Question 1

Derive the profit functions, average cost functions and net supply correspondences of the following firms, and draw their net supply correspondences

$$Q = \alpha X, \alpha > 0 \tag{1}$$

$$Q = \alpha X^{\frac{1}{2}}, \alpha > 0 \tag{2}$$

$$Q = \alpha X^{2}, \alpha > 0 \tag{3}$$

$$Q = \min(X_1, \alpha X_2), \alpha > 0 \tag{4}$$

Question 2

Consider the function $R^2_{++} \xrightarrow{f} R,\ f(p_1, p_2) = p_2 - 3p_1$

1. Is it a profit function? **Yes, it is convex and positive-homogeneous of degree 1**

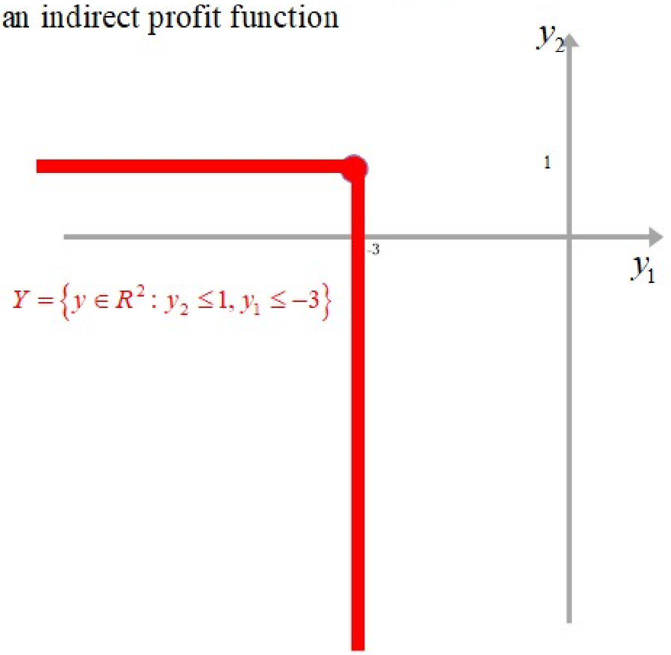
2. If it is, construct a rationalizing production set for f, and derive the corresponding net supply correspondence

The rationalizing set is $Y = \bigcap_{p \gg 0} Q(p)$ where $Q(p) = \{y \in R^2 : py \leq f(p)\}$ hence

$$Y = \{y \in R^2 : p_1 y_1 + p_2 y_2 \leq p_2 - 3p_1, \forall p_1 > 0, \forall p_2 > 0\}$$

Eliminating prices from the definition of Y we obtain $Y = \{y \in R^2 : y_2 \leq 1, y_1 \leq -3\}$

production set rationalizing $f(p) = p_2 - 3p_1$
as an indirect profit function



We can verify this by solving the maximization problem

$$\max py = [p_1 \quad p_2]\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = p_1 y_1 + p_2 y_2$$
$$\text{subject to } y_1 \le 1, y_2 \le -3$$
$$\text{variables:} y_i, i = 1, 2$$
$$\text{parameters:} p_i, i = 1, 2$$
$$\text{conditions on parameters:} p_i > 0, i = 1, 2$$

The maximum of this problem is $y = [-3 \quad 1]$,hence the profit function of $Y = \{y \in R^2 : y_1 \le -3, y_2 \le 1\}$ is $\pi(p) = [p_1 \quad p_2]\begin{bmatrix} -3 \\ 1 \end{bmatrix} = f(p)$,as claimed.

