$$r_t = \mu + \alpha r_{t-1} + \varepsilon_t \qquad \varepsilon_t \overset{iid}{\sim} (0, \sigma^2) \qquad t = 1, 2, \dots, n$$

AR(1)

ΣΥΝΘΗΚΗ ΣΤΑΣΙΜΟΤ. $2^{\underline{ης}}$ ΤΑΞΗΣ $\underline{iff}$ $|\alpha| < 1$

$$E(r_t) = \mu + \alpha E(r_{t-1}) + \underbrace{E(\varepsilon_t)}_{= 0} \Rightarrow$$

$$\Rightarrow E(r_t) = \mu + \alpha E(r_{t-1})$$

$$\xRightarrow{\text{STAT.}} E(r_t) = \mu + \alpha E(r_t) \Rightarrow E(r_t) = \frac{\mu}{1-\alpha}$$

$$V(r_t) = V(\mu + \alpha r_{t-1} + \varepsilon_t) \Rightarrow$$

$$V(r_t) = V(\alpha r_{t-1}) + V(\varepsilon_t) + 2\,Cov(\alpha r_{t-1}, \varepsilon_t)$$

$$\Rightarrow V(r_t) = \alpha^2 V(r_{t-1}) + \sigma^2 + 0$$

STAT $\Longrightarrow V(r_t) = \alpha^2 V(r_t) + \sigma^2 \Rightarrow$

$$V(r_t) = \frac{\sigma^2}{1-\alpha^2}$$

$$r_t = \mu + \alpha r_{t-1} + \varepsilon_t$$

$$r_{t-1} = \mu + \alpha r_{t-2} + \varepsilon_{t-1}$$

$$r_{t-2} = \mu + \alpha r_{t-3} + \varepsilon_{t-2}$$

$$\vdots$$

$$r_{t-1} = f(\varepsilon_{t-1}, \varepsilon_{t-2}, \varepsilon_{t-3}, \ldots)$$

$Cov(r_{t-1}, \varepsilon_t) = 0$ Δυτι ε_t i iid

$Cov(r_t, r_{t-k}) = \gamma_k \leftarrow$ ΕΞΑΡΤΑΤΑΙ ΜΟΝΟ ΑΠΟ ΤΟ k (ΟΧΙ ΑΠΟ ΤΟ t ΛΟΓΩ ΣΤΑΣΙΜ.)

$Cov(r_{t-v}, r_{t-v-k}) = \gamma_k$

Z, X, Y Τ.Μ. $\alpha, \beta \in \mathbb{R}$

$E(\alpha X + \beta Y) = \alpha E(X) + \beta E(Y)$

$V(\alpha X + \beta Y) = \alpha^2 V(X) + \beta^2 V(Y) + 2\alpha\beta\, Cov(X,Y)$

$Cov(\alpha X + \beta Y, Z) = \alpha\, Cov(X,Z) + \beta\, Cov(Y,Z)$

$$r_t = \mu + \alpha r_{t-1} + \varepsilon_t \Rightarrow$$

$$\operatorname{Cov}(r_t, r_{t-k}) = \operatorname{Cov}(\mu + \alpha r_{t-1} + \varepsilon_t, r_{t-k})$$

$$\Rightarrow \operatorname{Cov}(r_t, r_{t-k}) = \overbrace{\operatorname{Cov}(\mu, r_{t-k})}^{=0} + \alpha \operatorname{Cov}(r_{t-1}, r_{t-k}) + \operatorname{Cov}(\varepsilon_t, r_{t-k})$$

