PROBLEM 1

For all allowed values of the parameters, find all global maxima of the following maximization problem, or show that none exist.

Objective function $\alpha x - y$

Constraints

$y \le 1, x \le \omega y, x \ge 0, y \ge 0$

variables x, y

parameters α, ω

conditions on parameters $\alpha > 0, \omega > 0$

$$\boxed{\begin{array}{l}\underline{\text{answer}}\\ [x,y] = \begin{cases} [0,0] & \text{if} \quad \alpha\omega < 1 \\ [\omega y, y], 0 \le y \le 1 & \text{if} \quad \alpha\omega = 1 \\ [\omega, 1] & \text{if} \quad \alpha\omega > 1 \end{cases}\end{array}} \quad (1)$$

PROBLEM 2

For all allowed values of the parameters, find all global maxima of the following maximization problem, or show that none exist.

Objective function $x - \sqrt{y}$

Constraints

$y \le 1, x \le \omega y, x \ge 0, y \ge 0$

variables x, y

parameters ω

conditions on parameters $\omega > 0$

answer

$$[x,y]=\begin{cases}[0,0] & \text{if} \quad \omega<1\\ \{[0,0],[1,1]\} & \text{if} \quad \omega=1\\ [\omega,1] & \text{if} \quad \omega>1\end{cases} \tag{2}$$

PROBLEM 3

For all allowed values of the parameters, find all global maxima of the following maximization problem, or show that none exist.

Objective function Q

Constraints

$Q \le K+L-2$
$Q \le K+2L-3$
$Q \ge 0$

variables Q

parameters K,L

conditions on parameters $K \ge 0, L \ge 0$

The solution of this maximization problem is a function $Q=F(K,L)$ .Draw the better-than sets of this function. Is the function F quasi-concave?

answer

$$F(K,L)=\begin{cases}K+L-2 & \text{if} \quad L\ge 1 \text{ and } K+L\ge 2 \text{ and } K+2L\ge 3\\ K+2L-3 & \text{if} \quad L\le 1 \text{ and } K+L\ge 2 \text{ and } K+2L\ge 3\\ \text{undefined(no maximum exists)} & \text{if} \quad K+L<2 \text{ or } K+2L<3\end{cases} \tag{3}$$

The domain of F is the set $D=\{[K,L]\in\mathbb{R}^2 : K\ge 0, L\ge 0, K+L\ge 2, K+2L\ge 3\}$; D is a convex set(intersection of four half-spaces).The function $D \xrightarrow{F} \mathbb{R}$ is concave because $F(K,L)=\min\{K+L-2, K+2L-3\}$ is the lower envelope of two concave functions defined on the convex set D .

PROBLEM 4

For all allowed values of the parameters, find all global maxima of the following maximization problem, or show that none exist.

Objective function $x_1 + 2\theta\sqrt{x_2 x_3 \ldots x_L}$

Constraints $p_1 x_1 + p_2 x_2 + \ldots + p_L x_L \le m, x_1 \ge 0, x_2 \ge 0, \ldots, x_L \ge 0$

variables $x_1, x_2, \ldots, x_L \ge 0$

parameters $p_1, p_2, \ldots, p_L, m, L, \theta$

conditions on parameters
$p_1 > 0, p_2 > 0, \ldots, p_L > 0, m > 0, \theta > 0, L \text{ integer}, L \ge 1$

PROBLEM 5

For all allowed values of the parameters, find all global maxima of the following maximization problem, or show that none exist.

Objective function $pA - wX$

Constraints $X \ge F + \frac{c}{2} A^2, A \ge 0, X \ge 0$

variables A, X

parameters F, p, w, c

conditions on parameters $F > 0, p > 0, w > 0, c > 0$

answer

$$A = \frac{p}{cw}, X = F + \frac{c}{2}\left(\frac{p}{cw}\right)^2 \quad (4)$$

PROBLEM 6

For all allowed values of the parameters, find all global maxima of the following maximization problem, or show that none exist.

Objective function $\log x_1 + \eta \log(x_2 + A)$

constraints $p_1 x_1 + p_2 x_2 \le m, x_1 \ge 0, x_2 \ge 0$

variables x_1, x_2

parameters η, A, p_1, p_2, m

conditions on parameters $\eta > 0, A \ge 0, p_1 > 0, p_2 > 0, m > 0$

answer

$$[x_1, x_2] = \begin{cases} [\frac{m}{p_1}, 0] & \text{if} \quad \eta m - A p_2 \le 0 \\ [\frac{A p_2 + m}{(\eta + 1) p_1}, \frac{\eta m - A p_2}{p_2 (\eta + 1)}] & \text{if} \quad \eta m - A p_2 > 0 \end{cases} \tag{5}$$

PROBLEM 7

For all allowed values of the parameters, find all global maxima of the following maximization problem, or show that none exist.

Objective function $\theta_0 \log X_0 + \sum_{i=1}^{N} \theta_i \log A_i + \sum_{i=1}^{N} \theta_i \log X_i$

Constraints

$$A_1 + ... + A_N \le 2\sqrt{\hat{X}}$$
$$X_0 + X_1 + ... + X_N + \hat{X} \le N$$
$$X_0 \ge 0, X_1 \ge 0, ..., X_N \ge 0, A_1 \ge 0, ..., A_N \ge 0, \hat{X} \ge 0$$

variables $X_0, X_1, ..., X_N, A_1, ..., A_N, \hat{X}$

parameters $N, \theta_0, ..., \theta_N$

conditions on parameters

$$\theta_0 > 0, ..., \theta_N > 0, \sum_{i=0}^{N} \theta_i = 1, N \text{ integer}, N > 4$$

answer

$$\psi \triangleq \sum_{i=1}^{N} \theta_i$$

$$\hat{X} = \frac{\psi N}{2\theta_0 + 3\psi}$$

$$X_i = \frac{2\theta_i N}{2\theta_0 + 3\psi}, i \geq 0$$

$$A_i = \frac{2\theta_i \sqrt{N}}{\sqrt{2\theta_0 + 3\psi}\sqrt{\psi}}, i \geq 1 \tag{6}$$