Question 3

Consider the following price-quantity data (two commodities, three observations)

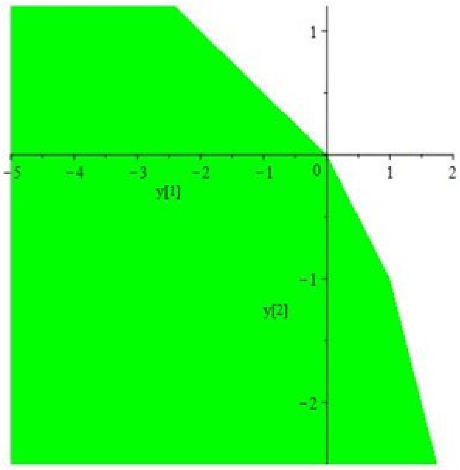
$$\text{quantity vectors} = \begin{bmatrix} 1 & 2 & -2 \\ -1 & -3 & 1 \end{bmatrix} = [y^1, y^2, y^3] \tag{5}$$

$$\text{price vectors} = \begin{bmatrix} 3 & 4 & 1 \\ 3 & 2 & \theta \end{bmatrix} = [p^1, p^2, p^3] \tag{6}$$

1. For which values of $\theta > 0$ do the data satisfy WAPM? $\theta \ge \frac{3}{2}$

2. Construct and draw the rationalizing production set YO when $\theta = 2$.

$YO = \{y \in R^2 : y_1 + y_2 \le 0, 2y_1 + y_2 \le 1, y_1 + 2y_2 \le 0\}$



3. Find the profit maximizing netput vectors of the firms with production sets $Y = \{y^1, y^2, y^3\}$ and $Y = YO$ at $p = [4, 3]$ when $\theta = 2$.

Case $Y=\{y^1,y^2,y^3\}$

$py^1=1, py^2=-1, py^3=-5$, hence $y=y^1$

Case $Y=YO$

The firm solves the following problem

$$\max py=[4 \quad 3]\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}=4y_1+3y_2$$
$$\text{subject to } y_1+y_2\le 0, 2y_1+y_2\le 1, y_1+2y_2\le 0$$
$$\text{variables:} y_i, i=1,2$$

The maximum is $y=[1,-1]$

4. Find the indirect profit functions and the net supply correspondences of the firms with production sets $Y=\{y^1,y^2,y^3\}$ and $Y=YO$ when $\theta=2$.

Case $Y=\{y^1,y^2,y^3\}$

The firm solves the following problem

$$\max py{=}[p_1 \quad p_2]\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}=p_1y_1+p_2y_2$$
$$\text{subject to } y\in\left\{y^1,y^2,y^3\right\}$$
$$\text{variables:} y_i, i=1,2$$
$$\text{parameters:} p_i, i=1,2$$
$$\text{conditions on parameters:} p_i>0, i=1,2$$

The firm's profits at each of its three choices are

$$\begin{bmatrix} py^1=p_1-p_2 \\ py^2=2p_1-3p_2 \\ py^3=p_2-2p_1 \end{bmatrix}$$

Hence y^1 is profit-maximizing if $p_1-p_2\ge 2p_1-3p_2, p_1-p_2\ge p_2-2p_1$ hence the indirect profit function must satisfy

$$\frac{\pi(p)}{p_1}=\begin{cases}1-\frac{p_2}{p_1} & \text{if } \frac{1}{2}\le\frac{p_2}{p_1}\le\frac{3}{2}\\ 2-3\frac{p_2}{p_1} & \text{if } ?\\ \frac{p_2}{p_1}-2 & \text{if } ?\end{cases}$$

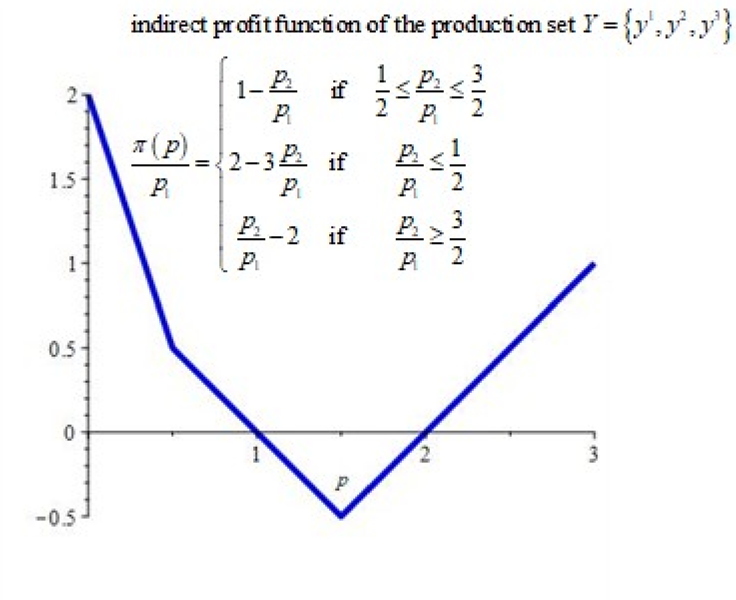
Netput y^2 is profit-maximizing if $2p_1-3p_2\ge p_1-p_2, 2p_1-3p_2\ge p_2-2p_1$ hence the indirect profit function must satisfy

