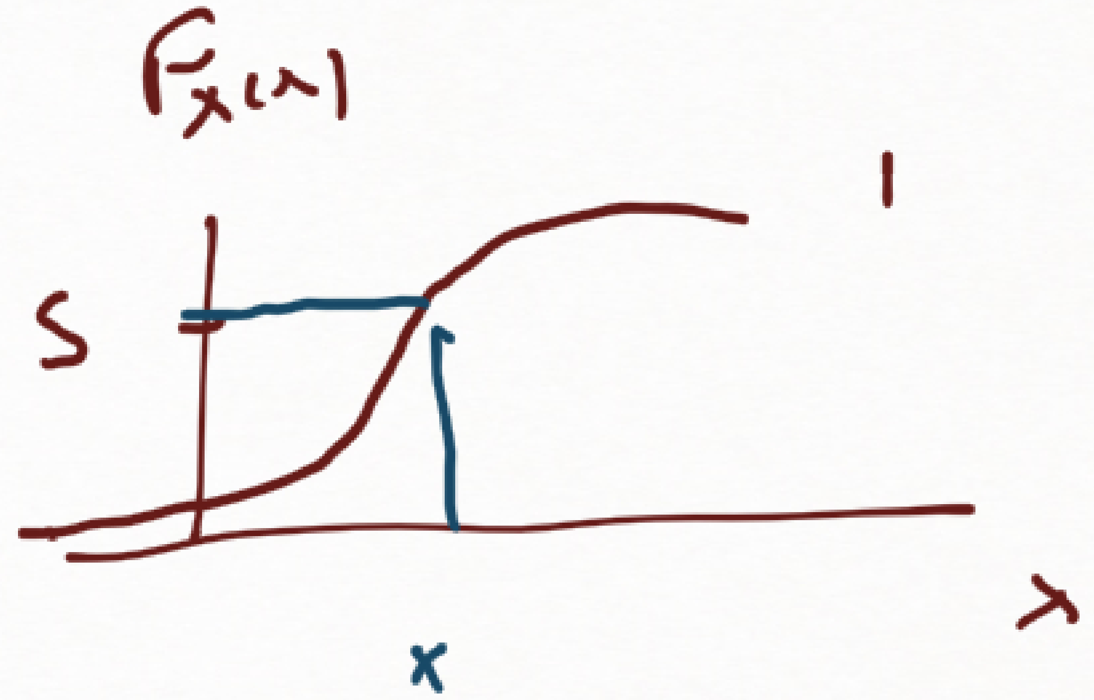
Quantile function

$F_X(x) = P(X(\omega) \le x)$

$x \mapsto F_X(x)$

$F_X \uparrow$



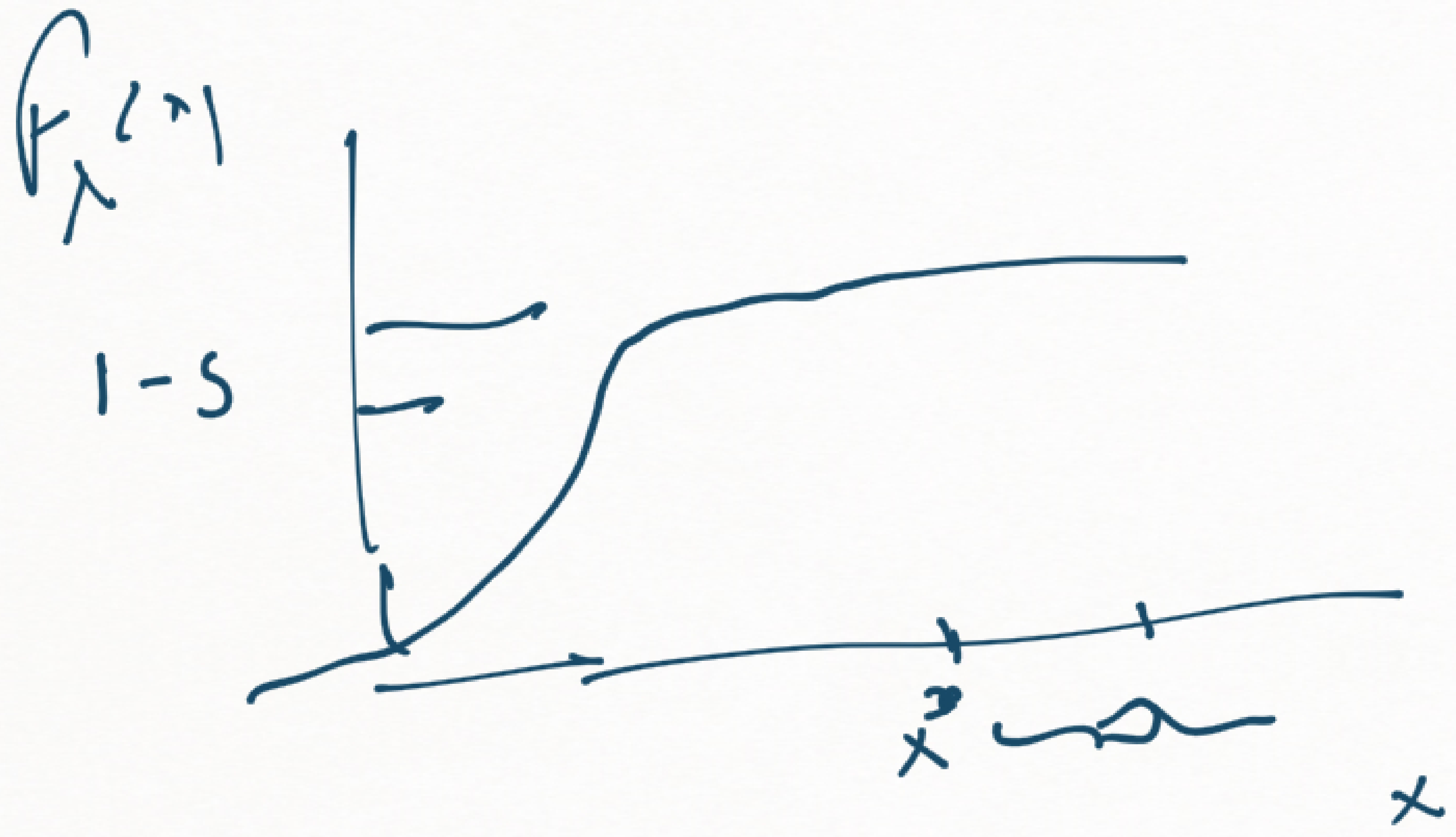
Quantile function — Analogous to $F_X(\cdot)$

