

TAKEHOME EXAM

Please answer all questions. Omitting calculations is OK.

PROBLEM 1

THE ECONOMY

Consider a three-period economy with

- two consumers, namely Ψ and Ω
- three goods, namely A, L, B
- Two firms, namely α and β

firm α produces good A out of good L with production function

$$A_\alpha = cL_\alpha, c > 0 \quad (1)$$

where L_α is the quantity of good L used as an input in the first period, and A_α is the quantity of good A produced in the second period, and c is a parameter.

firm β produces good B out of good A with production function

$$B_\beta = 4\sqrt{A_\beta} \quad (2)$$

where A_β is the quantity of good A used as an input in the second period, and B_β is the quantity of good B produced in the third period.

Consumer Ψ

- consumption set: all variables ≥ 0
- utility function

$$u_\Psi = A_2^\Psi B_3^\Psi \quad (3)$$

where A_2^Ψ is the quantity of good A consumed by Ψ in the second period, and B_3^Ψ is the quantity of good B consumed by Ψ in the third period.

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- endowment: $\bar{L} > 0$ units of good L in the first period only. No endowment of any other good in any period.
- profit shares: sole owner of firm α , no share in firm β

consumer Ω

- consumption set: all variables ≥ 0
- utility function

$$u_{\Omega} = L_1^{\Omega} \quad (4)$$

where L_1^{Ω} is the quantity of good L consumed by Ω in the first period.

- endowment: no endowment of any good in any period.
- profit shares: sole owner of firm β , no share in firm α

Policy

Firm α pays a tax $t_{\alpha} \geq 0$ per unit of input used.

Firm β receives a subsidy $s_{\beta} \geq 0$ per unit of input used.

Firm β pays a tax $\tau, 0 \leq \tau < 1$ per unit of profit earned.

QUESTIONS

Answer the following questions for all allowed parameter values

1. Compute all efficient points.
2. Compute all competitive equilibria.
3. For which parameter values are competitive equilibria efficient?

PROBLEM 2

THE ECONOMY

- Two goods, α and χ , written in this order. Good α is a public good; good χ is a private good.
- Two consumers, 1 and 2.
- One firm.

Consumer 1

- Consumption set R_+^2
- Endowment vector $\omega_1 = \overbrace{[0, 1]}^{\alpha, \chi}$
- Profit share $\theta_1 \geq 0$
- Utility function $u_1 = X_1 + \log(A)$

Consumer 2

- Consumption set R_+^2
- Endowment vector $\omega_2 = \overbrace{[0, \kappa]}^{\alpha, \chi}, \kappa > 1$
- Profit share $\theta_2 = 1 - \theta_1 \geq 0$
- Utility function $u_2 = X_2 + \rho \log(A), \rho > 1$

The firm produces good α out of good χ with technology described by the production function

$$\hat{A} = 2\hat{X}$$

QUESTIONS

Answer the following questions for all allowed parameter values

- Compute all efficient allocations. Compute and draw the Pareto frontier and the utility possibility set.
- Compute all competitive equilibria.
- For which values of the parameters, if any, are competitive equilibria efficient?

PROBLEM 3

THE ECONOMY

- Four goods, A, B, L, K
- One consumer
- Two firms, α and β

Consumer

- Consumption set R_+^4
- Endowment vector $\omega = [0, 0, \overbrace{\bar{L}, \bar{K}}^{A, B, L, K}]$
- Utility function $u = \alpha \log(A) + (1 - \alpha) \log(B), 0 < \alpha < 1$

Firm α produces good A out of goods K, L with production function

$$\hat{A} = \min \{ K_A, \rho L_A \}, \rho > 0$$

where K_A, L_A are the quantities of goods K, L, respectively, used as inputs in the production of good A, and \hat{A} is the quantity produced of good A.

Firm β produces good B out of goods K, L with technology described by the production function

$$\hat{B} = 2\sqrt{K_B L_B}$$

where K_B, L_B are the quantities of goods K, L, respectively, used as inputs in the production of good B, and \hat{B} is the quantity produced of good B.

QUESTIONS

Answer the following questions for all allowed parameter values

- Compute all competitive equilibria.
- For which values of the parameters, if any, do competitive equilibria exist?
- Plot equilibrium prices as a function of α , keeping all other parameters fixed
- Plot equilibrium prices as a function of \bar{K} , keeping all other parameters fixed

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PROBLEM 4

THE ECONOMY

- Two goods, 1 and 2, written in this order.
- Two consumers, A and B.

Consumer A

- Consumption set $X_A = \{(A_1, A_2) : A_1 + A_2 \geq 2, A_1 \geq 0, A_2 \geq 0\}$
- Endowment vector $\omega_A = [0, 2]$
- Utility function $u_A = A_1 A_2$

Consumer B

- Consumption set $X_B = R_+^2$
- Endowment vector $\omega_B = [\kappa, 0], \kappa > 0$
- Utility function $u_B = B_1 B_2$

QUESTIONS

Answer the following questions for all allowed parameter values

- Compute all competitive equilibria.
- For which values of the parameters, if any, do competitive equilibria exist?

PROBLEM 5

THE ECONOMY

Consider a five-period economy with

- one consumer
- two goods, namely A and K
- Two firms, namely γ and δ

firm γ produces good A out of good K with technology described by

$$\hat{A}_2 = 2\hat{K}_1, \hat{A}_3 = \hat{K}_1 \quad (5)$$

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where \hat{K}_1 is the quantity of good K used by firm γ as an input in the first period, and \hat{A}_2, \hat{A}_3 are the quantities of good A produced by firm γ in the second and third periods.

firm δ produces good A out of good K with technology described by

$$\hat{A}_4 = 2\hat{K}_3, \hat{A}_5 = \hat{K}_3 \quad (6)$$

where \hat{K}_3 is the quantity of good K used by firm δ as an input in the third period, and \hat{A}_4, \hat{A}_5 are the quantities of good A produced by firm δ in the fourth and fifth periods.

Consumer

- consumption set: all variables ≥ 0
- utility function

$$u = \sum_{t=1}^5 \log(A_t) + \sum_{t=1}^5 \log(K_t) \quad (7)$$

where A_t is the quantity of good A consumed by the consumer in period t , and K_t is the quantity of good K consumed by the consumer in period t

- endowment: one unit of good A in the first period only, and one unit of good K in each period.

QUESTIONS

Answer the following questions

- Compute all competitive equilibria.
- Plot equilibrium prices and quantities as functions of t

PROBLEM 6

THE ECONOMY

- Two goods, A and X, written in this order.
- One consumer
- One firm.

The Consumer

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- Consumption set R_+^2
- Endowment vector $\omega = [0, \bar{X}]$
- Profit share $\theta = 1$
- Utility function $u = \log A + \log X$

The firm produces good A out of good X with technology described by the production function

$$\hat{A} = \begin{cases} \hat{X}^2 & \text{if } 0 \leq \hat{X} \leq F \\ F^2 & \text{if } \hat{X} \geq F \end{cases}$$

Parameters: F, \bar{X} . Conditions on parameters: $0 \leq F < \bar{X}$

QUESTIONS

Answer the following questions for all allowed parameter values

- Compute all competitive equilibrium allocations E
- Compute all efficient allocations P
- First welfare theorem: For which parameter values, if any, is it true that $E \subset P$?
- Compute the set of decentralizable efficient allocations $P \cap E$
- Second welfare theorem: For which parameter values, if any, is it true that all efficient allocations are decentralizable, i.e., that $P = E$?