Monetary and fiscal policy interactions

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Outline

- **Topic:** Leeper (1991) studies monetary and fiscal policy interactions in a simple GE model.
- Apply Blanchard and Kahn (1980) method in Leeper (1991) model.
- Set up the Model.
- Solve for a Decentralized Equilibrium.
- Solve the first order system of difference equations.
- Find BK conditions in Matlab.
- Dynare implementation of the above.

Model

We follow Leeper (1991):

1. Households.
2. Government.
    - Monetary policy.
    - Fiscal policy.
    - Simple linear rules.

Households

A representative household chooses $\{c_t, b_t\}_{t=0}^{\infty}$ to solve:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \log(c_t)$$

subject to:

$$c_t + \frac{B_t}{P_t} = y + R_{t-1} \frac{B_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} - \tau_t$$

where:

- B_t are nominal government bonds with real value, $b_t \equiv \frac{B_t}{P_t}$.
- P_t is the aggregate price level with $\Pi_t \equiv \frac{P_t}{P_{t-1}}$.
- τ_t is lump-sum taxes.
- y a constant endowment.

First-order conditions

$$\frac{1}{R_t} = \beta \mathbf{E_t} \left[\frac{1}{\Pi_{t+1}} \right] \tag{1}$$

Government

- Monetary authority sets the nominal interest rate, $\{R_t\}_t^{\infty}$.

- Fiscal authority sets the fiscal instruments and satisfy the government budget constraint (see below).

Government Budget Constraint

The government budget constraint is given by:

$$\frac{B_t}{P_t} = R_{t-1}\frac{B_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t} + g - \tau_t \qquad (2)$$

- Fiscal policy has three fiscal instruments at its disposal, $\{b_t, g, \tau_t\}_{t=0}^{\infty}$.
- Each period t, it can set two independently, while one should adjust to satisfy equation (2).
- In what follows, fiscal policy sets exogenously, $\{g, \tau_t\}_{t=0}^{\infty}$, while $\{b_t\}_{t=0}^{\infty}$ will adjust residually.

Simple feedback policy rules

- Monetary rule:

$$R_t = \phi_0 + \phi_\pi \Pi_t + \psi_t \tag{3}$$

- Fiscal rule:

$$\tau_t = \gamma_0 + \gamma b_{t-1} + \theta_t \tag{4}$$

- where, ψ_t and θ_t are monetary and fiscal policy shocks, respectively.

Clearing market conditions

- The goods,

$$y = c_t + g \tag{5}$$

  and,

- the assets market clear.

Decentralized Equilibrium

The Decentralized Equilibrium is a dynamic path for the endogenous variables of the model $\{c_t, \Pi_t\}_{t=0}^{\infty}$, plus the residual policy instrument $\{b_t\}_{t=0}^{\infty}$,

- Such that equations (1), (2) and (5), are satisfied.
- Given initial conditions for the endogenous and exogenous state variables, $\{b_{-1}\}$, and $\{\psi_{-1}, \theta_{-1}\}$ of the model.
- Given policy variables $\{R_t\}_{t=0}^{\infty}$ and $\{\tau_t, g\}_{t=0}^{\infty}$.
- While, $\{R_t\}_{t=0}^{\infty}$, and, $\{\tau_t\}_{t=0}^{\infty}$, are set according to simple rules.

Equilibrium conditions

We end up with a dynamic system of 5 non-linear difference equations in 5 unknowns $\{c_t, \Pi_t, b_t, R_t, \tau_t\}_{t=0}^{\infty}$:

$$\frac{1}{R_t} = \beta \mathbf{E_t} \left[\frac{1}{\Pi_{t+1}} \right] \tag{DE1}$$

$$b_t = \frac{R_{t-1}}{\Pi_t} b_{t-1} + g - \tau_t \tag{DE2}$$

$$y = c_t + g \tag{DE3}$$

$$R_t = \phi_0 + \phi_\pi \Pi_t + \psi_t \tag{DE4}$$

$$\tau_t = \gamma_0 + \gamma b_{t-1} + \theta_t \tag{DE5}$$

Linearized System

$$\hat{R}_t = \beta \mathbf{E_t} \hat{\Pi}_{t+1} \tag{LE1}$$

$$b\hat{b}_t = \frac{Rb}{\Pi}\left(R_{t-1} - \Pi_t + b_{t-1}\right) - \tau\hat{\tau}_t \tag{LE2}$$