$s \in [0,1]$ Because 1. $x \in \mathbb{R}$ s.t. $F(x) = s$

$s \mapsto x$ [illegible] $F(x) = s$

$F_X^{-1} = Q$

X ζη



$VaR_s(X)$

$P(X > x^*) = 1-s$

s επίπεδο εμπιστοσύνης

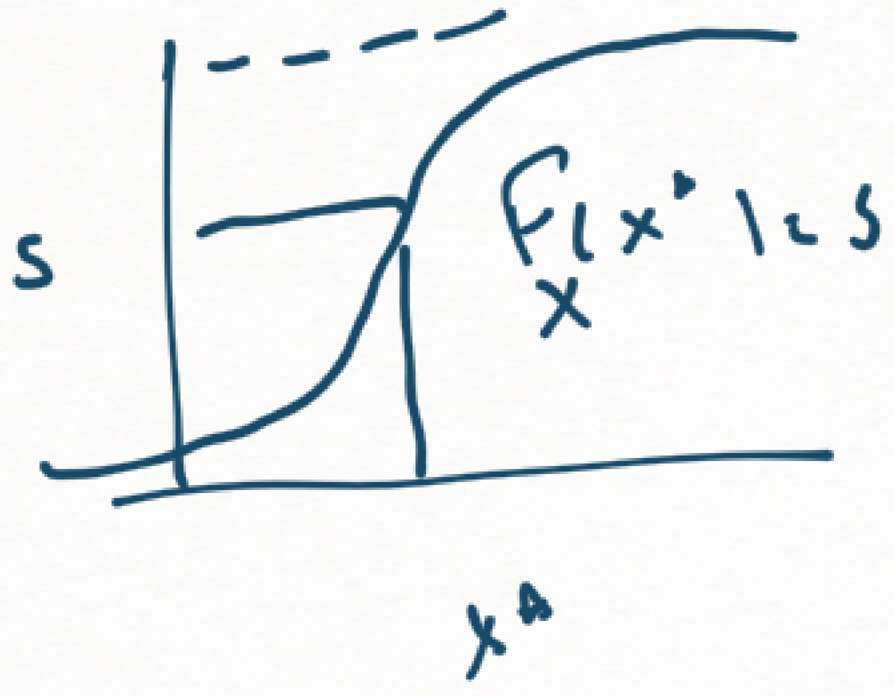
$s = 95\%$

x^* κίνδυνος ο οποίος αν πιθανότητα να τον υπερβώ είναι τ. π.χ.

