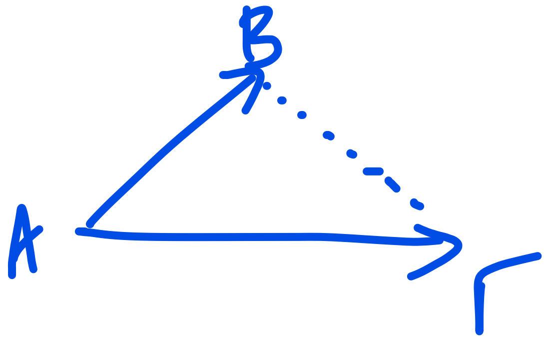
18 Μαρτ.

ΟΡΙΖΟΥΣΕΣ :-

(determinant)



= αριθμός $\in \mathbb{R}$.

$$\boxed{A^{n\times n}}$$

$\boxed{\det(A)}$ Απεικόνιση: $A^{n\times n} \longrightarrow \mathbb{R}$

Ιδιότητες: $\det(I_n) = 1$

$A^{n\times n}$ Αν $\det(A) = 0 \Leftrightarrow A^{-1}$ δεν υπάρχει.

Υπολογισμός:

$\boxed{A}$ 2×2 :

$$A = \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix} \qquad \det(A) = 12 - 15$$

$$= -3$$

$$\boxed{A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow A^{-1} = \frac{1}{\underbrace{\det(A)}_{\neq 0}} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}$$

$$\left(A_v \quad \det(A) = 0 \; ? \quad \cancel{A^{-1}} \qquad A^{+} \right)$$

$n\times 2$

$$A = \begin{bmatrix} 3 & 12 \\ 6 & 5 \end{bmatrix} \quad \det(A) = 15 - 72 = -57$$

3×3 Sarrus

$n\times 1$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ -1 & -2 & 1 \end{bmatrix}$$

$\det(A)$?

$$\begin{array}{ccc:cc} + & + & + & & \\ 1 & 2 & 3 & 1 & 2 \\ 0 & 1 & 4 & 0 & 1 \\ -1 & -2 & 1 & -1 & -2 \\ - & - & - & & \end{array}$$

$$1 - \cancel{8} + 0 + 3 + \cancel{8} + 0 = \underline{\underline{4}}$$

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nx2

$$\begin{bmatrix} 3 & 5 & -2 \\ 1 & 6 & -1 \\ 4 & 2 & -3 \end{bmatrix} = B$$

$\det(B)$?

$$\begin{array}{ccccc} + & + & + & & \\ 3 & 5 & -2 & 3 & 5 \\ 1 & 6 & -1 & 1 & 6 \\ 4 & 2 & -3 & 4 & 2 \\ - & - & - & & \end{array}$$

$$-54-20-4+48+6+15=-9$$

1x3

$$\begin{array}{ccccc}
+ & + & + & & \\
1 & 2 & 3 & 1 & 2 \\
3 & 4 & 5 & 3 & 4 \\
1 & 1 & 2 & 1 & 1 \\
- & - & - & &
\end{array}$$

$$8+10+9-12-5-12$$

$$27-29=-2$$

$$\begin{array}{ccccc}
2 & -4 & 1 & 2 & -4 \\
6 & 5 & 0 & 6 & 5 \\
3 & -1 & 3 & 3 & -1
\end{array}$$

$$30-6-15+72=81$$

Τυχαίο 3x3 πίνακα: Ανάπτυγμα της ορίζουσας

κατά γραμμή ή στήλη:

$$\begin{array}{ccc} + & - & + \\ \end{array}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$+1 \cdot \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} - 2 \cdot \begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3 \cdot \begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix}$$

(υποορίζουσα)

$$1 \cdot (45 - 48) - 2(36 - 42) + 3 \cdot (32 - 35)$$

$$-3 + 12 - 9 = 0$$

4x4:

$$\begin{array}{cccc} + & - & + & - \\ \end{array}$$
$$\begin{vmatrix} 3 & 2 & -1 & 5 \\ 4 & 0 & 6 & 2 \\ 1 & 2 & 3 & -4 \\ 0 & -2 & 5 & 9 \end{vmatrix} = r$$

$$+3\begin{vmatrix} 0 & 6 & 2 \\ 2 & 3 & -4 \\ -2 & 5 & 9 \end{vmatrix} - 2\begin{vmatrix} 4 & 6 & 2 \\ 1 & 3 & -4 \\ 0 & 5 & 9 \end{vmatrix} - 1\begin{vmatrix} 4 & 0 & 2 \\ 1 & 2 & -4 \\ 0 & -2 & 9 \end{vmatrix} - 5\begin{vmatrix} 4 & 0 & 6 \\ 1 & 2 & 3 \\ 0 & -2 & 5 \end{vmatrix}$$

