

**ΟΙΚΟΝΟΜΙΚΟ  
ΠΑΝΕΠΙΣΤΗΜΙΟ  
ΑΘΗΝΩΝ**



ATHENS UNIVERSITY  
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# ECONOMETRICS

## Panel Data Models

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# Panel Data Models

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# Panel data models

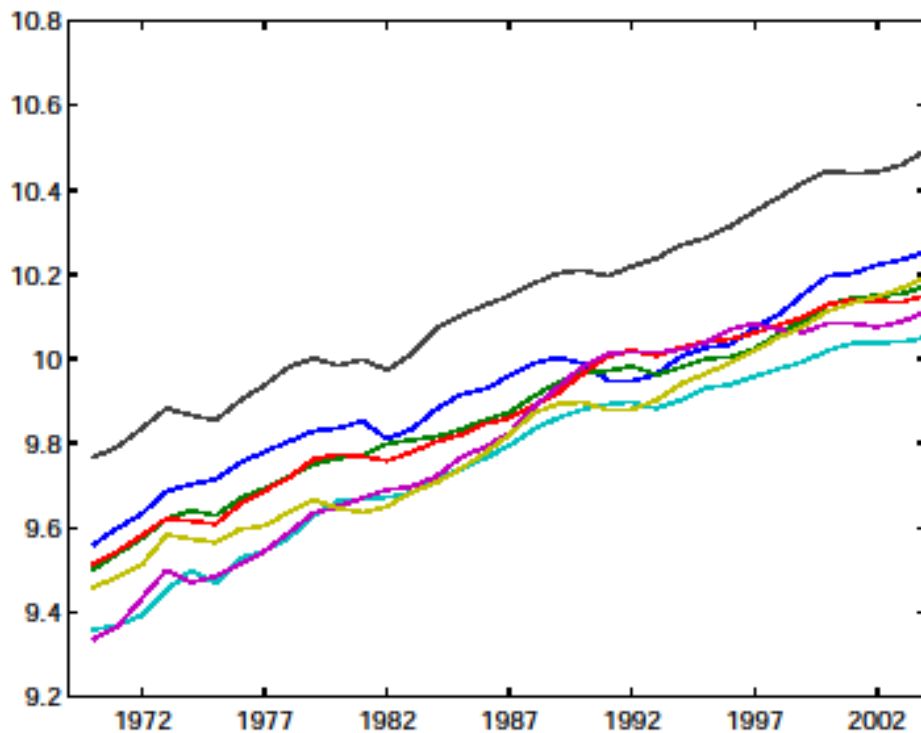
## Introduction and motivation

Advantages:

- Larger number of data points (more degrees of freedom)
- Combine cross section and time series data
- Limitation of the omitted variable problem
- Takes into account common and cross sectional explanatory variables
- May account for cross sectional dependence

# Panel data

## GDP series for the G7 countries



# Panel data

## Dependent variables and Specific cross sectional explanatory variables

- Panel data:

**Dependent variable:**  $Y_{it}$ ,  $i = 1, \dots, N$  (cross section unit)  
 $t = 1, \dots, T$  (time period)

**Explanatory variables** (specific cross sectional unit variables, i.e. different for each unit)

$X_{ijt}$ ,  $i = 1, \dots, N$  (cross section unit)  
 $j = 1, \dots, k$  (explanatory variables)  
 $t = 1, \dots, T$  (time period)

E.g.:  $Y_{it}$ : GDP for different countries across time,  $X_{ijt}$ : investments, industrial production, unemployment for different countries across time