$1-s = 5\%$

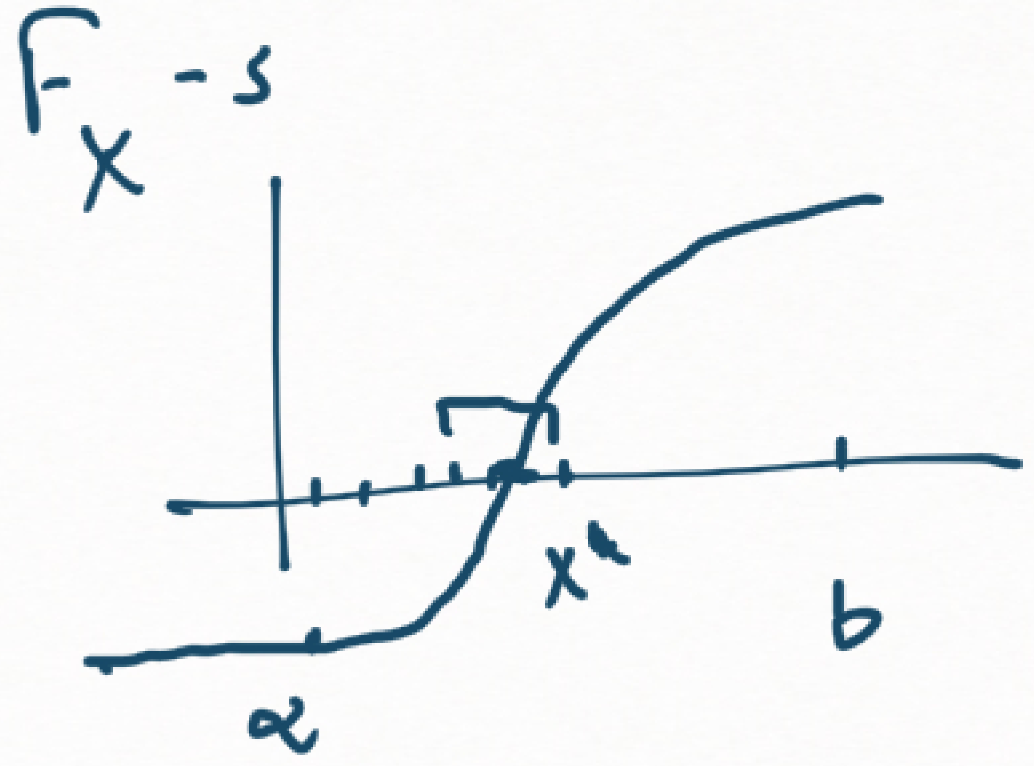
$$F_X^{-1}(s) = x^*$$

$$F_X(x^*) = s$$

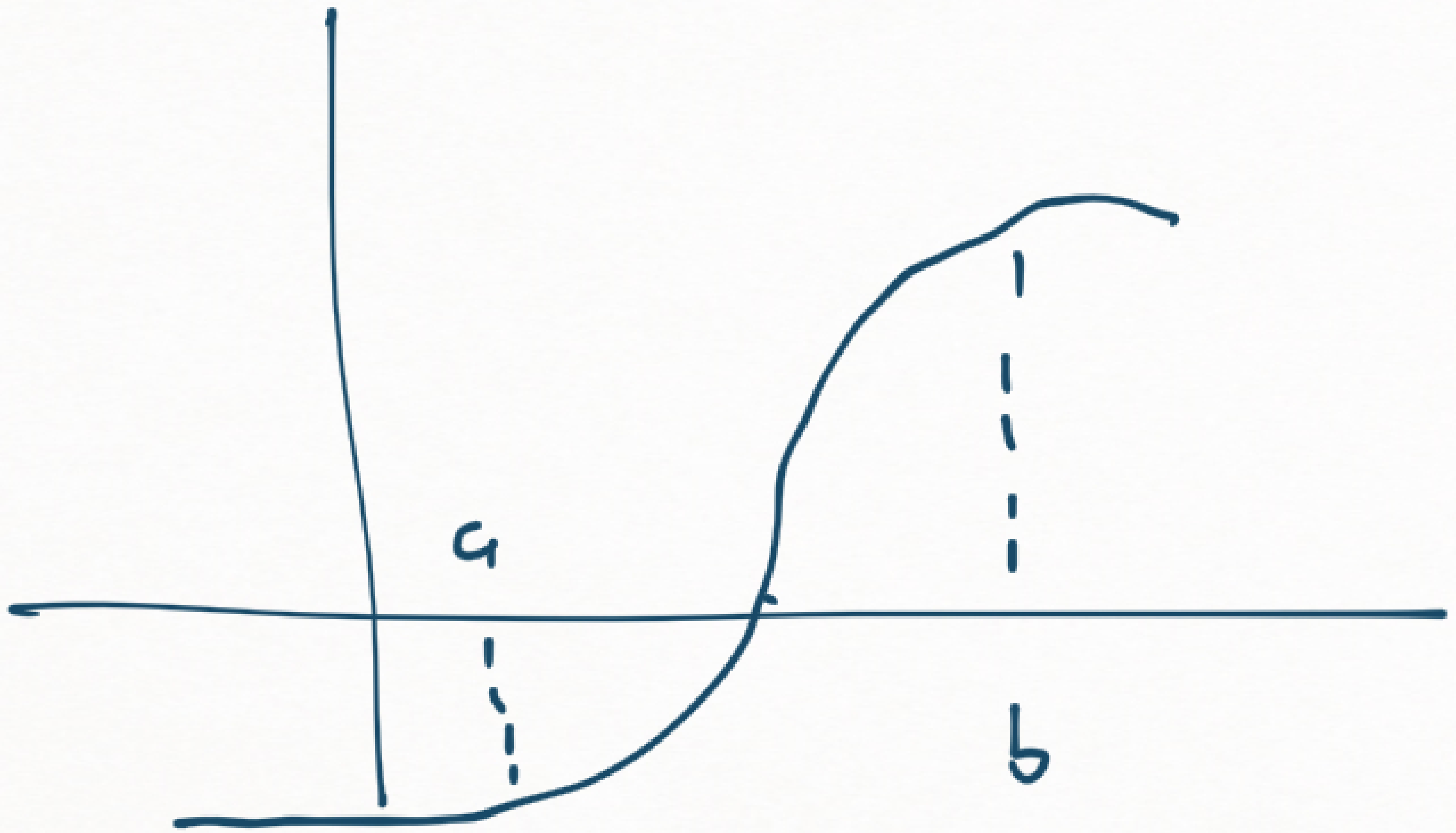


Then run a F_X [illegible] algorithm.