$\boxed{\text{Sarrus 4 φορές}}$

$$\ldots\ldots = \underline{-668}$$

$\det(A)$ ή $|A|$

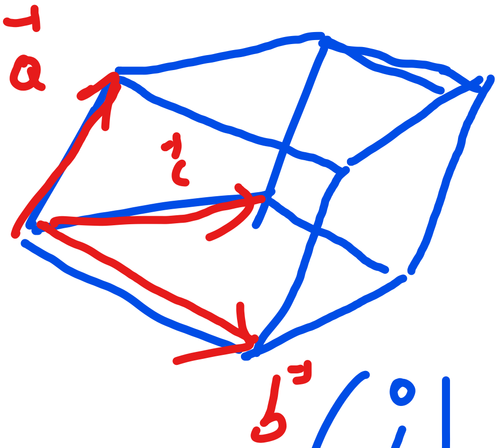
$A : (2\times 2) \longrightarrow |\det(A)| = $ Εμβαδόν

$\begin{pmatrix}1\\2\end{pmatrix}, \quad \begin{pmatrix}3\\4\end{pmatrix}$

$$\begin{vmatrix}1 & 2\\ 3 & 4\end{vmatrix} = -2$$

$A^{3\times 3}: \ |\det(A)| = $ Όγκος

$\vec{a} = \begin{pmatrix}1\\2\\3\end{pmatrix}$



$\vec{b} = \begin{pmatrix}0\\1\\4\end{pmatrix}$

$\vec{c} = \begin{pmatrix}-1\\-2\\1\end{pmatrix}$

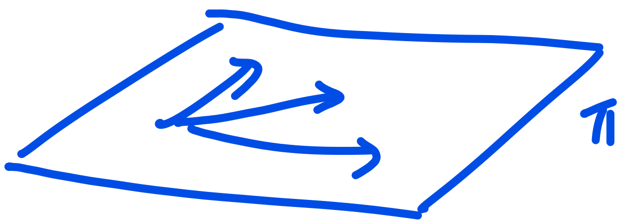
$$V = \left\| \begin{matrix}1 & 2 & 3\\ 0 & 1 & 4\\ -1 & -2 & 1\end{matrix} \right\| = 2$$

$\vec{a}, \ \vec{b}, \ \vec{c}$

$$\begin{vmatrix}\cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot\end{vmatrix} = 0 \quad ?$$

$$\Rightarrow V = 0 \Rightarrow ?$$

ΣΥΝΕΠΙΠΕΔΑ



ΙΔΙΟΤΗΤΕΣ:

- $\det(A) = \det(A^T)$

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = -2 \quad , \quad \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = -2$$

- Αν $A =$ διαγώνιος ή τριγωνικός

$\det(A) =$ γινόμενο διαγωνίου.

π.χ. $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ $\quad \det(B) = -6$

$\Gamma = \begin{bmatrix} 3 & 4 & 5 \\ 0 & 2 & 6 \\ 0 & 0 & -4 \end{bmatrix}$ $\quad \det(\Gamma) = -24$

$\bullet\ \det(AB) = \det(A)\cdot\det(B)$

$(AB \neq BA\ ?)$ $= \det(B)\cdot\det(A)$

$A = LU$
$A = QR$

Άρα $A\cdot A^{-1} = I \Rightarrow \det(A\cdot A^{-1}) = \det I$

$\rightarrow \det(A)\cdot\det(A^{-1}) = 1 \Rightarrow$

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

$\bullet$ 7 από pdf

$k\cdot$ γραμμή $\rightarrow k\cdot\det(A)$

$$\left|\begin{bmatrix} 3 & 5 \\ 8 & 9 \end{bmatrix}\right| = -13, \quad \left|\begin{bmatrix} 6 & 10 \\ 8 & 9 \end{bmatrix}\right| = -26$$

• $\det(A^k) = [\det(A)]^k$

$\det(A^k) = \det(A \cdot A \cdot A \ldots A) =$
$\det(A) \cdot \det(A) \cdot \det(A) \ldots \det(A) =$
$[\det(A)]^k$

• Αν γραμμή ή στήλη $= 0 \Rightarrow$
$\det(A) = 0$

• Αν αλλάξω θέση 2 γραμμές στον A
$\longrightarrow$ νέος: A'

$$\det(A') = -\det(A)$$