$$\Rightarrow \quad \gamma_k = \alpha \gamma_{k-1} + 0$$

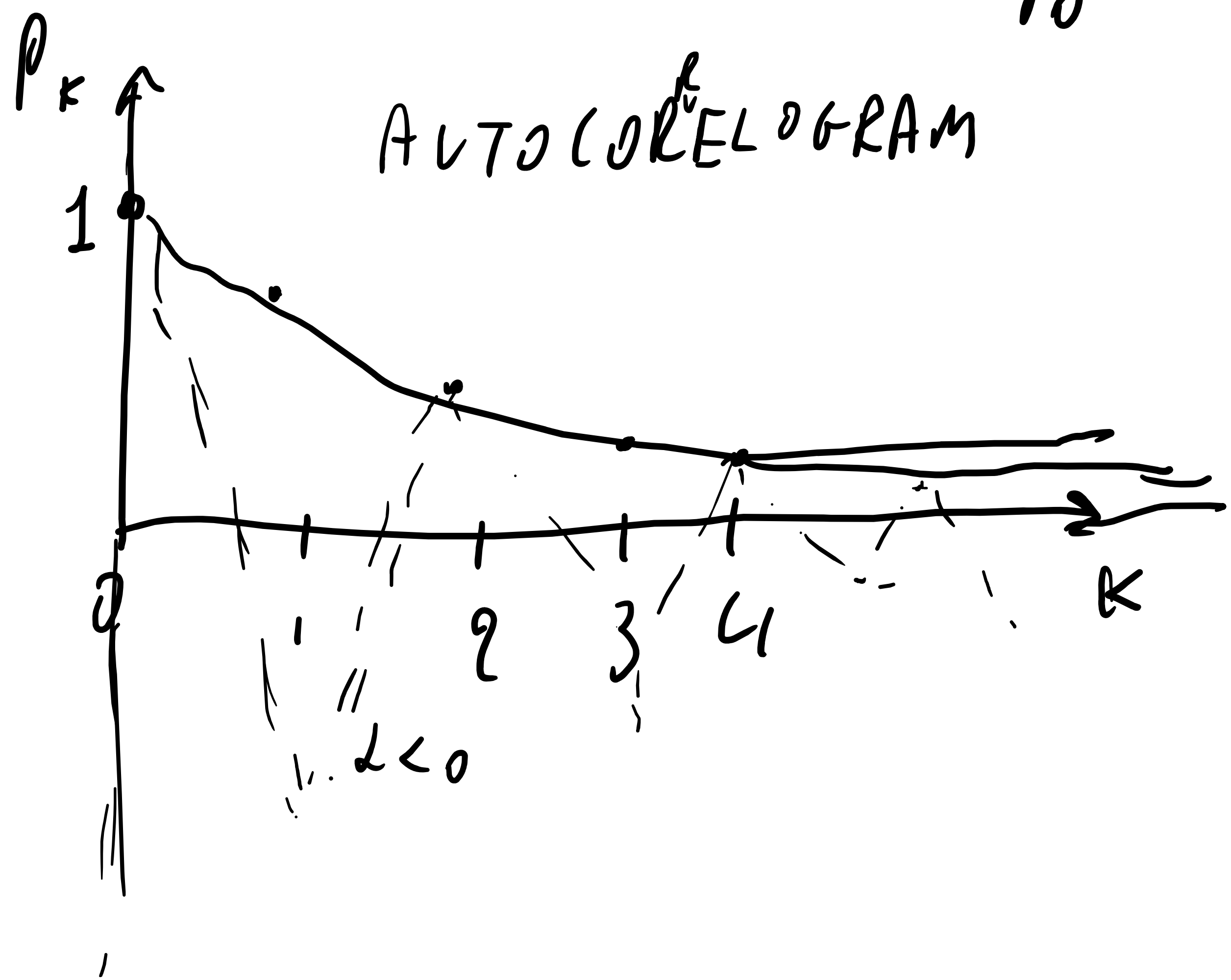
$$\Rightarrow \gamma_1 = \alpha \gamma_0 = \alpha V(r_t)$$

$$\gamma_2 = \alpha \gamma_1 = \alpha^2 V(r_t)$$

$$\vdots$$

$$\gamma_k = \alpha \gamma_{k-1} = \alpha^k V(r_t)$$

$$\text{Corr}(r_t, r_{t-k}) = \rho_k = \frac{\gamma_k}{\gamma_0} = \frac{\alpha^k \gamma_0}{\gamma_0} = \underline{\underline{\alpha^k}}$$



$\alpha > 0$

PARTIAL CORRELATION ΤΑΞΕΩΣ κ (ρ_k^*)

ΜΕΡΙΚΗ ΑΥΤΟΣΥΣΧΕΤΙΣΗ > > > .