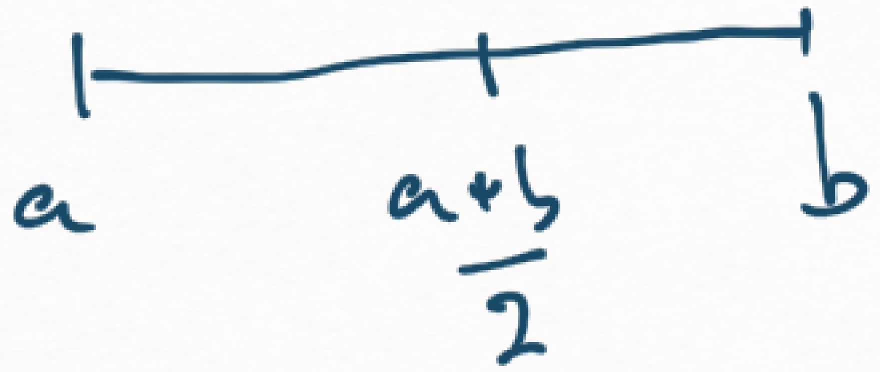
$$F_X(x^*) = s \iff F_X(x^*) - s = 0$$

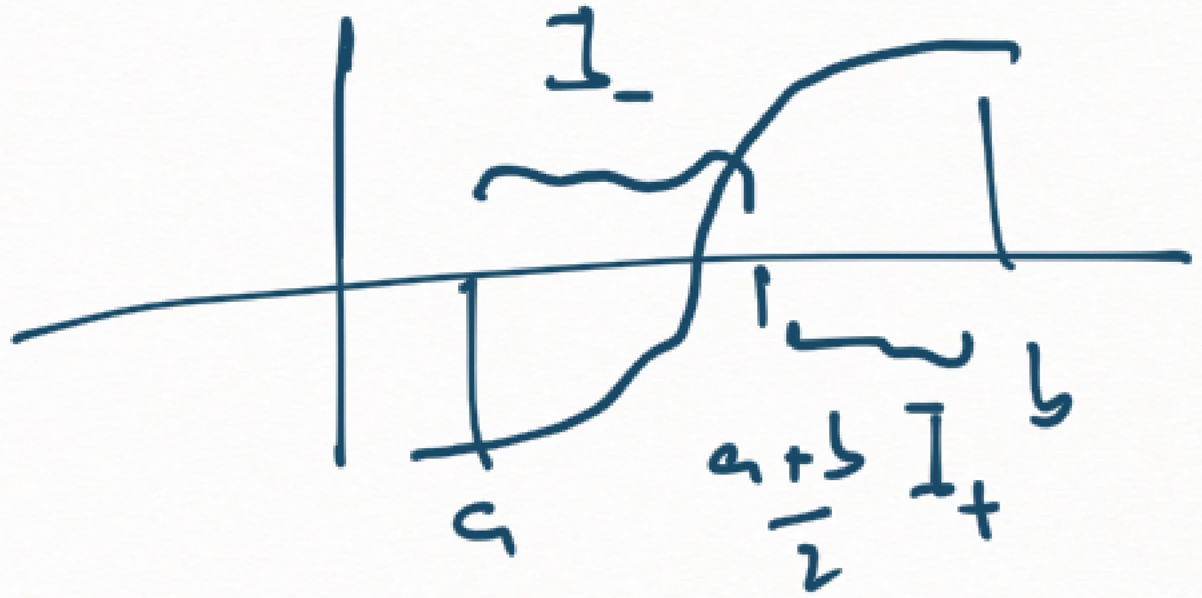


$F_X - s$

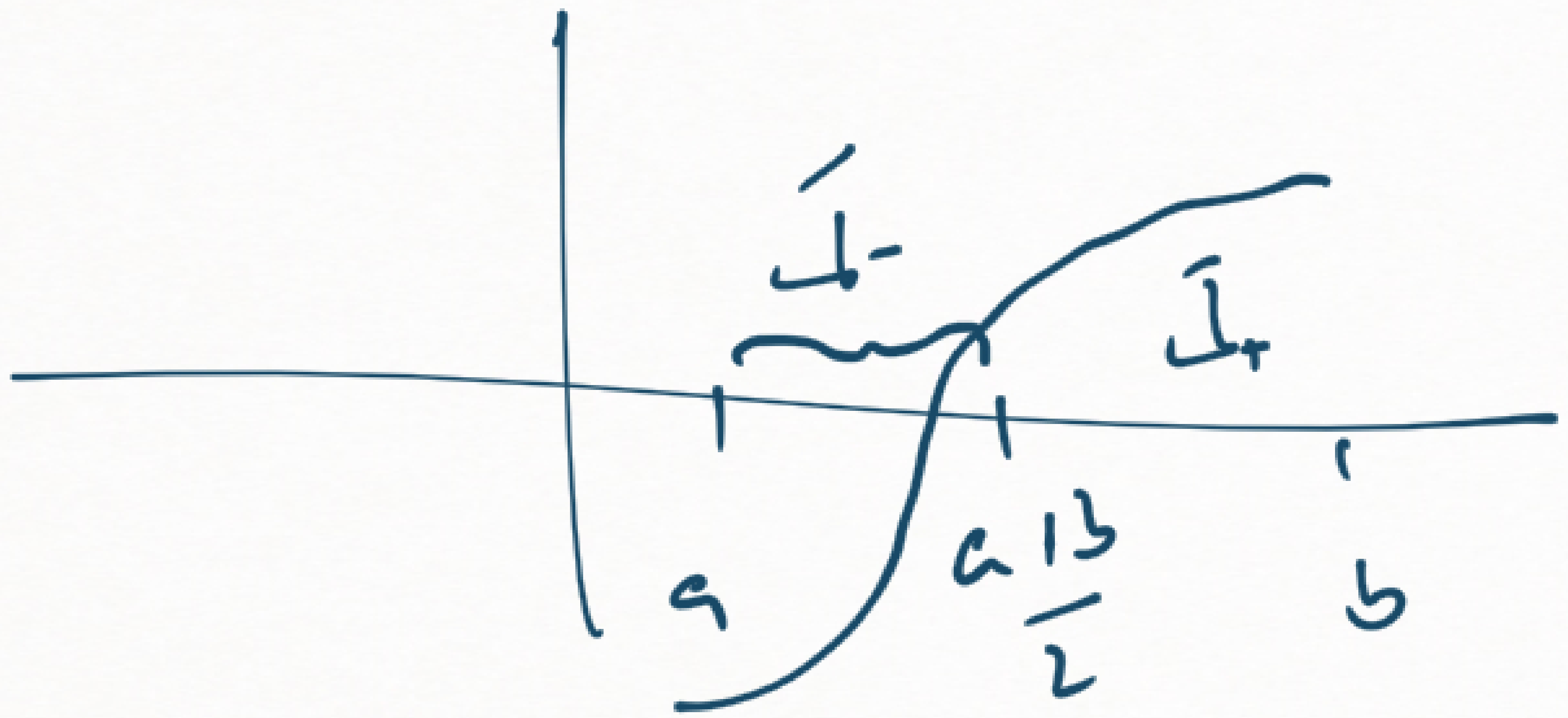


$(F_X(a)-s)(F_X(b)-s) < 0$ Sms $\exists\ x^* \in [a, b)$ T.~. $F_X(x^*)-s=0$





$f = F_x - s$



(1) $f(a)\,f\left(\frac{a+b}{2}\right) < 0$ Υπάρχει σημείο μηδενισμού στ. $I_- = \left(a, \frac{a+b}{2}\right)$

(2), ή $f\left(\frac{a+b}{2}\right) f(b) < 0$ " $I_+ = \left(\frac{a+b}{2}, b\right)$

Αν (1) $I_- = \left(a, \frac{a+b}{2}\right)$ Έχω εγκλωβίσει τ. x^* σ. I_- $\rightsquigarrow$ μεγαλύτερη ακρίβεια από πριν

Αν (2) $I_+ = \left(\frac{a+b}{2}, b\right)$ " I_+ "

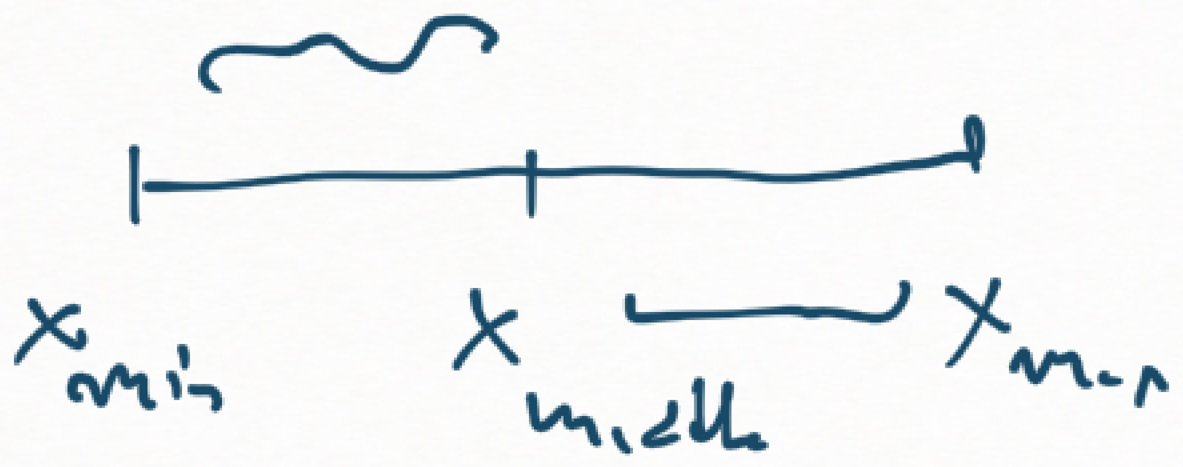
Αν (1)

I_- Το χωρίζω στην μέση →

Αν (2)

I_+ " →

Η μέθοδος της διχοτόμου Bisection Method.