$$\hat{c}_t = 0 \tag{LE3}$$

$$R\hat{R}_t = \phi_\pi \Pi \hat{\Pi}_t + \psi\hat{\psi}_t \tag{LE4}$$

$$\tau\hat{\tau}_t = \gamma\hat{b}_{t-1} + \theta\hat{\theta}_t \tag{LE5}$$

Final system

Combining the above equations (i.e., variable reduction), we end up with the following 2nd order difference equation system:

$$\mathbf{E_t}\hat{\Pi}_{t+1} = \phi_\pi \beta \hat{\Pi}_t + \frac{\theta}{R}\hat{\theta}_t \tag{1*}$$

$$\hat{b}_t = \left(\frac{1}{\beta} - \gamma\right)\hat{b}_{t-1} - \frac{1}{\beta}\hat{\Pi}_t + \frac{1}{\beta}\phi_\pi\left(\frac{\pi}{R}\right)\hat{\Pi}_{t-1} + \frac{1}{\beta}\left(\frac{\theta}{R}\right)\hat{\theta}_{t-1} + \frac{g}{b}\hat{g}_t - \frac{\psi}{b}\hat{\psi}_t \tag{2*}$$

Reduce it to first order

$$E_t\hat{\Pi}_{t+1} = \phi_\pi \beta \hat{\Pi}_t + \frac{\theta}{R}\hat{\theta}_t$$

$$\hat{b}_t = \left(\frac{1}{\beta} - \gamma\right)\hat{b}_{t-1} - \frac{1}{\beta}\hat{\Pi}_t + \frac{1}{\beta}\phi_\pi\left(\frac{\pi}{R}\right)\hat{x}_t + \frac{1}{\beta}\left(\frac{\theta}{R}\right)\hat{\theta}_{t-1} + \frac{g}{b}\hat{g}_t - \frac{\psi}{b}\hat{\psi}_t$$

$$\hat{x}_{t+1} = \hat{\Pi}_t$$

In matrix form

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} E_t \begin{bmatrix} \hat{\Pi}_{t+1} \\ \hat{b}_t \\ \hat{x}_{t+1} \end{bmatrix} = \begin{bmatrix} \phi_\pi \beta & 0 & 0 \\ -\frac{1}{\beta} & \frac{1}{\beta} - \gamma & \phi_\pi \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{\Pi}_t \\ \hat{b}_{t-1} \\ \hat{x}_t \end{bmatrix} + \begin{bmatrix} 0 & \frac{\theta}{R} \\ \frac{1}{\beta}\frac{\theta}{R} & \frac{\psi}{b} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\theta}_t \\ \hat{\psi}_t \end{bmatrix}$$

In matrix form(cont'ed)

$$A\mathrm{X}_t = B\mathrm{X}_{t-1} + C\mathrm{Z}_t$$

$$\mathrm{X}_t = A^{-1}B\mathrm{X}_{t-1} + A^{-1}C\mathrm{Z}_t$$

$$\mathrm{X}_t = Q\mathrm{X}_{t-1} + W\mathrm{Z}_t \qquad \text{(BK)}$$

Blanchard-Kahn Method

Blanchard-Kahn method

1. Apply the Jordan decomposition to the matrix Q:

$$Q = PJP^{-1}$$

   where J is a matrix that contains the eigenvalues in the main diagonal and P the associated eigenvectors.

2. Equation (BK) can be written as:

$$\mathrm{X}_t = PJP^{-1}\mathrm{X}_{t-1} + W\mathrm{Z}_t$$

$$P^{-1}\mathrm{X}_t = JP^{-1}\mathrm{X}_{t-1} + P^{-1}W\mathrm{Z}_t$$

Blanchard-Kahn method(cont'ed)

$$P^{-1}\mathrm{X}_t = JP^{-1}\mathrm{X}_{t-1} + P^{-1}W\mathrm{Z}_t$$

$$P^{-1}\mathrm{X}_t = \begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{bmatrix} P^{-1}\mathrm{X}_{t-1}$$

Suppose:

$$P^{-1} = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}$$

to simplify notation,

$$\tilde{X}_t = P^{-1}\mathrm{X_t}$$

Blanchard-Kahn method(cont'ed)

$$\tilde{X}_t = \begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{bmatrix} \tilde{X}_{t-1} \tag{6}$$

where, for algebraic simplicity, I set $Z_t = 0$ for all t.

