Econometrics 2
Lecture 18: Asymptotic Properties of the Maximum Likelihood Estimator

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Note: These notes digitize the handwritten derivations from class and are intended to be studied in conjunction with the slides: probit.pdf.

1. General Framework

We return to the general theory of Maximum Likelihood Estimation (MLE) in order to derive its asymptotic properties. To achieve this, we will utilize our general theory regarding the asymptotic properties of **M-type estimators**.

For the sake of simplicity, let us assume that we are working within a framework of **independence**. Under this assumption, the sample average log-likelihood function is defined as:

$$l_n(\beta) := \frac{1}{n}\sum_{i=1}^{n} \ln p(z_{(i)}, \beta) \tag{1}$$

Example: In the specific case of the **Probit model**, the density for an observation i takes the form:

$$p(z_{(i)}, \beta) = \Phi(x_{(i)}\beta)^{Y_{(i)}} \left(1 - \Phi(x_{(i)}\beta)\right)^{1-Y_{(i)}} \tag{2}$$

2. Consistency of the MLE

By applying our general theory, we will attempt to examine (without entering into excessive technical detail) the conditions that ensure the **consistency** of the estimator.

We recall that the Maximum Likelihood Estimator, $\hat{\beta}_n$, is defined as the value that maximizes the log-likelihood function. This optimization problem can be equivalently represented as a minimization problem:

$$\begin{aligned}\hat{\beta}_n &\in \arg\max_{\beta\in\Theta} l_n(\beta) \\ &= \arg\min_{\beta\in\Theta} (-l_n(\beta)) \\ &= \arg\min_{\beta\in\Theta} (l_n(\beta_0) - l_n(\beta))\end{aligned}$$

where β_0 represents the true value of the parameter. This last representation is particularly useful for establishing consistency, as it allows us to analyze the difference between the likelihood evaluated at the true parameter and at any other candidate value.