QR DECOMPOSITION

$$A = QR = \begin{bmatrix} Q \end{bmatrix} \begin{bmatrix} \diagdown & \\ 0 & \diagdown \end{bmatrix}$$

ORTHOGONAL $\quad$ UPPER TRIANGULAR

$$Q^T Q = I$$

If A is non singular this factorization is unique

$$A = QR \Rightarrow Q^T A = Q^T Q R = R$$

QR and Gram-Schmidt

$$A = [\, a_1 \quad a_2 \quad \cdots \quad a_n \,]$$

$$u_1 = a_1 \qquad e_1 = u_1 / \|u_1\|$$

$$u_2 = a_2 - \langle a_2, e_1 \rangle e_1 \qquad e_2 = u_2 / \|u_2\|$$

$$\vdots$$

$$u_k = a_k - \langle a_k, e_1 \rangle e_1 - \cdots - \langle a_k, e_{k-1} \rangle e_{k-1} \qquad e_k = u_k / \|u_k\|$$

$$A = [a_1, \cdots, a_n] = \underbrace{[e_1, \cdots, e_n]}_{Q} \underbrace{\begin{bmatrix} \langle a_1, e_1 \rangle & \langle a_2, e_1 \rangle & \cdots & \langle a_n, e_1 \rangle \\ 0 & \langle a_2, e_2 \rangle & \cdots & \langle a_n, e_2 \rangle \\ 0 & 0 & \cdots & \langle a_n, e_3 \rangle \\ & & & \\ 0 \cdot \quad 0 & \cdots & 0 & \langle a_n, e_n \rangle \end{bmatrix}}_{R}$$