Y <sub>1</sub>	Y <sub>2</sub>	...	Y <sub>N</sub>	X <sub>11</sub>	X <sub>12</sub>	...	X <sub>1k</sub>	...	X <sub>N1</sub>	X <sub>N2</sub>	...	X <sub>Nk</sub>
Y <sub>11</sub>	Y <sub>21</sub>	...	Y <sub>N1</sub>	X <sub>111</sub>	X <sub>121</sub>	...	X <sub>1k1</sub>	...	X <sub>N11</sub>	X <sub>N21</sub>	...	X <sub>Nk1</sub>
Y <sub>12</sub>	Y <sub>22</sub>	...	Y <sub>N2</sub>	X <sub>112</sub>	X <sub>122</sub>	...	X <sub>1k2</sub>	...	X <sub>N12</sub>	X <sub>N22</sub>	...	X <sub>Nk2</sub>
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	...	⋮	⋮	⋮	⋮
Y <sub>1T</sub>	Y <sub>2T</sub>	...	Y <sub>NT</sub>	X <sub>11T</sub>	X <sub>12T</sub>	...	X <sub>1kT</sub>	...	X <sub>N1T</sub>	X <sub>N2T</sub>	...	X <sub>NkT</sub>

# Panel data

## Dependent variables and Common explanatory variables

- Panel data:

**Dependent variable:**  $Y_{it}$ ,  $i = 1, \dots, N$  (cross section unit)  
 $t = 1, \dots, T$  (time period)

**Explanatory variables** (common variables, i.e. common for each unit)

$X_{jt}$ ,  $j = 1, \dots, k$  (explanatory variables)  
 $t = 1, \dots, T$  (time period)

E.g.:  $Y_{it}$ : GDP for different countries across time,  $X_{jt}$ : global volatility index, global panic index across time (common explanatory variables for all units)

$Y_1$	$Y_2$	...	$Y_N$	$X_1$	$X_2$	...	$X_k$
$Y_{11}$	$Y_{21}$	...	$Y_{N1}$	$X_{11}$	$X_{21}$	...	$X_{k1}$
$Y_{12}$	$Y_{22}$	...	$Y_{N2}$	$X_{12}$	$X_{22}$	...	$X_{k2}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$Y_{1T}$	$Y_{2T}$	...	$Y_{NT}$	$X_{1T}$	$X_{2T}$	...	$X_{kT}$

# Panel data models

## A general dynamic panel data model

- Consider the following dynamic panel data model:

$$Y_{it} = \delta_i + \beta_i t + \sum_{k=1}^K \alpha_{ik} X_{kt} + \sum_{q=1}^Q \gamma_{iq} X_{iqt} + \varepsilon_{it},$$
$$\varepsilon_{it} = \varphi_i \varepsilon_{i,t-1} + u_{it},$$

where

- $i = 1, \dots, N$  denotes the cross-sectional units of the panel (e.g. countries)
- $t = 1, \dots, T$  denotes the time period
- $Y_{it}$  denotes the economic or financial dependent variable (e.g. gross domestic product)
- $X_{kt}$  is a set of  $K$  exogenous explanatory factors or covariates (e.g. a global volatility index, a market or a commodity index) which are common for all cross-sectional units of the panel, and affect the dependent variables of the panel through separate/different coefficients  $\alpha_{ik}$
- $X_{iqt}$  is a set of  $Q$  of cross-sectional specific factors (explanatory variables) which are different for each cross-sectional unit with separate coefficients  $\gamma_{iq}$
- $\delta_i$  are the intercept coefficients of the model, which are different for each cross sectional unit
- $\beta_i$  are the trend coefficients, which are different for each cross sectional unit
- $\varepsilon_{it}$  is a zero-mean autoregressive one, AR(1), process which captures the dynamics of the panel data model. The error term  $u_{it}$  is assumed not to be serially correlated, i.e.  $E(u_{is}u_{it}) = 0$ , for all  $t \neq s$  and for all  $i$ , but it is heterogeneous and correlated across  $i$ , i.e.  $E(u_{it}u_{jt}) \neq 0$  for all  $i$  and  $j$ , and therefore, allows for cross sectional dependence across units

# Panel data models

## Basic linear panel data model

- The **basic linear panel data model** used in econometric literature can be described through suitable restrictions of the following general model:

$$Y_{it} = \alpha_{it} + \beta_{it}X_{it} + u_{it},$$

where

- $i = 1, \dots, N$  denotes the cross-sectional units of the panel
- $t = 1, \dots, T$  denotes the time period
- $Y_{it}$  denotes the economic or financial dependent variable
- $X_{it}$  is the explanatory factors or covariates
- $\alpha_{it}$  is the intercept, which is different for each cross sectional unit, and across time
- $\beta_{it}$  are the explanatory variables coefficients, which are different for each cross sectional unit, and across time
- $u_{it}$  is the error term
  
