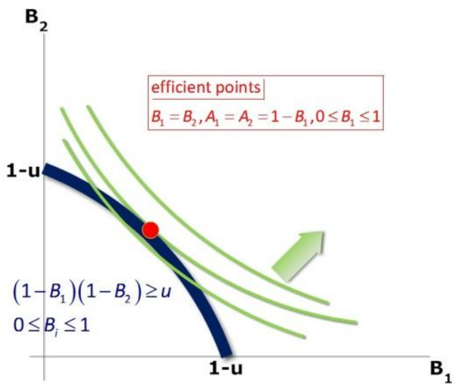
TAXATION AND EFFICIENCY

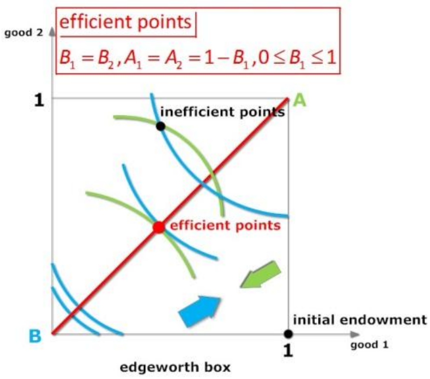
- Consumers A,B
- Goods 1, 2.
- Preferences $u_A = 2\sqrt{A_1 A_2}, u_B = 2\sqrt{B_1 B_2}$
- Endowments $e_A = [1,0], e_B = [0,1]$

EFFICIENT POINTS

$\max u_B = \log B_1 + \log B_2$

subject to $\log A_1 + \log A_2 \geq u, A_1 + B_1 \leq 1, A_2 + B_2 \leq 1, A_1 \geq 0, B_1 \geq 0, A_2, B_2 \geq 0$





$$\boxed{\begin{array}{l} \text{efficient points} \\ B_1 = B_2, A_1 = A_2 = 1 - B_1, 0 \leq B_1 \leq 1 \\ \text{efficient frontier} \\ u_A + u_B = 2, 0 \leq u_B \leq 2 \end{array}} \quad (1)$$

COMPETITIVE EQUILIBRIUM WITH TAXATION

Consumers pay a tax $-1 < t$ for each unit of good 1 they buy.Tax revenue R is distributed to consumers with lump-sum transfers T_A, T_B

1. NAME THE PRICE OF EACH GOOD

$p_i =$ price of good i. *Normalize* $p_1 = 1$. *Let* $p = p_2$

2. DEFINE CONSUMER INCOMES

$$M_A = 1 + T_A, M_B = p + T_B \quad (2)$$

3. SOLVE THE OPTIMIZATION PROBLEMS OF CONSUMERS

$\max U_A = \log A_1 + \log A_2$, subject to $A_1 + pA_2 \le M_A$
$\max U_B = \log B_1 + \log B_2$, subject to $(1+t)B_1 + pB_2 \le M_B$

The solutions are

$$(A_1, A_2) = \left(\frac{M_A}{2}, \frac{M_A}{2p}\right) \tag{3}$$

$$(B_1, B_2) = \left(\frac{M_B}{2(1+t)}, \frac{M_B}{2p}\right) \tag{4}$$

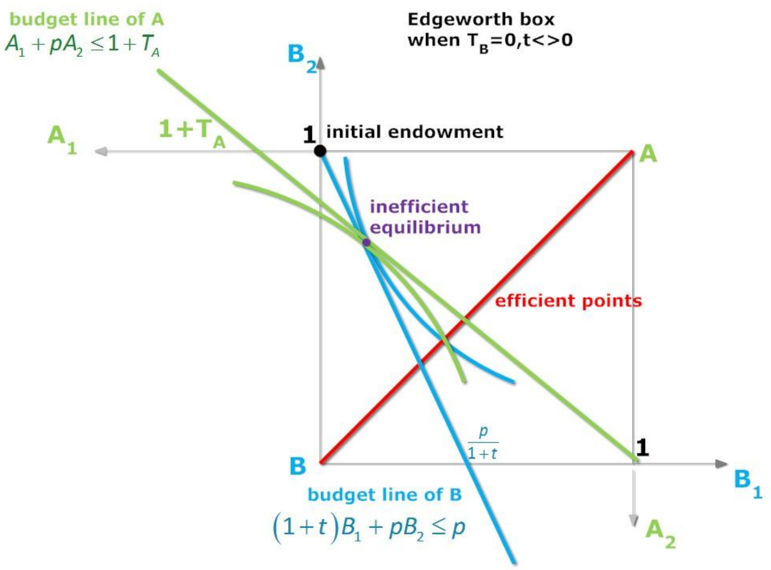
5. SOLVE THE EQUILIBRIUM CONDITIONS

$$1 = A_1 + B_1, 1 = A_2 + B_2, T_A + T_B = tB_1 \tag{5}$$

There is a unique solution, described by

competitive equilibria with unit taxation t>-1,t$\neq$0 equilibria are inefficient.

$$p = \frac{2+2t+tT_B}{2+t},\ T_A = \frac{t-2T_B}{2+t}, -1 < T_B < 1+t$$
$$A_1 = \frac{1+t-T_B}{2+t},\ A_2 = \frac{1+t-T_B}{2(1+t)+tT_B}$$
$$B_1 = \frac{1+T_B}{2+t}, B_2 = \frac{(1+t)(1+T_B)}{2(1+t)+tT_B} \tag{6}$$
$$R = t\frac{1+T_B}{2+t}$$



competitive equilibria with t=0.
every equilibrium is efficient.
every efficient point can be obtained as an equilibrium

$$p=1,\ T_A+T_B=0\ ,-1<T_B<1, t=0$$ (7)

$$A_1=\frac{1-T_B}{2}=A_2$$

$$B_1=\frac{1+T_B}{2}=B_2$$

competitive equilibria with t>-1
consumer A gets all tax revenue

$$p=\frac{2+2t}{2+t},\ T_A=\frac{t}{2+t}, T_B=0$$

$$A_1=\frac{1+t}{2+t},\ A_2=\frac{1}{2}$$ (8)

$$B_1=\frac{1}{2+t}, B_2=\frac{1}{2}$$

$$R=\frac{t}{2+t}$$

competitive equilibria with t>-1
consumer B gets all tax revenue

$$p=1+\frac{t}{2},\ T_A=0, T_B=\frac{t}{2}$$

$$A_1=\frac{1}{2},\ A_2=\frac{1}{2+t}$$ (9)

$$B_1=\frac{1}{2}, B_2=\frac{1+t}{2+t}$$

$$R=\frac{t}{2}$$