0. $x_{min}=a$ $\quad x_{max}=b$ $\quad x_{middle}=\frac{a+b}{2}$

1. Το σημείο μηδενισμού είναι σε $\quad I_- = (x_{min}, x_{middle})$ $\quad$ (1a)

ή $\quad$ " $\quad I_+ = (x_{middle}, x_{max})$ $\quad$ (1b)

2. Αν (1a) χειρίζομαι μόνο μέσα το I_- $\quad \left.\begin{array}{l} x_{min}=a \to x_{min} \\ x_{max}=x_{middle} \end{array}\right\}$ $\quad x_{middle}=\frac{x_{min}+x_{max}}{2}$

Αν (1b) $\quad$ " $\quad I_+$ $\quad \left.\begin{array}{l} x_{min}=x_{middle} \\ x_{max}=x_{max} \end{array}\right\}$ $\quad$ "

Η μέθοδος της αντιστροφής

F_X συνάρτηση κατανομής

$U \sim \mathcal{U}[0,1]$ δηλ. $P(U \leq u) = u \quad \forall u \in [0,1]$

$Z = F_X^{-1}(U)$ $\qquad P(Z \leq z) = P(F_X^{-1}(U) \leq z) = P(U \leq F_X(z)) = F_X(z)$

$\downarrow$
μονότονη

δηλ. — $U \sim \mathcal{U}[0,1] \Rightarrow Z = F_X^{-1}(U) \sim F_X$

$$\boxed{U \sim \mathcal{U}[0,1]}$$

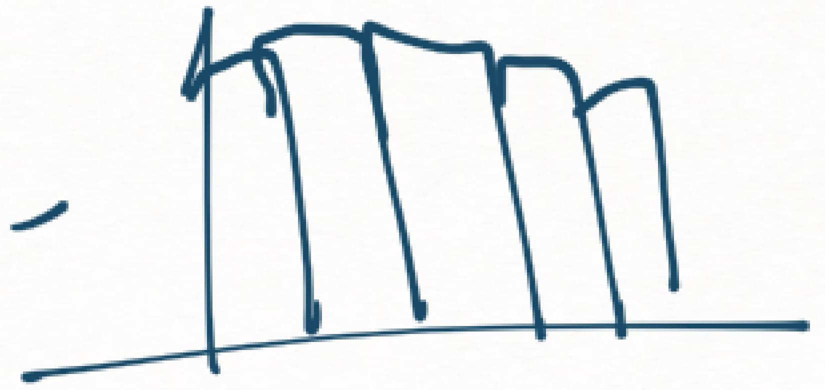
$U \sim \mathcal{U}[0,1]$ ψευδοτυχαίοι

$x_0 \qquad x_{n+1} = \varphi(x_n)$ φ υπολογίσιμη

$x_N = U$

$x_0 + \epsilon \qquad x_{n+1} = \varphi(x_n)$

$x'_N = U'$



x_N

Chaos

ευαίσθητο στις αρχικές συνθήκες

$$\mathbb{E}[X] = \int_0^1 x \underbrace{f(x)}_{\text{pdf}} dx = 1/2 \simeq \frac{1}{N}\sum_{i=1}^{N} x_i$$

$f(x) = 1$

$$Var(X) = \mathbb{E}\left[(X - E[X])^2\right] = \int_0^1 (x - 1/2)^2 \underbrace{f(x)}_{\text{pdf}} dx = 1/12 \simeq \frac{1}{N}\sum_{i=1}^{N} (x_i - E[X])^2$$

$f(x) = 1$

Εκθετική

$$F_X(x) = 1 - e^{-\lambda x}$$

$$s = F_X(x) = 1 - e^{-\lambda x} \iff 1 - s = e^{-\lambda x}$$

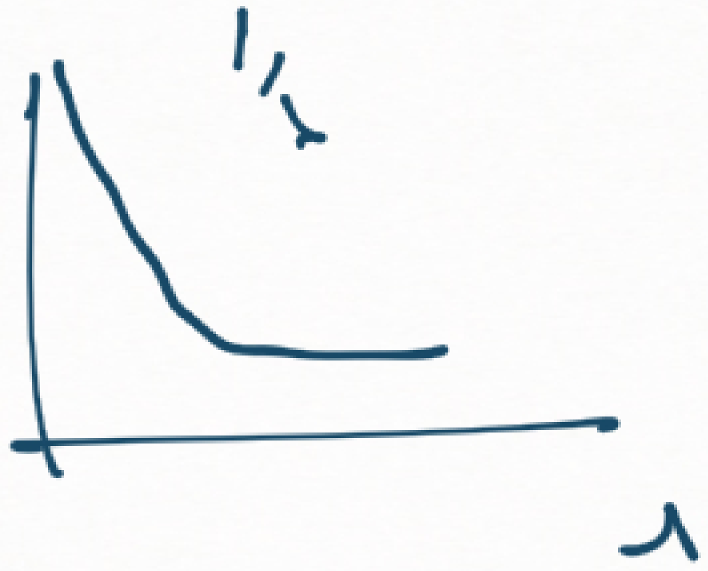
$$\iff \ln(1-s) = -\lambda x \iff x = -\frac{1}{\lambda}\ln(1-s)$$

$$F_X^{-1}(s) = -\frac{1}{\lambda}\ln(1-s)$$

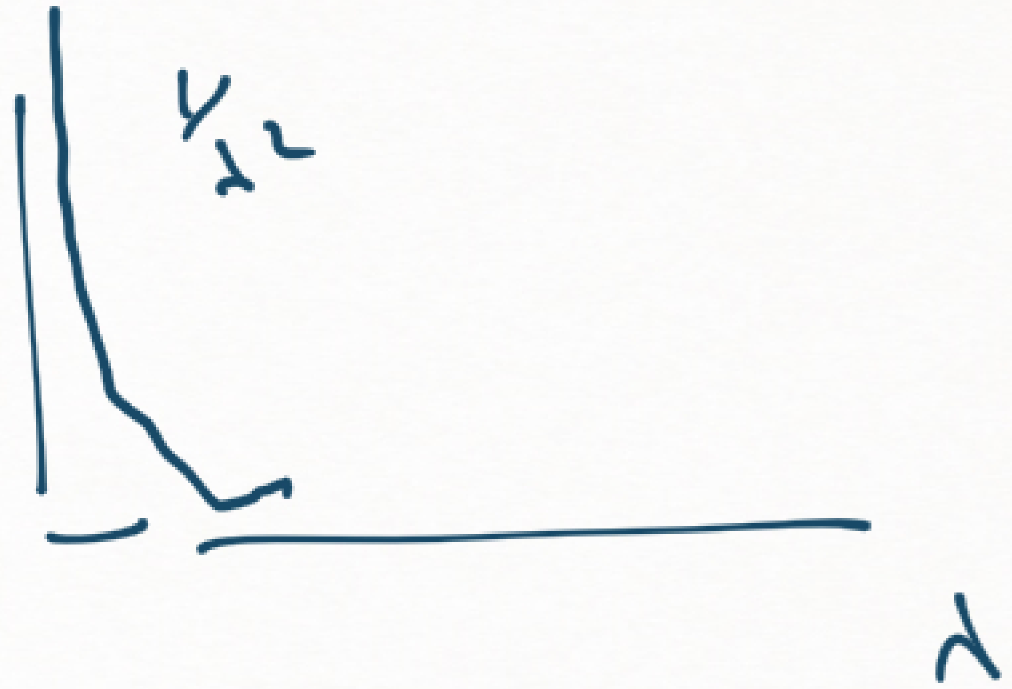
$$\mathbb{E}[X] = 1/\lambda$$

$$Var(X) = 1/\lambda^2$$

$\mathbb{E}[X]$



$\cancel{\mathbb{E}[X^2]}$ $Var(X)$.