- Number of observations:  $NT$
- Number of parameters to be estimated:  $2NT$
- Can not be estimated !! Thus, **suitable restrictions are imposed** on  $\alpha_{it}$  and  $\beta_{it}$



# Panel data models

## A simple panel data model (model I)

- The simplest panel data model assumes parameter homogeneity. In the general linear panel model  $Y_{it} = \alpha_{it} + \beta_{it}X_{it} + u_{it}$  impose the following restrictions:  $\alpha_{it} = \alpha$  and  $\beta_{it} = \beta$  for all  $i, t$ . The resulting model can be written in the form:

$$Y_{it} = a + \beta X_{it} + u_{it},$$

where

- $i = 1, \dots, N$  denotes the cross-sectional units of the panel
- $t = 1, \dots, T$  denotes the time period
- $Y_{it}$  denotes the economic or financial dependent variable
- $X_{it}$  is the explanatory factors or covariate
- $a$  is the intercept of the model, which is common for all cross sectional units and across time
- $\beta$  is the explanatory variable coefficient, which is common for all cross sectional units across time
- $u_{it}$  is the error term
  
- This simple panel model does not account for the heterogeneity of the cross sectional units (common  $a$  and  $\beta$  parameter)
- Number of observations:  $NT$
- Number of parameters to be estimated: 2

# Panel data models

## Fixed effects panel data model (model II)

- In the general linear panel data model  $Y_{it} = \alpha_{it} + \beta_{it}X_{it} + u_{it}$  impose the following restrictions:  $\alpha_{it} = \alpha_i$  for all  $t$ , and  $\beta_{it} = \beta$  for all  $i$  and  $t$ . The resulting model is called the **fixed effects model**:

$$Y_{it} = \alpha_i + \beta X_{it} + u_{it},$$

where

- $i = 1, \dots, N$  denotes the cross-sectional units of the panel
- $t = 1, \dots, T$  denotes the time period
- $Y_{it}$  denotes the economic or financial dependent variable
- $X_{it}$  is the explanatory factors or covariate
- $\alpha_i$  is the intercept of the model, which is different for each cross sectional unit
- $\beta$  is the slope, which is common for all cross sectional units
- $u_{it}$  is the error term
  
- This panel model accounts for the heterogeneity of the cross sectional units (different intercept coefficients  $\alpha_i$ , common  $\beta$  parameter)
- Number of observations:  $NT$
- Number of parameters to be estimated:  $N+1$

# Panel data models

## Fixed effects panel data model (model III)

- In the general linear panel data model  $Y_{it} = \alpha_{it} + \beta_{it}X_{it} + u_{it}$  impose the following restrictions:  $\alpha_{it} = \alpha_i$ , and  $\beta_{it} = \beta_i$  for all  $t$ . The resulting model is:

$$Y_{it} = \alpha_i + \beta_i X_{it} + u_{it},$$

- $i = 1, \dots, N$  denotes the cross-sectional units of the panel
- $t = 1, \dots, T$  denotes the time period
- $Y_{it}$  denotes the economic or financial dependent variable
- $X_{it}$  is the explanatory factors or covariate
- $\alpha_i$  is the intercept of the model, which is different for each cross sectional unit
- $\beta_i$  is the slope, which is different for each cross sectional unit
- $u_{it}$  is the error term
  
- This panel model accounts for the heterogeneity of the cross sectional units (different intercept coefficients  $\alpha_i$ , and different  $\beta_i$  coefficients)
- Different parameters across units, but constant across time
- Number of observations:  $NT$
- Number of parameters to be estimated:  $2N$
- To be able to estimate this model:  $T > 2$

