December 1, 2016 [EXAM]

Consider the following economy

Consumers: 1 and 2

Goods: A, B, C.

Technology

Good A is produced out of good C with production function $A = 2\theta\sqrt{C}, 0 < \theta < 1$

Preferences/endowments/consumption sets

$u_1 = B_1, e_1 = (0, 0, \gamma), B_1 \geq 0$
$u_2 = \min(A_2, B_2), e_2 = (0, \beta, 0), A_2 \geq 0, B_2 \geq 0$

(Goods in the endowment vectors are written in the order A, B, C).

Consumer 2 is the sole owner of the firm.

1. Compute all competitive equilibria for all values of the parameters $\beta > 0, \gamma > 0, 0 < \theta < 1$,

2. For which values of the parameters β, γ, if any, will an increase in the productivity θ of agent 1 reduce his equilibrium utility?

ANSWERS

① P_A = price of A, P_B = price of B, P_C = price of C

② Incomes $M_1 = \gamma P_C$ $M_2 = \beta P_B + \pi$

③ Consumer demand functions (UMAX)

$$\boxed{A_1 = C_1 = 0, \quad B_1 = M_1 / P_B \qquad C_2 = 0, \quad A_2 = B_2 = \frac{M_2}{P_A + P_B}}$$

④ Profit max

$$\text{MAX } \pi = 2\theta P_A \sqrt{C} - P_c C$$

$$\boxed{c = \frac{\theta^2 P_A^2}{P_c^2}, \quad a = \frac{2\theta^2 P_A}{P_c}, \quad \pi = \frac{\theta^2 P_A^2}{P_c}}$$

⑥ Equilibrium conditions

$A_1 + A_2 = A$	$A_2 = A$
$B_1 + B_2 = \beta$	$B_1 + B_2 = \beta$
$C_1 + C_2 + C = \gamma$	$C = \gamma$

COMPETITIVE EQUILIBRIUM, $\theta < \frac{\beta}{2\sqrt{\gamma}}$

$$P_A/P_c = (\sqrt{\gamma})/\theta, \quad P_B/P_c = \frac{\gamma}{\beta - 2\theta\sqrt{\gamma}}, \quad \pi = P_c\sqrt{\gamma}$$

$$B_1 = \beta - 2\theta\sqrt{\gamma}, \quad A_1 = C_1 = 0, \quad A = 2\theta\sqrt{\gamma}$$

$$A_2 = B_2 = 2\theta\sqrt{\gamma}, \quad C_2 = 0, \quad C = \gamma$$

$$u_1^E = \beta - 2\theta\sqrt{\gamma}, \quad u_2^E = 2\theta\sqrt{\gamma}$$

Hence when $\beta > 2\theta\sqrt{\gamma}$, an increase in the productivity of consumer 1 leaves consumer 1 worse off and consumer 2 better off.

If $\beta \leq 2\theta\sqrt{\gamma}$ we have to change the definition of equilibrium to allow for free goods. The equilibria will be

COMPETITIVE EQUILIBRIUM $\theta \geq \beta/2\sqrt{\gamma}$

$$P_A = P_C = 0, \quad P_B > 0, \quad \pi = 0,$$

$$A_1 = B_1 = C_1 = 0, \quad A_2 = B_2 = A = \beta, \quad C_2 = 0$$

$$C = \frac{\beta^2}{4\theta^2} \leq \gamma$$

$$u_1^E = 0, \quad u_2^E = \beta$$