Blanchard-Kahn conditions

For each model with m control variables and n state variables the BK conditions are:

1. If the number of eigenvalues of matrix J outside the unit circle, say k, is equal to the number of controls, m, the solution of this system is unique, i.e., $k = m$. (Uniqueness).
2. If $k > m$, there is no solution to the system (Instability).
3. If $k < m$, there is an infinite number of solutions (Indeterminacy).

When BK for uniqueness are satisfied, i.e., $k = m$

The solution to the system is given by:

$$\begin{bmatrix} E_t \tilde{\Pi}_{t+1} \\ \tilde{B}_t \end{bmatrix} = \begin{bmatrix} V_1 & 0 \\ 0 & V_2 \end{bmatrix} \begin{bmatrix} \tilde{\Pi}_t \\ \tilde{B_{t-1}} \end{bmatrix}$$

Since, all the elements of V_1 are outside the unit circle, we solve the first equation forwards and we end up with:

$$\tilde{\Pi}_t = 0$$

Since, all the elements of V_2 are inside the unit circle, we solve the second equation backwards and we end up with:

$$\tilde{B}_t = V_2 \tilde{B}_{t-1}$$

BK method in our model

We have to solve $\det(Q - \lambda I) = 0$, where:

$$Q - \lambda I = \begin{bmatrix} \phi_\pi \beta - \lambda & 0 & 0 \\ -\frac{1}{\beta} & \frac{1}{\beta} - \gamma - \lambda & \phi_\pi \\ 0 & 0 & 1 - \lambda \end{bmatrix}$$

$$\det(Q - \lambda I) = (\phi_\pi \beta - \lambda) \left[\left(\frac{1}{\beta} - \gamma - \lambda \right) (1 - \lambda) \right]$$

The eigenvalues of Q are

- $\phi_\pi \beta$
- $\frac{1}{\beta} - \gamma$
- 1

When BK conditions for uniqueness are satisfied in our model?

In our model we have one control variable, Π_t, and one state variable, b_t.

- BK conditions for a unique equilibrium are satisfied when one eigenvalue lies inside the unit circle and one eigenvalue lies outside.
- Our model has a unique equilibrium in two cases:
  - When $|\phi_\pi \beta| < 1$ and $\left|\frac{1}{\beta} - \gamma\right| > 1$.
  - When $|\phi_\pi \beta| > 1$ and $\left|\frac{1}{\beta} - \gamma\right| < 1$.

Active and Passive policies à la Leeper (1991)

- An **active authority** is not constrained by the current budgetary conditions, i.e., it is free to choose a decision rule that depends on past, current, or expected future variables.
- A **passive authority** is constrained by household optimization and the active authority's actions, as such it must satisfy the government budget.
- The passive authority's **decision rule** necessarily depends on the current state of government debt.

Monetary and fiscal policy interactions

The policy parameter space is divided in 4 regions:

1. Region I: AM/PF, $|\phi_\pi \beta| > 1$ and $\left|\frac{1}{\beta} - \gamma\right| < 1$. **Uniqueness**.
2. Region II: PM/AF, $|\phi_\pi \beta| < 1$ and $\left|\frac{1}{\beta} - \gamma\right| > 1$. **Uniqueness**.
3. Region III: PM/PF, $|\phi_\pi \beta| < 1$ and $\left|\frac{1}{\beta} - \gamma\right| < 1$. **Indeterminacy**.
4. Region IV: AM/AF, $|\phi_\pi \beta| > 1$ and $\left|\frac{1}{\beta} - \gamma\right| > 1$. **Instability**.

Properties of Region I

- MP actively pursues price stability by reacting strongly to inflation (i.e., satisfies the Taylor principle, $\phi_\pi > \frac{1}{\beta}$).