= ΣΥΣΧΕΤΙΣΗ (ΓΡΑΜΜ.) ΠΑΡΑΤΗΡΗΣΕΩΝ ΠΟΥ ΔΙΑΦΕΡΟΥΝ ΚΑΤΑ κ ΠΕΡΙΟΔΟΥΣ ΑΦΟΥ ΛΑΒΩ ΥΠΟΨΙΝ ΜΟΥ ΟΛΕΣ ΤΙΣ ΠΡΟΗΓΟΥΜΕΝΕΣ (ΠΡΟΗΓΟΥΜΕΝΗΣ ΤΑΞΗΣ) ΑΥΤΟΣΥΣΧΕΤΙΣΕΙΣ.

$$r_t = \mu + \alpha r_{t-1} + \varepsilon_t \Rightarrow$$

$$r_t = \mu + \alpha\mu + \alpha^2 r_{t-2} + \varepsilon_t + \alpha\varepsilon_{t-1}$$

$$\vdots$$

$$r_t = \mu + \alpha\mu + \alpha^2\mu + \dots + \alpha^{k-1}\mu + \alpha^k r_{t-k} + \varepsilon_t + \alpha\varepsilon_{t-1} + \alpha^2\varepsilon_{t-2} + \dots + \alpha^k\varepsilon_{t-k}$$

$$\rho_k = \alpha^k$$

$$r_t = \mu + \alpha r_{t-1} + \delta_2 r_{t-2} + \delta_3 r_{t-3} + \dots + \rho_k^* r_{t-k} + u_t$$