$U_1, U_2 \sim U(0,1)$

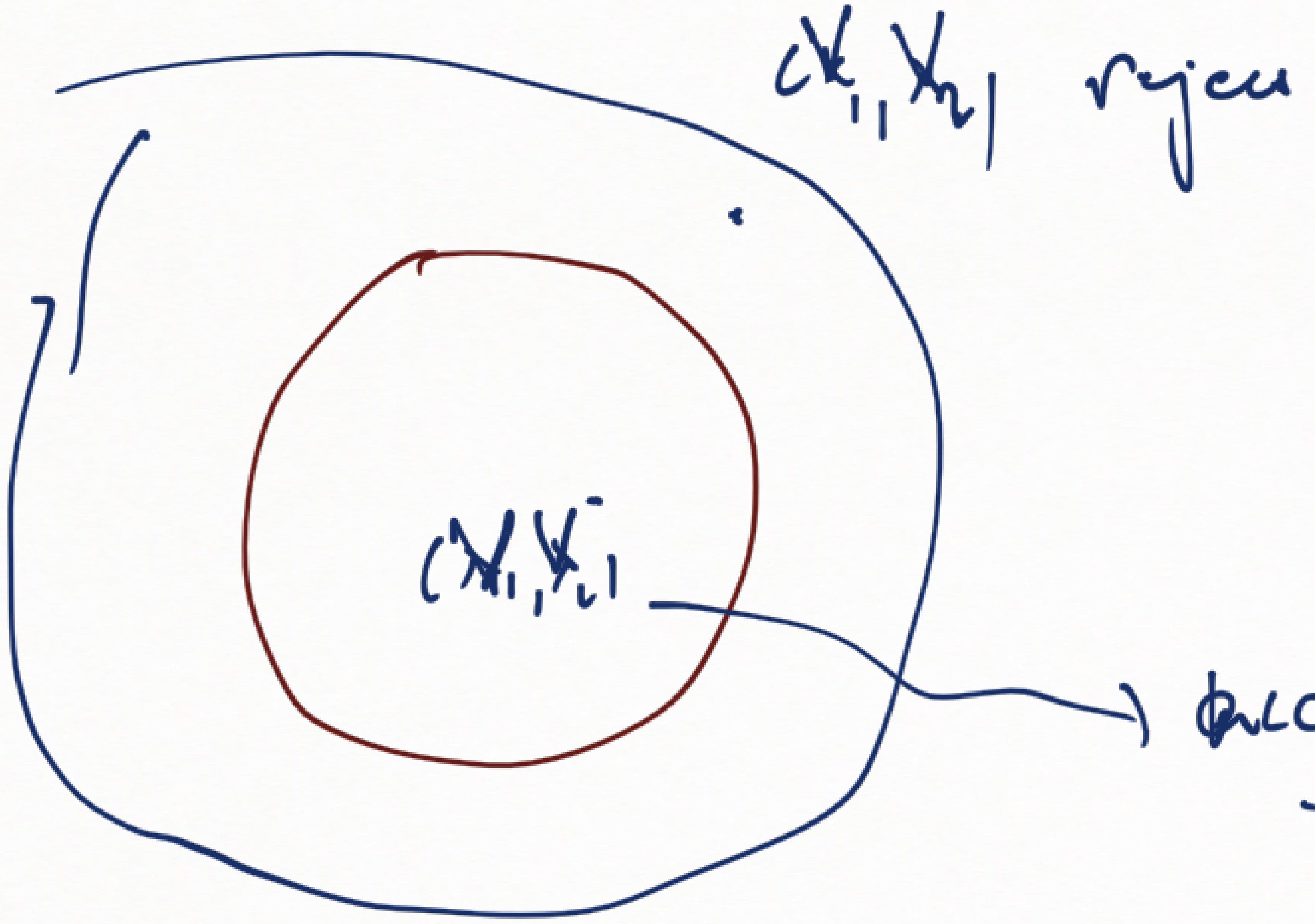
$V_1 = 2U_1 - 1$

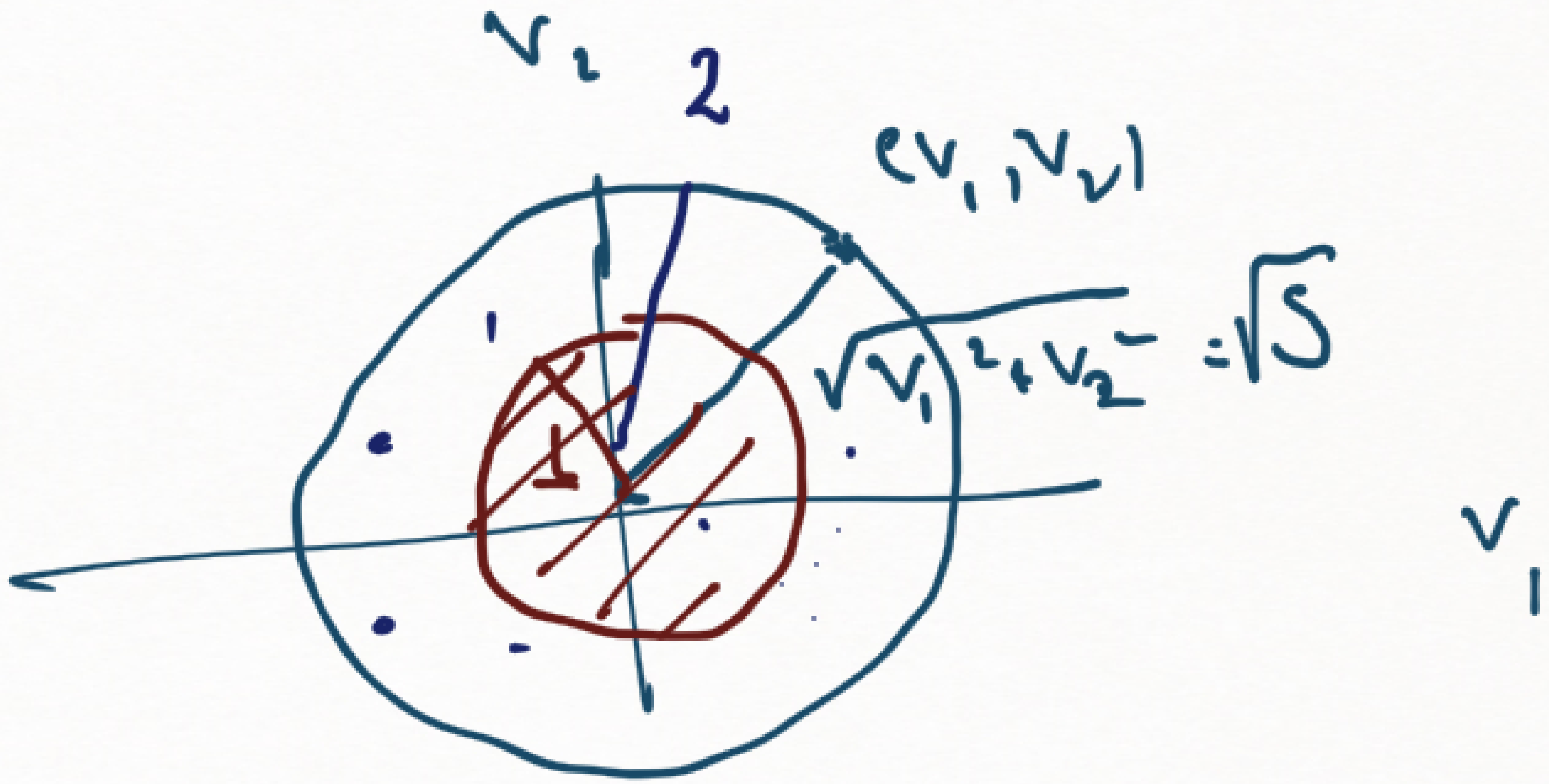
$V_2 = 2U_2 - 1$

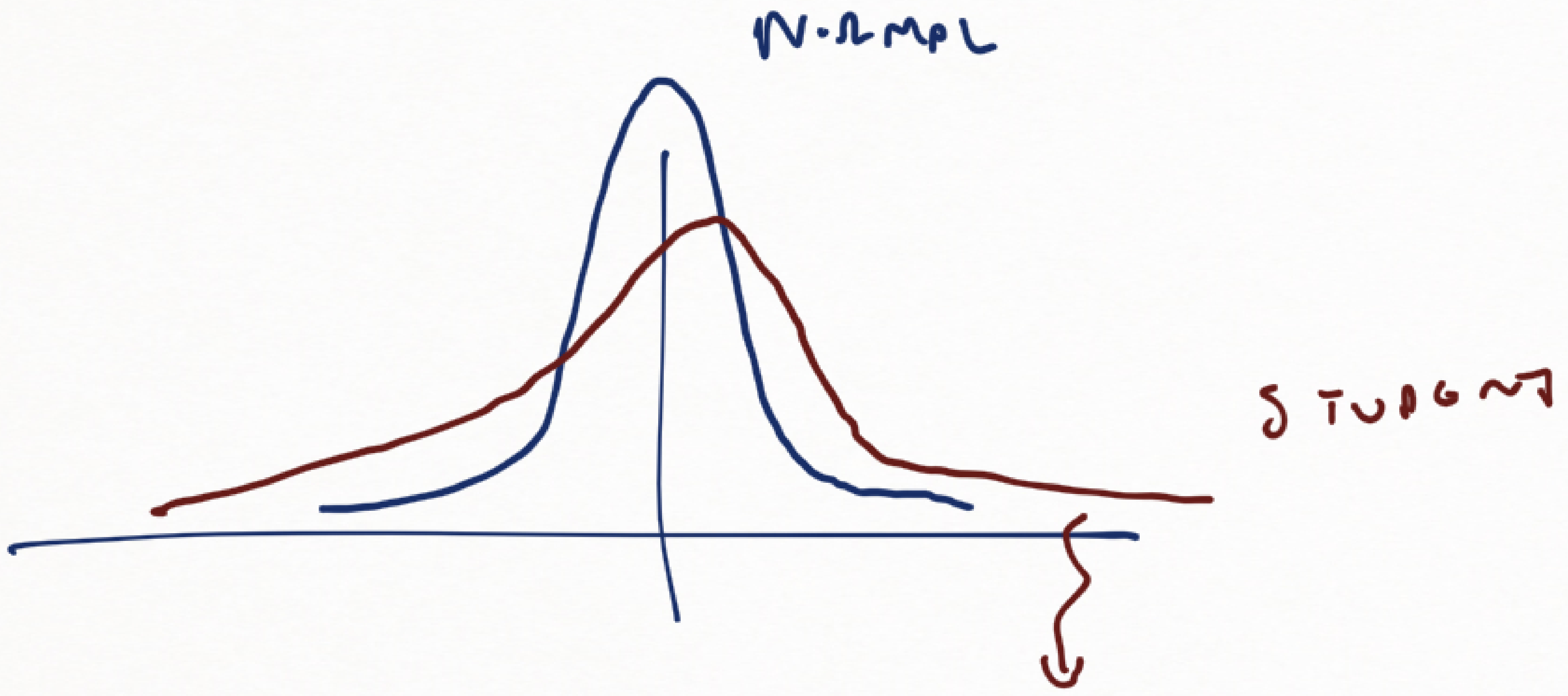
0 1

-1 $+1$

$S = V_1^2 + V_2^2$





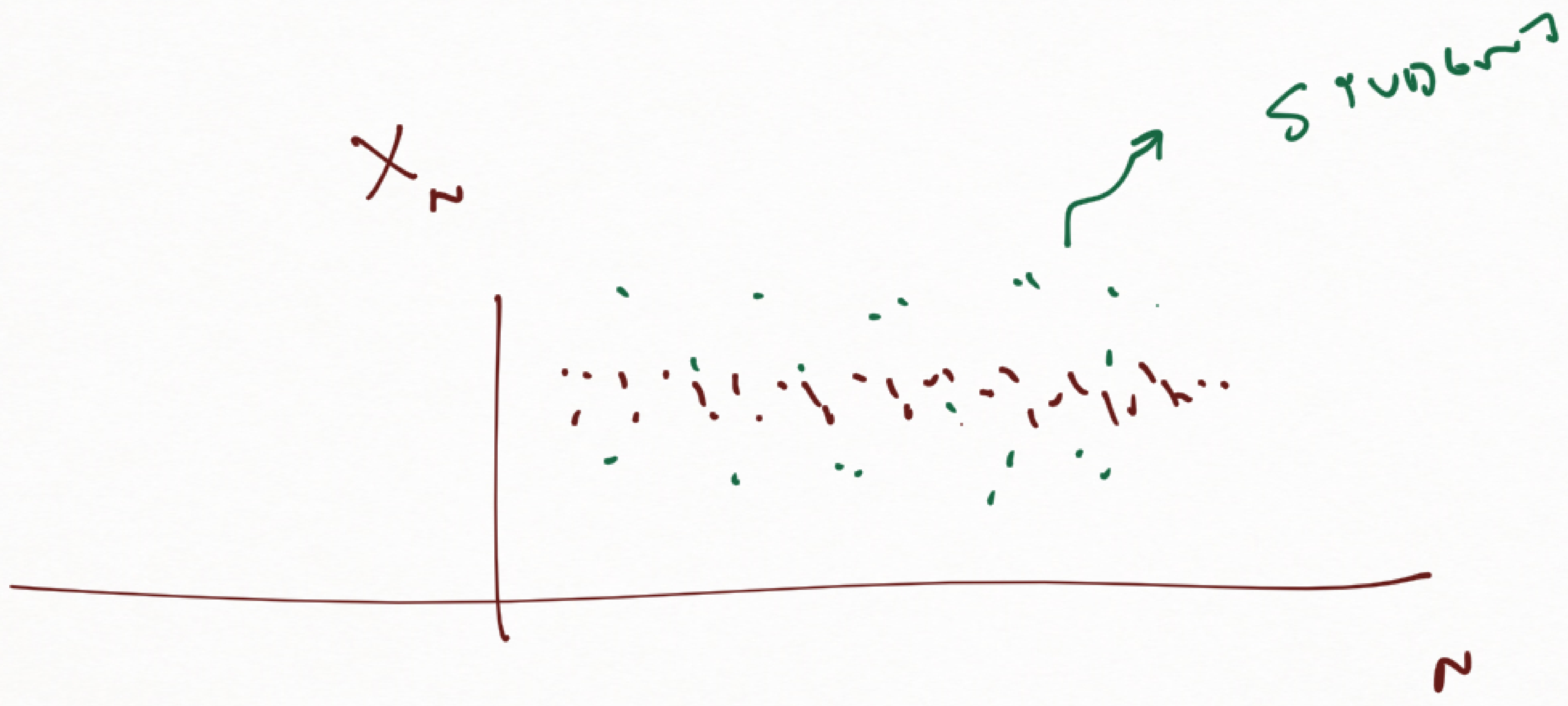


ΠΙΟ ΑΡΓΗ ΜΕΙΩΣΗ → ΜΙΑ ΠΙΟ ΒΑΡΙΑ ΟΥΡΑ.

→ ΜΕΓΑΛΥΤΕΡΗ ΠΙΘΑΝΟΤΗΤΑ ΓΙΑ ΠΑΡΑΤΗΡΗΣΕΙΣ ΜΑΚΡΙΑ ΑΠΟ ΤΗΝ ΜΕΣΗ

Όσο το ν μεγαλώνει τόσο πιο <u>λεπτή</u> γίνεται η ουρά

$\nu \to \infty \qquad t_\nu \to N(0,1)$


Students
X_N
N

Black - Scholes

$S(t)$ τιμή μιας μετοχής την χρονική στιγμή t

Οι λογαριθμικές αποδόσεις

$$R(t) = \ln\left(\frac{S(t)}{S(0)}\right) \sim N(\mu t, \sigma^2 t) \qquad t > 0$$

$P(t) \longrightarrow$ τιμή παραγώγου

$$[x(1) \quad x(2) \quad x(3) \quad \cdots \quad x(t)]$$

$.\mathrm{diff}()$

$$[x(1) - x(0), \quad x(2) - x(1), \quad \cdots$$

