0 is rejected as -3.3389<-1.645 and the GARCH(1,1) is stationary

Table 5 (EGARCH (1,1) Estimation)

Dependent Variable: FTSE
Method: ML - ARCH
Sample: 1 1500
Included observations: 1500
Convergence achieved after 16 iterations
LOG(GARCH) = C(2) + C(3)*ABS(RESID(-1)/@SQRT(GARCH(-1))) +
C(4)*RESID(-1)/@SQRT(GARCH(-1)) + C(5)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.057269	0.044175	1.296407	0.1948
		Variance Equation		
C(2)	-0.075715	0.018955	-3.994403	0.0001
C(3)	0.172689	0.025583	6.750243	0.0000
C(4)	-0.092823	0.015844	-5.858708	0.0000
C(5)	0.951746	0.010960	86.84060	0.0000

R-squared	-0.000131	Mean dependent var	0.079125
Adjusted R-squared	-0.002807	S.D. dependent var	1.912061
S.E. of regression	1.914743	Akaike info criterion	4.011784
Sum squared resid	5481.028	Schwarz criterion	4.029494
Log likelihood	-3003.838	Durbin-Watson stat	2.028758

7. **Is the above model second order stationary? Are the estimated coefficients significant?**

The EGARCH(1,1) is stationary if c(5), the coefficient of $\ln(\sigma^2_{t-1})$ is less than 1. Hence let H_0: **c(5)≥1** versus H_1: **c(5)<1**. The t statistic is given by $t = \frac{C(5)-1}{s.e.(c(5))} = \frac{0.951746-1}{0.01096} = -4.40$

Hence H_0 is rejected as -4.40<-1.645 and the EGARCH(1,1) is stationary

8. **What is the relative advantage(s) of the EGARCH model above as compared to the GARCH model in table 3? What is the estimated value of the coefficient which produces this relative advantage(s).**

The main advantage of the EGARCH model is the model is able to explain the dynamic asymmetry possibly present at the data. In fact the parameter C(4) is the dynamic asymmetry parameter and if negative the model explains the leverage effect. In our case it is negative, -0.092823, and p-value/2=0.000/2=0.00<5%, explaining thus the leverage effect of the data. Minor advantages are that the EGARCH models do not require positivity constraints and stationarity requires only one parameter to be less than 1, as opposed to the GARCH model which requires, for atstionarity, the sum of the two coefficients must be less than 1.

9. Compare the GARCH(1,1) with the EGARC(1,1) models (Tables 3 and 5)
The models are non-nested. Hence, we employ the information criteria. Both, the Schwarz and Akaike criteria are smaller for the EGARCH model. Consequently the EGARCH is a better model.

Assume that you have two random variables x_t and z_t.

Table 6: Null Hypothesis: X has a unit root

Exogenous: Constant

Lag Length: 0 (Automatic based on SIC, MAXLAG=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.587089	0.8688
Test critical values:	1% level	-3.474265	
	5% level	-2.880722	
	10% level	-2.577077	

*MacKinnon (1996) one-sided p-values.

Table 7: Null Hypothesis: Z has a unit root

Exogenous: Constant

Lag Length: 1 (Automatic based on SIC, MAXLAG=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.672676	0.4431
Test critical values:	1% level	-3.474265	
	5% level	-2.880722	
	10% level	-2.577077	

*MacKinnon (1996) one-sided p-values.

10. What are Tables 6 and 7 presenting? What are the conclusions of the tests?

They present the results of Unit Root tests. In both cases we do not reject the null of a unit root.

Table 8: Dependent Variable: X

Method: Least Squares

Included observations: 150

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.678573	0.260691	-2.602976	0.0102
Z	1.176853	0.027309	43.09356	0.0000

R-squared	0.926187	Mean dependent var	-11.09386
Adjusted R-squared	0.925688	S.D. dependent var	4.389548
S.E. of regression	1.196602	Akaike info criterion	3.210093
Sum squared resid	211.9148	Schwarz criterion	3.250234
Log likelihood	-238.7569	F-statistic	1857.055
Durbin-Watson stat	1.549489	Prob(F-statistic)	0.000000

Table 9: Null Hypothesis: RESID has a unit root (RESID are the residuals from the regression of Table 8).

Exogenous: Constant

Lag Length: 0 (Automatic based on SIC, MAXLAG=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-9.918846	0.0000
Test critical values:	1% level	-3.474567	
	5% level	-2.880853	
	10% level	-2.577147	

*MacKinnon (1996) one-sided p-values.

11. Based of the results in Tables 8 and 9 are x_t and z_t cointegrated? If the answer is yes, what is the dynamic relationship between x_t and z_t?

Yes they are cointegrated, as both are integrated (have unit roots) and the residuals of regression in Table 8 are stationary (the null of unit root is rejected Table 9). Hence, the regression in Table 8 represents the long-run relationship between the variables. As now they are cointegrated the is an ECM representing the short-time relationship between the variables, i.e. **Δx_t depends possibly, on Δz_t, Δx_{t-i}, Δz_{t-j}, and, surely, on the lagged residual of the regression in Table 8 with a negative coefficient.**

You could use the following:
Critical Value of the Standard Normal leaving 10% at the right tail is 1.285, leaving 5% is 1.645, and 2.5% is 1.960.

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