ρ_k^* ΘΕΩΡΗΤΙΚΟΣ ΣΥΝΤΕΛΕΣΤΗΣ ΠΟΛΛΑΠΛΗΣ

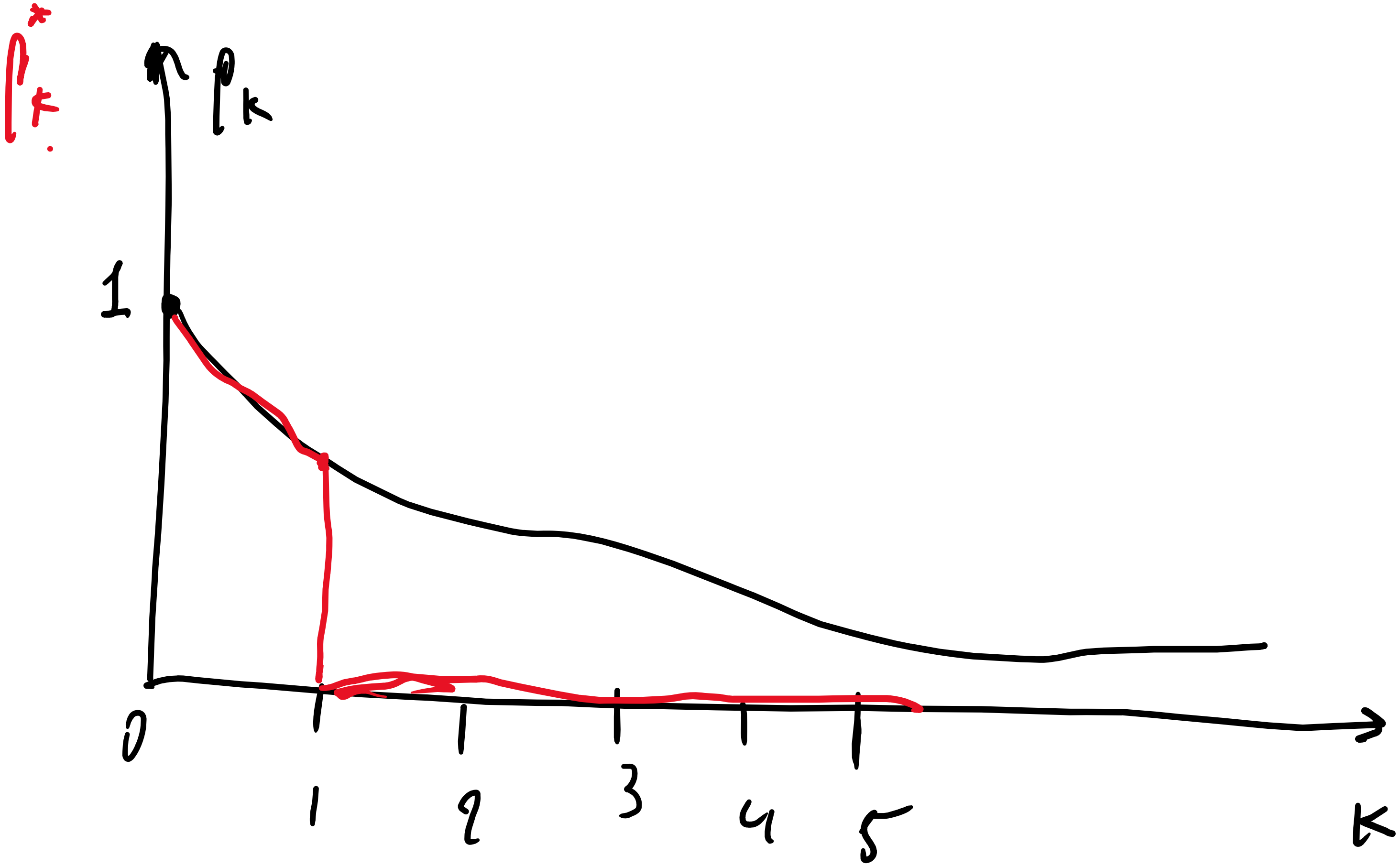
$$\rho_1^* = \rho_1 = \alpha \qquad (r_t = \mu + \alpha r_{t-1} + \varepsilon_t)$$

$$r_t = \mu + \alpha r_{t-1} + \rho_2^* r_{t-2} + u_t =$$

$$\boxed{\rho_2^* = 0}$$

$$r_t = \mu + \alpha r_{t-1} + \rho_2^* r_{t-2} + \rho_3^* r_{t-3} + u_t$$

$$\rho_3^* = 0 \quad \Rightarrow \quad \rho_k^* = \begin{cases} \rho_1 = \alpha & k = 1 \\ 0 & k \geq 2 \end{cases}$$



ΤΑ ρ_k Ι ρ_k^* ΧΑΡΑΚΤΗΡΙΖΟΥΝ ΜΟΝΑΔΙΚΑ
ΤΑ ΓΡΑΜΜΙΚΑ ΔΥΝΑΜΙΚΑ ΜΟΝΤΕΛΑ.

$$r_t = \mu + \alpha r_{t-1} + \varepsilon_t \Rightarrow r_t = \mu + \alpha\mu + \alpha^2 r_{t-2} + \varepsilon_t + \alpha\varepsilon_{t-1}$$

$$\cdots\cdots \Rightarrow r_t = \mu + \alpha\mu + \alpha^2\mu + \alpha^3\mu + \dots + \varepsilon_t + \alpha\varepsilon_{t-1} + \alpha^2\varepsilon_{t-2} + \alpha^3\varepsilon_{t-3} + \dots$$

$$\Rightarrow r_t = \mu(1+\alpha+\alpha^2+\alpha^3+\dots\dots) + \varepsilon_t + \alpha\varepsilon_{t-1} + \alpha^2\varepsilon_{t-2} + \dots$$