# Panel data models: Fixed Effects

## Least Squares Dummy variables (LSDV)

- Consider the following fixed effects panel data model:

$$Y_{it} = \alpha_i + \beta X_{it} + u_{it}$$

- Fixed parameters  $\alpha_i$  (non stochastic), different for each sectional unit, common  $\beta$
- We are interested in:
  - Estimate the model parameters
  - Perform hypothesis testing (for the heterogeneity of intercept parameters)
- Re-write the model by using/constructing appropriate dummy variables

$$Y_{it} = \alpha + \beta X_{it} + \gamma_2 d_{2t} + \gamma_3 d_{3t} + \cdots + \gamma_N d_{Nt} + u_{it}$$

- Number of observations:  $NT$ , number of parameters:  $2 + (N - 1) = N + 1$

- The model implies that:

$$i = 1: Y_{1t} = \alpha + \beta X_{1t} + u_{1t}$$

$$i = 2: Y_{2t} = \alpha + \beta X_{2t} + \gamma_2 d_{2t} + u_{2t} \Rightarrow Y_{2t} = (\alpha + \gamma_2) + \beta X_{2t} + u_{2t}$$

$$i = 3: Y_{3t} = \alpha + \beta X_{3t} + \gamma_3 d_{3t} + u_{3t} \Rightarrow Y_{3t} = (\alpha + \gamma_3) + \beta X_{3t} + u_{3t}$$

⋮

$$i = N: Y_{Nt} = \alpha + \beta X_{Nt} + \gamma_N d_{Nt} + u_{Nt} \Rightarrow Y_{Nt} = (\alpha + \gamma_N) + \beta X_{Nt} + u_{Nt}$$

# Panel data models: Fixed Effects

## Construct the dummy variables

$i$	$t$	$Y$	$X$	$D_2$	$D_3$
1	1	$Y_{11}$	$X_{11}$	0	0
	2	$Y_{12}$	$X_{12}$	0	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	$T$	$Y_{1T}$	$X_{1T}$	0	0
2	1	$Y_{21}$	$X_{21}$	1	0
	2	$Y_{22}$	$X_{22}$	1	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	$T$	$Y_{2T}$	$X_{2T}$	1	0
3	1	$Y_{31}$	$X_{31}$	0	1
	2	$Y_{32}$	$X_{32}$	0	1
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	$T$	$Y_{3T}$	$X_{3T}$	0	1

Example: number of units  $N = 3$  (i.e. construct  $N - 1 = 2$  dummy variables), time period  $T$ , dependent variable  $Y_{it}$ , explanatory variable  $X_{it}$

# Panel data models: Fixed Effects

## Hypothesis testing (t-test)

- For the fixed effects panel data model of the form:

$$Y_{it} = \alpha + \beta X_{it} + \gamma_2 d_{2t} + \gamma_3 d_{3t} + \dots + \gamma_N d_{Nt} + u_{it}$$

we can perform two types of hypothesis tests:

- **(i) t-test:**

$$H_0: \gamma_i = 0$$

$$H_1: \gamma_i \neq 0$$

by using the following test statistic:

$$T = \frac{\hat{\gamma}_i}{se(\hat{\gamma}_i)}$$

The hypothesis testing about parameters  $\gamma_i$  is very important, since if we reject the null hypothesis ( $H_0: \gamma_i = 0$ ), this implies that parameter  $\gamma_i$  is statistically significant at level  $\alpha$ , and the corresponding intercept parameter for unit  $i$ , is  $\alpha + \gamma_i$ , and is statistically different than the intercept ( $\alpha$ ) of the baseline cross sectional unit 1.

# Panel data models: Fixed Effects

## Hypothesis testing (F-test)

- (ii) F-test:

$$H_0: \gamma_2 = \gamma_3 = \dots = \gamma_N = 0$$

$$H_1: \text{not } H_0$$

The null hypothesis implies the following **restricted model**:

$$H_0: Y_{it} = \alpha + \beta X_{it} + u_{it}$$

while the alternative hypothesis implies the **unrestricted model**:

$$H_1: Y_{it} = \alpha + \beta X_{it} + \gamma_2 d_{2t} + \gamma_3 d_{3t} + \dots + \gamma_N d_{Nt} + u_{it}$$

