

$$A = \begin{bmatrix} 1 & 2 & 8 & 2 \\ 3 & 9 & -1 & 2 \\ -1 & 2 & 6 & 3 \\ 1 & 5 & 3 & 2 \end{bmatrix}$$

$$E_{24} A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 8 & 2 \\ 3 & 9 & -1 & 2 \\ -1 & 2 & 6 & 3 \\ 1 & 5 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 8 & 2 \\ 1 & 5 & 3 & 2 \\ -1 & 2 & 6 & 3 \\ 3 & 9 & -1 & 2 \end{bmatrix}$$

Εντάξει
 ξε-rrs
 2 και 4

$$E_{43}^{(-3)} E_{24} A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 8 & 2 \\ 1 & 5 & 3 & 2 \\ -1 & 2 & 6 & 3 \\ 3 & 9 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 8 & 2 \\ 1 & 5 & 3 & 2 \\ -1 & 2 & 6 & 3 \\ 0 & 15 & 17 & 11 \end{bmatrix}$$

Αγαμέμνην - 3
 ξε-rrs Την
 ξε-rrs 3 εν. Την 4

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

$$\xrightarrow{R_2 = R_2 - \left(\frac{a_{21}}{a_{11}}\right) R_1}$$

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ 0 & a_{22} & \dots & a_{2n} \\ a_{31} & a_{32} & \dots & a_{3n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

$$\xrightarrow{R_3 = R_3 - \left(\frac{a_{31}}{a_{11}}\right) R_1}$$

$$E_{21}\left(\frac{a_{21}}{a_{11}}\right) A$$

$$E_{31}\left(\frac{a_{31}}{a_{11}}\right) \left\{ E_{21}\left(\frac{a_{21}}{a_{11}}\right) A \right\}$$

.....
 If we follow this pattern again (1st column)

$$E_{n1}\left(\frac{a_{n1}}{a_{11}}\right) \dots E_{31}\left(\frac{a_{31}}{a_{11}}\right) E_{21}\left(\frac{a_{21}}{a_{11}}\right) A =$$

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ 0 & a_{22} & \dots & a_{2n} \\ 0 & a_{32} & \dots & a_{3n} \\ \vdots & \vdots & & \vdots \\ 0 & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

Σ υε x1/vpt με 2η στήλη (2nd column)

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ 0 & a_{22} & \dots & a_{2n} \\ 0 & a_{32} & \dots & a_{3n} \\ \vdots & \vdots & & \vdots \\ 0 & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

$$\xrightarrow{R_3 = R_3 - \left(\frac{a_{32}}{a_{22}}\right) R_2}$$

$$E_{32}\left(\frac{a_{32}}{a_{22}}\right)$$

$$\begin{bmatrix} a_{11} & a_{12} & * \\ 0 & a_{22} & * \\ 0 & 0 & * \\ \vdots & * & \vdots \\ 0 & * & \vdots \end{bmatrix}$$

$$\xrightarrow{R_4 = R_4 - \left(\frac{a_{42}}{a_{22}}\right) R_2}$$

$$E_{42}\left(\frac{a_{42}}{a_{22}}\right) \left\{ E_{32}\left(\frac{a_{32}}{a_{22}}\right) \right\}$$

→

$$E_{n2}\left(\frac{a_{n2}}{a_{22}}\right) \dots E_{42}\left(\frac{a_{42}}{a_{22}}\right) E_{32}\left(\frac{a_{32}}{a_{22}}\right) A = E_{n2}\left(\frac{a_{n2}}{a_{22}}\right) \dots E_{42}\left(\frac{a_{42}}{a_{22}}\right) E_{32}\left(\frac{a_{32}}{a_{22}}\right) \times$$

$$\left\{ E_{n1}\left(\frac{a_{n1}}{a_{11}}\right) \dots E_{31}\left(\frac{a_{31}}{a_{11}}\right) \right\} A$$

Transform (in total)

$$E_{n,n-1} \left(\frac{a_{n,n-1}}{a_{n-1,n-1}} \right) \times \dots \times \left[E_{ni} \left(\frac{a_{ni}}{a_{ii}} \right) \dots E_{i+1,i} \left(\frac{a_{i+1,i}}{a_{ii}} \right) \right] \rightarrow i\text{th column}$$

$$\times \dots \times \left[E_{n2} \left(\frac{a_{n2}}{a_{22}} \right) \dots E_{32} \left(\frac{a_{32}}{a_{22}} \right) \right] \rightarrow 2\text{nd column}$$

$$\times \left[E_{n1} \left(\frac{a_{n1}}{a_{11}} \right) \dots E_{21} \left(\frac{a_{21}}{a_{11}} \right) \right] \rightarrow 1\text{st column}$$

i -th column

$$E_i = E_{ni} \left(\frac{a_{ni}}{a_{ii}} \right) \dots E_{i+1,i} \left(\frac{a_{i+1,i}}{a_{ii}} \right)$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \vdots & 0 \\ \vdots & \vdots \\ \vdots & \vdots \\ 0 & \vdots \end{bmatrix}$$

i -column



$$\begin{bmatrix} 0 & 0 \\ \vdots & \vdots \\ 1 & 0 \\ -\frac{a_{i+1,i}}{a_{ii}} & 1 \\ \vdots & 0 \\ -\frac{a_{ni}}{a_{ii}} & \vdots \end{bmatrix}$$

i -row