$$\log x = [\log x(1) \quad \log(x_2) \quad \log(x_3) \quad \cdots \quad \log(x_t)]$$

$$\log x.\mathrm{diff}() = [\underset{\text{NAN}}{\downarrow}, \quad \log(x_2) - \log(x_1) \quad \log(x_3) - \log(x_2), \quad \cdots, \quad \log(x_t) - \log(x_{t-1})]$$

$$\log(x_t) - \log(x_{t-1}) = \log(x_t / x_{t-1})$$

$$R(t) = \ln\left(\frac{S(t+1)}{S(t)}\right) = X_t \sim F$$

$X_1, X_2, \ldots, X_t$ [illegible]

$$\ln \frac{S(t)}{S(0)} = \ln\left[\frac{S(t)}{S(t-1)} \frac{S(t-1)}{S(t-2)} \cdots \frac{S(2)}{S(1)} \frac{S(1)}{S(0)}\right] =$$

$$\ln\left[\frac{S(t)}{S(t-1)}\right] + \ln\left[\frac{S(t-1)}{S(t-2)}\right] + \ldots + \ln\left[\frac{S(2)}{S(1)}\right] + \ln\left[\frac{S(1)}{S(0)}\right]$$

$$= X_t + X_{t-1} + \ldots + X_2 + X_1$$

[illegible]

[illegible]

Ķ.Ο.Θ.