- The F-test statistic can be used:

$$F = \frac{(RSS_R - RSS_{Unr}) / (df_R - df_{Unr})}{RSS_{Unr} / df_{Unr}} \sim F_{N-1, NT-N-1}$$

- $RSS_R$  and  $RSS_{Unr}$  are the residual sum of squares of the restricted and the unrestricted model, respectively

- $df_R = NT - 2$  and  $df_{Unr} = NT - 2 - (N - 1) = NT - N - 1$  are the degrees of freedom of the restricted and the unrestricted model, respectively, and  $df_R - df_{Unr} = [NT - 2] - [NT - N - 1] = N - 1$

# Panel data models: Fixed Effects

## Within Estimator

- Consider the following fixed effects panel data model:

$$Y_{it} = \alpha_i + \beta X_{it} + u_{it} \quad (1)$$

- Define

$$\bar{Y}_i = \frac{1}{T} \sum_{t=1}^T Y_{it}, \quad \bar{X}_i = \frac{1}{T} \sum_{t=1}^T X_{it}, \quad \bar{u}_i = \frac{1}{T} \sum_{t=1}^T u_{it}$$

- Then

$$\begin{aligned} Y_{it} &= \alpha_i + \beta X_{it} + u_{it} \Rightarrow \\ \Rightarrow \frac{1}{T} \sum_{t=1}^T Y_{it} &= \frac{1}{T} \sum_{t=1}^T \alpha_i + \frac{1}{T} \beta \sum_{t=1}^T X_{it} + \frac{1}{T} \sum_{t=1}^T u_{it} \Rightarrow \\ &\Rightarrow \bar{Y}_i = \frac{1}{T} T \alpha_i + \beta \bar{X}_i + \bar{u}_i \Rightarrow \\ &\Rightarrow \bar{Y}_i = \alpha_i + \beta \bar{X}_i + \bar{u}_i \quad (2) \end{aligned}$$



# Panel data models: Fixed Effects

## Within Estimator

- By taking the difference (1)-(2) in equations (1) and (2)

$$Y_{it} = \alpha_i + \beta X_{it} + u_{it} \quad (1)$$

and

$$\bar{Y}_i = \alpha_i + \beta \bar{X}_i + \bar{u}_i \quad (2)$$

we obtain:

$$Y_{it} - \bar{Y}_i = \beta(X_{it} - \bar{X}_i) + (u_{it} - \bar{u}_i)$$

$$Y_{it}^* = \beta X_{it}^* + u_{it}^* \quad (3)$$

- **Within Estimator steps:**
  - Apply OLS in equation (3) and estimate  $\beta$ , i.e. obtain  $\hat{\beta}$
  - Using equation (2), estimate  $\alpha_i$ , i.e.  $\hat{\alpha}_i = \bar{Y}_i - \hat{\beta} \bar{X}_i$
  - Computationally easy
  - Can not conduct hypothesis testing

# Panel data models: Fixed Effects

## First-difference Estimator

- Another way of estimating panel data models is by first-differencing the data: lagging the model and subtracting, the time-invariant components are eliminated, and the model

$$\Delta Y_{i,t} = \beta \Delta X_{i,t} + \Delta u_{i,t}$$

can be consistently estimated by pooled OLS. This is called the first-difference estimator

- The differences are defined as follows  $\Delta Y_{i,t} = Y_{i,t} - Y_{i,t-1}$ ,  $\Delta X_{i,t} = X_{i,t} - X_{i,t-1}$ ,  $\Delta u_{i,t} = u_{i,t} - u_{i,t-1}$ , for  $t = 2, \dots, T$
- Its relative efficiency, and so reasons for choosing it against other consistent alternatives, depends on the properties of the error term. The first-difference estimator is usually preferred if the errors  $u_{it}$  are strongly persistent in time, because then the  $\Delta u_{i,t}$  will tend to be serially uncorrelated.

