Micro exam

Consider the following economy

Consumers: 1 and 2

Goods: A, B, C.

Technology

There is one firm, producing good A out of good C with production function

$$A = \theta\sqrt{2C}, 0 < \theta < 1$$

Preferences

$$u_1 = \ln(B_1^{2456} + 1)$$
$$u_2 = \left(\min(A_2^{3456}, B_2^{3456})\right)^{23/57}$$

Consumption sets

$$X_A = X_B = \mathbb{R}_+^2$$

Endowments

$$e_1 = \begin{bmatrix} 0 \\ 0 \\ 1/2 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

where goods are written in the order A, B, C. Consumer 2 is the owner of the firm.

1. Compute all competitive equilibria for all values of the parameters.

Hint: replace the utility functions with simpler ones that represent the same preferences

2. Suppose that the productivity θ of consumer 1 increases. Who benefits and who loses?

Answers

The utility function $u_1 = \ln(B_1^{2456} + 1)$ represents the same preferences as the utility function $v_1 = B_1$,because u_1 is a monotonic increasing transformation of v_1 .

The utility function $u_2 = \left(\min(A_2^{3456}, B_2^{3456})\right)^{23/57}$ represents the same preferences as the utility function $v_2 = \min(A_2, B_2)$,for the same reason.

Micro exam

Computation of competitive equilibria

1. NAME the price of each good

$p_A = price\ of\ A, p_B = price\ of\ B, p_C = price\ of\ C$

Normalize $p_C = 1$

2.DEFINE consumer incomes

$$\begin{aligned} m_1 &= 1/2 \\ m_2 &= p_B + \Pi \end{aligned} \tag{1}$$

3. SOLVE the optimization problems of firms

profit $\Pi = p_A A - p_C C = p_A A - \frac{A^2}{2\theta^2}$ is maximized at

$$\begin{bmatrix} A \\ C \\ \Pi \end{bmatrix} = \begin{bmatrix} \theta^2 p_A \\ \frac{\theta^2 p_A^2}{2} \\ \frac{\theta^2 p_A^2}{2} \end{bmatrix} \tag{2}$$

4. SOLVE the optimization problems of consumers

$\max\ v_1 = B_1$, subject to

$p_A A_1 + p_B B_1 + p_C C_1 \le m_1, A_1 \ge 0, B_1 \ge 0, C_1 \ge 0$

variables : A_1, B_1, C_1

$\max\ v_2 = \min(A_2, B_2)$, subject to

$p_A A_2 + p_B B_2 + p_C C_2 \le m_2, A_2 \ge 0, B_2 \ge 0, C_2 \ge 0$

variables : A_2, B_2, C_2

The solutions are

$$\begin{bmatrix} A_1 \\ B_1 \\ C_1 \end{bmatrix} = \begin{bmatrix} 0 \\ m_1 / p_B \\ 0 \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} A_2 \\ B_2 \\ C_2 \end{bmatrix} = \begin{bmatrix} m_2/(p_A + p_B) \\ m_2/(p_A + p_B) \\ 0 \end{bmatrix} \tag{4}$$

5. SOLVE the equilibrium conditions

$$\begin{aligned} A_1 + A_2 &= A \\ B_1 + B_2 &= 1 \\ C_1 + C_2 + C &= 1/2 \end{aligned} \tag{5}$$

There is a unique solution, described by

$$\boxed{\begin{aligned} & p_A = 1/\theta,\ p_B = \frac{1}{2(1-\theta)} \\ & m_1 = 1/2,\ m_2 = \frac{2-\theta}{2-2\theta} \\ & \begin{bmatrix} A_1 \\ B_1 \\ C_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1-\theta \\ 0 \end{bmatrix}, \begin{bmatrix} A_2 \\ B_2 \\ C_2 \end{bmatrix} = \begin{bmatrix} \theta \\ \theta \\ 0 \end{bmatrix} \\ & v_1 = 1-\theta, v_2 = \theta \\ & \Pi = 1/2, A = \theta, C = 1/2 \end{aligned}} \tag{6}$$

Consumer 1 becomes worse off the more productive he gets; all productivity gains go to consumer 2.