$$Z = \frac{\sum_{i=1}^{n} X_i - n\overbrace{E[X_1]}^{\mu}}{\sqrt{n}\,\underbrace{\sqrt{Var(X_1)}}_{\sigma}} \sim N(0,1)$$

$$n\mu + \sqrt{n}\,\sigma Z = \sum_{i=1}^{n} X_i$$

$Z \sim N(0,1)$ $\qquad n\mu + \sqrt{n}\,\sigma Z \sim N(n\mu, \sigma^2 n)$

$t = n$ $\Rightarrow$

$$\ln(S(t)/S(0)) \sim N(t\mu, \underbrace{\sigma^2}_{\text{volatility}} t)$$

$$\ln\left(\frac{S(t)}{S(0)}\right) \sim N(\mu t, \sigma^2 t)$$

$$\ln\left(\frac{S(t)}{S(0)}\right) = Z_t = \mu t + \sigma\sqrt{t}\, Z \quad \Rightarrow \quad S(t) = S(0)\, \underbrace{\exp\left[\mu t + \sigma\sqrt{t}\, Z\right]}$$

$$Z \sim N(0,1)$$

$Z \sim N(0,1)$ $\qquad$ $\alpha Z + \beta Z \sim N(\alpha, \beta^2)$

$t = 0, 1, 2, \ldots$ $\qquad$ for each t

$$\begin{cases} ① & Z \sim N(0,1) \\ ② & \mu t + \sigma\sqrt{t}\, Z = Z_t \\ ③ & S(t) = S(0)\exp(Z_t) \end{cases}$$