# Panel data models: Random Effects

## The Random effects model

- Consider the following panel data model:

$$Y_{it} = \alpha_i + \beta X_{it} + u_{it} \quad (1),$$

$$u_{it} \sim iid(0, \sigma^2)$$

- Suppose that  $a_i \sim D(a, \omega^2)$ , that is:  $E(a_i) = a$  and  $V(a_i) = \omega^2$
- Then, that  $a_i = a + \mu_i$  (2),  $\mu_i \sim iid D(0, \omega^2)$ , and  $E(\mu_i) = 0$ , and  $V(\mu_i) = \omega^2$
- In this model, we have two parameters ( $a, \omega^2$ ) instead of N parameters ( $a, N - 1$  dummy variables) with respect to the intercept
- The model can be written:

$$\begin{aligned} Y_{it} &= \alpha_i + \beta X_{it} + u_{it} \Rightarrow \\ \Rightarrow Y_{it} &= a + \mu_i + \beta X_{it} + u_{it} \Rightarrow \\ \Rightarrow Y_{it} &= a + \beta X_{it} + (\mu_i + u_{it}) \Rightarrow \\ \Rightarrow Y_{it} &= a + \beta X_{it} + v_{it}, \quad \text{where } v_{it} = \mu_i + u_{it} \end{aligned}$$

# Panel data models: Random Effects

## The Random effects model

- For the process

$$v_{it} = \mu_i + u_{it}$$

we observe that:  $v_{i1} = \mu_i + u_{i1}$ ,  $v_{i2} = \mu_i + u_{i2}$ , ...,  $v_{iT} = \mu_i + u_{iT}$ . Therefore,  $v_{it}$  contains a common component,  $\mu_i$ , across time, and thus  $v_{it}$  is auto-correlated.

- Its mean is: 
$$E(v_{it}) = E(\mu_i + u_{it}) = E(\mu_i) + E(u_{it}) = 0$$

- The variance is:

$$V(v_{it}) = V(\mu_i + u_{it}) = V(\mu_i) + V(u_{it}) = \omega^2 + \sigma^2$$

- The covariance of  $v_{it}$  with  $v_{is}$ ,  $t \neq s$ , is:

$$\text{Cov}(v_{it}, v_{is}) = E(v_{it}v_{is}) = E[(\mu_i + u_{it})(\mu_i + u_{is})] =$$

$$= E[\mu_i^2 + \mu_i u_{is} + u_{it} \mu_i + u_{it} u_{is}] =$$

$$= E[\mu_i^2] = V(\mu_i) = \omega^2 \neq 0, \text{ there is auto-correlation at } v_{it}$$

# Panel data models: Random Effects

## The Random effects model

- Therefore, the covariance matrix of  $\mathbf{v}_i = (v_{i1}, v_{i2}, \dots, v_{iT})'$  is

$$\mathbf{\Omega} = \begin{bmatrix} \omega^2 + \sigma^2 & \omega^2 & \dots & \omega^2 \\ \omega^2 & \omega^2 + \sigma^2 & \dots & \omega^2 \\ \vdots & \vdots & \ddots & \vdots \\ \omega^2 & \omega^2 & \dots & \omega^2 + \sigma^2 \end{bmatrix}$$

- However, there is not heteroskedasticity. The covariances between the elements of  $\mathbf{v}_i = (v_{i1}, v_{i2}, \dots, v_{iT})'$  and  $\mathbf{v}_j = (v_{j1}, v_{j2}, \dots, v_{jT})'$  are given by:

$$\begin{aligned} \text{Cov}(v_{it}, v_{jt}) &= E(v_{it}v_{jt}) = E[(\mu_i + u_{it})(\mu_j + u_{jt})] = \\ &= E[\mu_i\mu_j + \mu_i u_{jt} + u_{it}\mu_j + u_{it}u_{jt}] = 0 \end{aligned}$$

- Thus,

$$\text{Cov} \begin{pmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \vdots \\ \mathbf{v}_N \end{pmatrix} = \begin{bmatrix} \mathbf{\Omega} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{\Omega} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{\Omega} \end{bmatrix}$$