$$\frac{\pi(p)}{p_1}=\begin{cases}1-\frac{p_2}{p_1} & \text{if } \frac{1}{2}\le\frac{p_2}{p_1}\le\frac{3}{2}\\ 2-3\frac{p_2}{p_1} & \text{if } \frac{p_2}{p_1}\le\frac{1}{2}\\ \frac{p_2}{p_1}-2 & \text{if } ?\end{cases}$$

Netput y^3 is profit-maximizing if $p_2-2p_1\ge p_1-p_2, p_2-2p_1\ge 2p_1-3p_2$ hence the indirect profit function must satisfy

indirect profit function when $Y=\{y^1,y^2,y^3\}$

$$\frac{\pi(p)}{p_1}=\begin{cases}1-\frac{p_2}{p_1} & \text{if } \frac{1}{2}\le\frac{p_2}{p_1}\le\frac{3}{2}\\ 2-3\frac{p_2}{p_1} & \text{if } \frac{p_2}{p_1}\le\frac{1}{2}\\ \frac{p_2}{p_1}-2 & \text{if } \frac{p_2}{p_1}\ge\frac{3}{2}\end{cases}$$



Case $Y = YO$

The firm solves the following problem

$$\max\ py=[p_1 \quad p_2]\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = p_1y_1 + p_2y_2$$

subject to $y_1 + y_2 \le 0, 2y_1 + y_2 \le 1, y_1 + 2y_2 \le 0$

variables:$y_i, i = 1,2$

parameters:$p_i, i = 1,2$

conditions on parameters:$p_i > 0, i = 1,2$

Solving we obtain that

For $\frac{p_2}{p_1} < \frac{1}{2}$ the netput vectors $y^t = \begin{bmatrix} \frac{1+t}{2} & -t \end{bmatrix}, t = 1,2..$ are feasible and

$py^t = p_1\left(\frac{1}{2} + \left(\frac{1}{2} - \frac{p_2}{p_1}\right)t\right) \to \infty$ as $t \to \infty$,hence $\pi(p) = \infty$

For $\frac{1}{2} \le \frac{p_2}{p_1} \le 1$ the netput vector $y = [1 \quad -1]$ is a maximum, hence

$\pi(p) = p_1 - p_2$

For $1 \le \frac{p_2}{p_1} \le 2$ the netput vector $y = [0 \quad 0]$ is a maximum, hence $\pi(p) = 0$

For $\frac{p_2}{p_1} > 2$ the netput vectors $y^t = \begin{bmatrix} -t & \frac{t}{2} \end{bmatrix}, t = 1, 2..$ are feasible and

$py^t = \frac{p_1}{2}\left(\frac{p_2}{p_1} - 2\right)t \to \infty$ as $t \to \infty$,hence $\pi(p) = \infty$

indirect profit function of the production set $Y = YO$

$$\frac{\pi(p)}{p_1} = \begin{cases} 1 - \frac{p_2}{p_1} & \text{if } \frac{1}{2} \le \frac{p_2}{p_1} \le 1 \\ 0 & \text{if } 1 \le \frac{p_2}{p_1} \le 2 \\ \infty & \text{otherwise} \end{cases}$$