$1+\alpha+\alpha^2+\alpha^3+\dots$ ΣΥΓΚΛΙΝΕΙ IFF ΦΘΙΝΟΥΣΑ

IFF $|\alpha|<1 \Rightarrow 1+\alpha+\alpha^2+\alpha^3+\dots = \frac{1}{1-\alpha}$

ΑΝ $\mu=0$

ΤΟΤΕ $V(r_t) = V(\varepsilon_t) + \alpha^2 V(\varepsilon_{t-1}) + \alpha^4 V(\varepsilon_{t-2}) + \dots$
$+ \sum\sum \mathrm{cov}(\underbrace{\varepsilon_t}_{i,j}, \varepsilon_{t-i})$

$$\Rightarrow V(r_t) = \sigma^2 + \alpha^2\sigma^2 + \alpha^4\sigma^2 + \alpha^6\sigma^2 + \dots$$

$$= \sigma^2\left(1 + \alpha^2 + \alpha^4 + \alpha^6 + \dots\right)$$

ΥΠΑΡΧΕΙ IFF $\alpha^2 < 1 \Leftrightarrow |\alpha| < 1$

$$\Rightarrow V(r_t) = \frac{\sigma^2}{1-\alpha^2}$$

ΑΡΑ $|\alpha| < 1$ ΣΥΝΘΗΚΗ ΣΤΑΣΙΜΟΤΗΤΑΣ 2ης ΤΑΞΗΣ.

$$r_t = \mu + \alpha_1 r_{t-1} + \alpha_2 r_{t-2} + \dots + \alpha_v r_{t-v} + \varepsilon_t$$

$AR(v)$

L lag OPERATOR. $\Leftrightarrow$ $r_t L^k = r_{t-k}$

$$r_{t+5} L^3 = r_{t+5-3} = r_{t+2}, \quad r_{t-3} L^2 = r_{t-3-2} = r_{t-5}$$

$$(L^v)^k = L^{v \cdot k}, \quad L^v \cdot L^k = L^{v+k}$$

$$r_t = \mu + \alpha_1 r_t L + \alpha_2 r_t L^2 + \dots + \alpha_v r_t L^v + \varepsilon_t$$

$$\Rightarrow (1 - \alpha_1 L - \alpha_2 L^2 - \alpha_3 L^3 - \dots - \alpha_v L^v) r_t = \mu + \varepsilon_t$$

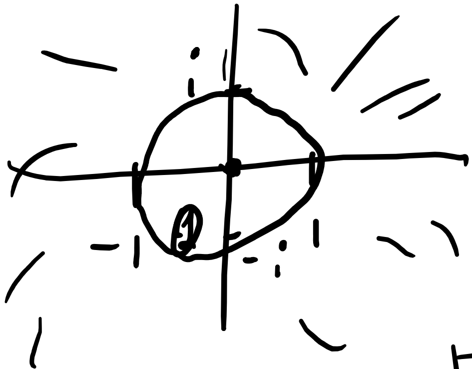
$$\Rightarrow r_t = \frac{\mu}{1-\alpha_1-\alpha_2-\cdots-\alpha_v} + \frac{1}{1-\alpha_1 L-\alpha_2 L^2-\cdots-\alpha_v L^v}\,\varepsilon_t$$

$$1-\alpha_1 L-\alpha_2 L-\alpha_3 L^3-\cdots-\alpha_v L^v = 0$$

$\hookrightarrow x_1, x_2, \ldots, x_v$ ριζες

$$(1-x_1 L)(1-x_2 L)(1-x_3 L)\cdots(1-x_v L)$$

$$|x_i| > 1$$



AR(1) $\quad 1-\alpha_1 L = 0 \Rightarrow L = \left|\frac{1}{\alpha}\right| > 1 \Rightarrow \boxed{|\alpha|<1}$