- FP is constrained by private and monetary policy behavior and passively adjusts lump-sum taxes to balance its budget, (i.e., $\gamma > \frac{1-\beta}{\beta}$). Note that in a steady state with zero inflation $\frac{1-\beta}{\beta} = R - 1$.

Properties of Region II

- FP does not adjust lump-sum taxes strongly to balance the budget (i.e., $\gamma < \frac{1-\beta}{\beta}$).

- MP is now constrained by private and fiscal policy behavior and passively adjusts the interest rate to deflate public debt, (i.e., does not satisfy the Taylor principle, $\phi_\pi < \frac{1}{\beta}$).

Region I & II

- Active policy uniquely determines the equilibrium inflation rate.

- Passive policy prevents an explosive path of government debt.

A large body of ‘never-ending’ literature on this topic, a non-exhaustive list includes, Sargent and Wallace (1975), Woodford (1998), Sims (2004), Davig and Leeper (2007), Kirsanova et al. (2009), Farmer et al. (2010), Leeper and Leith (2016), Angeletos and Lian (2023), Cochrane (2023), Kaplan et al. (2023).

Properties of Region III

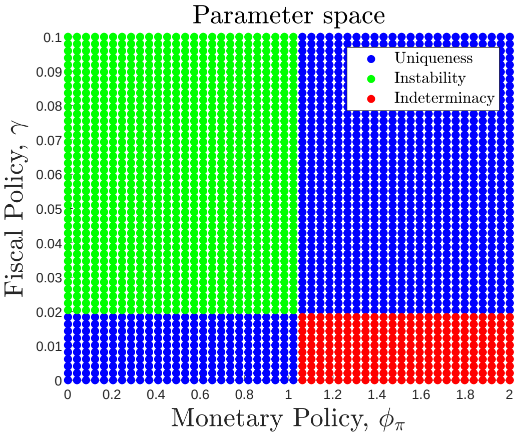
- Each authority behaves passively to prevent from an explosive path of public debt.

- Indeterminate equilibria.

Properties of Region IV

- Each authority actively disregards the budget constraint.
- Public debt explodes. Instability.

Numerical Implementation



References I

Angeletos, G.-M. and Lian, C. (2023). Determinacy without the taylor principle. *Journal of Political Economy*, 131(8):2125–2164.

Blanchard, O. J. and Kahn, C. M. (1980). The solution of linear difference models under rational expectations. *Econometrica*, 48(5):1305–1311.

Cochrane, J. H. (2023). *The Fiscal Theory of the Price Level*. Princeton University Press.

Davig, T. and Leeper, E. (2007). Generalizing the taylor principle. *American Economic Review*, 97(3):607–635.

Farmer, R. E. A., Waggoner, D. F., and Zha, T. (2010). Generalizing the taylor principle: Comment. *American Economic Review*, 100(1):608–17.

Kaplan, G., Nikolakoudis, G., and Violante, G. L. (2023). Price level and inflation dynamics in heterogeneous agent economies. Working Paper 31433, National Bureau of Economic Research.

References II

Kirsanova, T., Leith, C., and Wren-Lewis, S. (2009). Monetary and fiscal policy interaction: The current consensus assignment in the light of recent developments. *The Economic Journal*, 119(541):F482–F496.

Leeper (1991). Equilibria under 'active' and 'passive' monetary and fiscal policies. *Journal of Monetary Economics*, 27(1):129–147.

Leeper, E. and Leith, C. (2016). Chapter 30 - understanding inflation as a joint monetary–fiscal phenomenon. volume 2 of *Handbook of Macroeconomics*, pages 2305–2415. Elsevier.

Sargent, T. J. and Wallace, N. (1975). "rational" expectations, the optimal monetary instrument, and the optimal money supply rule. *Journal of Political Economy*, 83(2):241–254.

Sims, C. A. (2004). *7. Limits to Inflation Targeting*, pages 283–310. University of Chicago Press, Chicago.

Woodford, M. (1998). *Control of the Public Debt: A Requirement for Price Stability?*, pages 117–158. Palgrave Macmillan UK, London.