Lecture 8

Transactions across time

Consider an economy with two consumers, A and B, and one good, say corn. Consumers live for two periods, 1 and 2. A has one unit of corn in period 1, and nothing else. B has one unit of corn in period 2, and nothing else. Competitive equilibrium assumes that in this economy there will be two commodities traded: corn to be delivered in period 1, and corn to be delivered in period 2, and that consumers will have preferences among them. Here we let A_t, B_t denote the consumptions of A, B in period t, and we assume preferences of the form

$$U_A = A_1 A_2^{\delta}$$

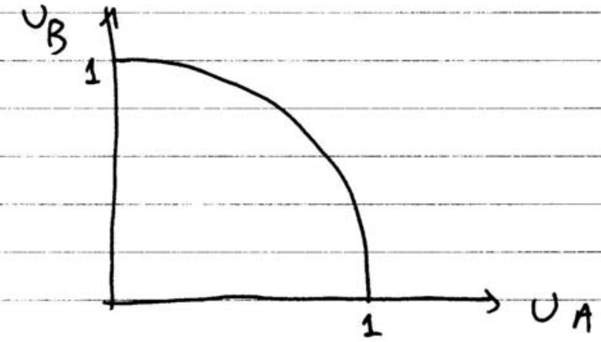
$$U_B = B_1 B_2^{\delta} \qquad 0 < \delta < 1$$

<u>Pareto points</u> are those that satisfy

$$A_t = A, \quad B_t = 1 - A, \quad 0 \le A \le 1$$

The <u>Pareto frontier</u> is

$$U_B = \left(1 - U_A^{\frac{1}{1+\delta}}\right)^{1+\delta}, \quad 0 \le U_A \le 1$$



Note that Pareto optimality dictates equal consumption in each period.

In competitive equilibrium, we have two markets, for commodities 1 and 2. Their prices will be p_1, p_2. Both markets operate in period 1; p_t is the price of one unit of corn to be delivered in period t.

The budget constraints are

$$p_1 A_1 + p_2 A_2 \le p_1 , \qquad p_1 B_1 + p_2 B_2 \le p_2$$

Hence when A buys A_2 in period 1, he buys the <u>promise</u> of delivery of A_2 units of corn in period 2. Normalize $p_1 = 1$. Let $p_2 = p$.

<u>Demand functions</u>

$$A = \left(\frac{1}{1+\delta} , \frac{\delta}{1+\delta} \frac{1}{p} \right)$$

$$B = \left(\frac{p}{1+\delta} , \frac{\delta}{1+\delta} \right)$$

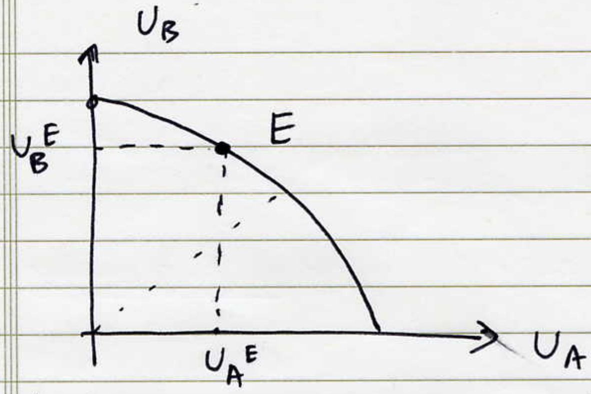
<u>Equilibrium conditions</u>

$$A_1 + B_1 = 1$$
$$A_2 + B_2 = 1$$

<u>Equilibrium values</u> $\quad p = \delta$

$$A = \left(\frac{1}{1+\delta} , \frac{1}{1+\delta} \right) \qquad U_A^E = \frac{1}{(1+\delta)^{1+\delta}}$$

$$B = \left(\frac{\delta}{1+\delta} , \frac{\delta}{1+\delta} \right) \qquad U_B^E = \left(\frac{\delta}{1+\delta} \right)^{1+\delta}$$



<u>Implicit interest rate</u> : In period 1,

A lends to B $1 - A_1 = 1 - \frac{1}{1+\delta} = \frac{\delta}{1+\delta}$

units of corn. In return, in period 2

B gives back to A $1 - B_1 = 1 - \frac{\delta}{1+\delta} = \frac{1}{1+\delta}$

units of corn. Hence, the return for A

is $r = \frac{(1-B_1) - (1-A_1)}{(1-A_1)} = \frac{\frac{1}{1+\delta} - \frac{\delta}{1+\delta}}{\frac{\delta}{1+\delta}} = \frac{1-\delta}{\delta} = \frac{1}{\delta} - 1$

We call r the implicit interest rate. Note

that $p = \delta$, so $r = \frac{1}{p} - 1$ ie

$p = \frac{1}{1+r}$ ie $P_2 = \frac{P_1}{1+r}$, the

well-known discounting formula.