# Panel data models:

## Fixed or Random Effects Model (Hausman test)

- **Decide between fixed or random effects model: we can run the Hausman test**
- Null hypothesis: the preferred model is random effects vs. the alternative the fixed effects (see Green, 2008, chapter 9).
- Steps:
  - Run a fixed effects model and save the estimates
  - Run a random model and save the estimates
  - Perform Hausman test. If the p-value is significant (for example  $\alpha=0.05 > p\text{-value}$ ) then use fixed effects, if not use random effects
- R command:
  - `fixed <- plm(y ~ sf1, data=data, index=c("country"), model="within")`
  - `random <- plm(y ~ sf1, data=data, index=c("country"), model="random")`
  - `phptest(fixed, random)`

# Panel data models:

## Tests - Diagnostics

- **Breusch-Pagan Lagrange Multiplier for random effects**
- The LM test helps you decide between a random effects regression and a simple OLS regression
- The null hypothesis in the LM test: no panel effect (i.e. OLS better). That is, no significant difference across units (i.e. no panel effect)
  
- R command:
  - `pool <- plm(y ~ sf1, data = data, model="pooling")`
  - `plmtest(pool, type=c("bp"))`

# Panel data models:

## Tests - Diagnostics

- **Breusch-Pagan Lagrange Multiplier test for cross-sectional dependence in panels**
- Cross-sectional dependence is a problem in panel data (especially with long time series)
- The null hypothesis in the Breusch-Pagan/LM Cross-sectional dependence tests is that residuals across units are not correlated
- Breusch-Pagan/LM (cross-sectional dependence) tests are used to test whether the residuals are correlated across units
  
- R commands:
  - Breusch-Pagan Lagrange Multiplier test for cross-sectional dependence in panels
  - `fixed <- plm(y ~ sf1, data=data, index=c("country"), model="within")`
  - `pcdtest(fixed, test = c("lm"))`
  
  - Pesaran test for cross-sectional dependence in panels
  - `pcdtest(fixed, test = c("cd"))`



# Panel data models:

## Testing for serial correlation

- **Breusch-Godfrey/Wooldridge test for serial correlation in panel models**
- Serial correlation tests apply to panel data. The null is that there is not serial correlation
- R commands: (required package: lmtest)
  - `fixed <- plm(y ~ sf1, data=data, index=c("country"), model="within")`
  - `pbgttest(fixed)`

# Panel data models:

## Testing for heteroskedasticity

- **Breusch-Pagan test for heteroskedasticity**
- The null hypothesis for the Breusch-Pagan test is homoskedasticity
- If heteroskedasticity is detected we can use robust covariance matrix to account for it, or model the conditional variances (using, for example, ARCH/GARCH type models)
- R commands: (required package: lmtest)
  - `bptest(y ~ sf1 + factor(country), data = data, studentize=F)`

# Panel data models:

## Controlling for heteroskedasticity

- Robust covariance matrix estimation (Sandwich estimator)
- The 'vcovHC' function estimates three heteroskedasticity-consistent covariance estimators:
  - "white1": for general heteroskedasticity but no serial correlation (recommended for random effects)
  - "white2": is "white1" restricted to a common variance within groups (recommended for random effects)
  - "arellano": both heteroskedasticity and serial correlation (recommended for fixed effects)
- The following options can be applied:
  - HC0 : heteroskedasticity consistent (default)
  - HC1,HC2, HC3 : Recommended for small samples. HC3 gives less weight to influential observations
  - HC4 : small samples with influential observations
  - HAC : heteroskedasticity and autocorrelation consistent (type ?vcovHAC for more details)
- R commands: (required package: lmtest)
  - `fixed <- plm(y ~ sf1, data=data, index=c("country"), model="within")`
  - `coefest(fixed) # Original coefficients`
  - `coefest(fixed, vcovHC) # Heteroskedasticityconsistent coefficients`
  - `coefest(fixed, vcovHC(fixed, method = "arellano")) # Heteroskedasticity consistent coefficients (Arellano)`
  - `random <- plm(y ~ sf1, data=data, index=c("country"), model="random")`
  - `coefest(random) # Original coefficients`
  - `coefest(random, vcovHC) # Heteroskedasticity consistent coefficients`

# Panel data models: Application to R

- Several panel data models will be implemented in R
- See corresponding R-file



**Thank you**