A4 : Probabilistic Voting

→ $U_i(g_A) > U_i(g_B)$: Voter i votes PARTY A. for sure. (Economic Voting)

BASIC CONDITION

↳ Under this condition the median voter was the important / crucial voter

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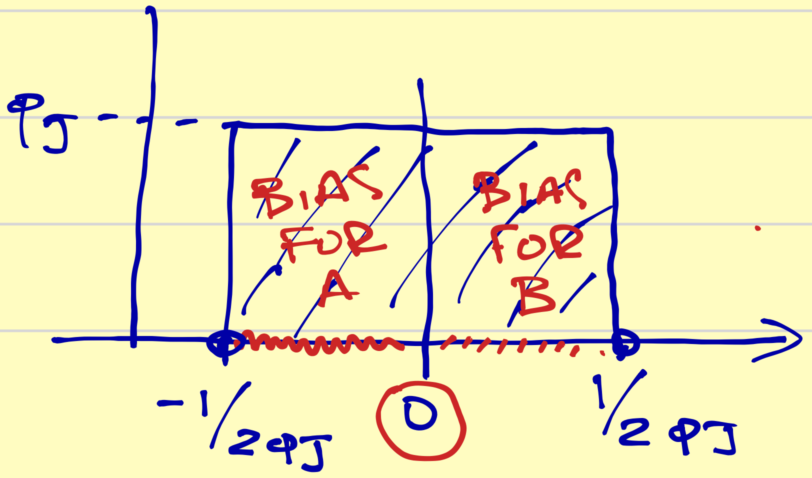
→ However in the real world voters cast their votes by having some kind of Biases.

Voter i votes PARTY A. if and only if
$U_i(y_A) > U_i(y_B) + \sigma_i$

* $\sigma_i > 0$ (positive) ⇒ BIAS IN FAVOR OF B
* $\sigma_i < 0$ (negative) ⇒ BIAS IN FAVOR OF A

$\sigma_i \sim$ uniform distribution



$\left[-\frac{1}{2\phi_J}, \frac{1}{2\phi_J}\right]$

Area = 1.

Basic Assumptions [Lindbeck + Weibull, 1987]

* Office seekers politicians / Parties

$\boxed{2.}$

* A society consisted of three groups of voters : (i) Rich (R), (ii) Middle income group (M), (iii) Poor (P) $y_R > y_M > y_P$

* Each group is a fraction a_J of the total population

$$\sum_{J=1}^{3} a_J = 1.$$

* $U_J(g_A) > U_J(g_B) + \sigma_{ij}$

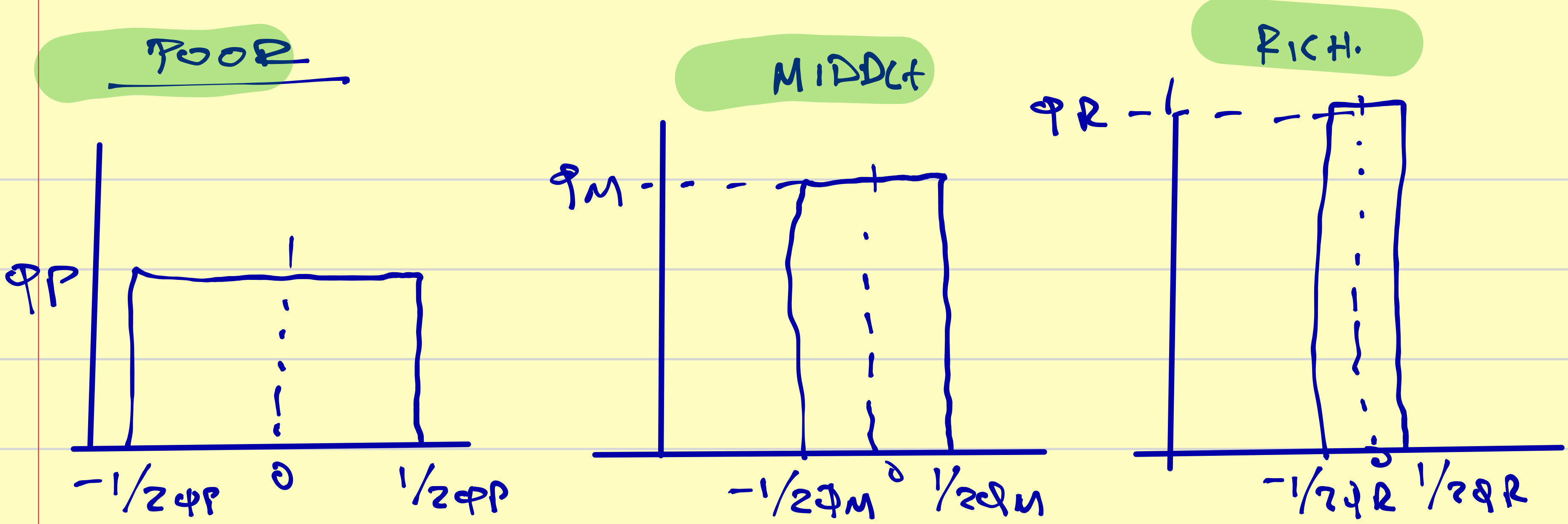
$U_P \rightarrow$ common for the poor
$U_M \rightarrow$ " " " middle
$U_R \rightarrow$ common " " Rich

U_J: COMMON UTILITY WITHIN THE GROUP

σ_{iJ}: individual Bias

* $\sigma_{iJ} \sim$ Uniform distribution common for the group. $\left[-\frac{1}{2\varphi_J}, \frac{1}{2\varphi_J}\right]$

$\varphi_P < \varphi_M < \varphi_R$



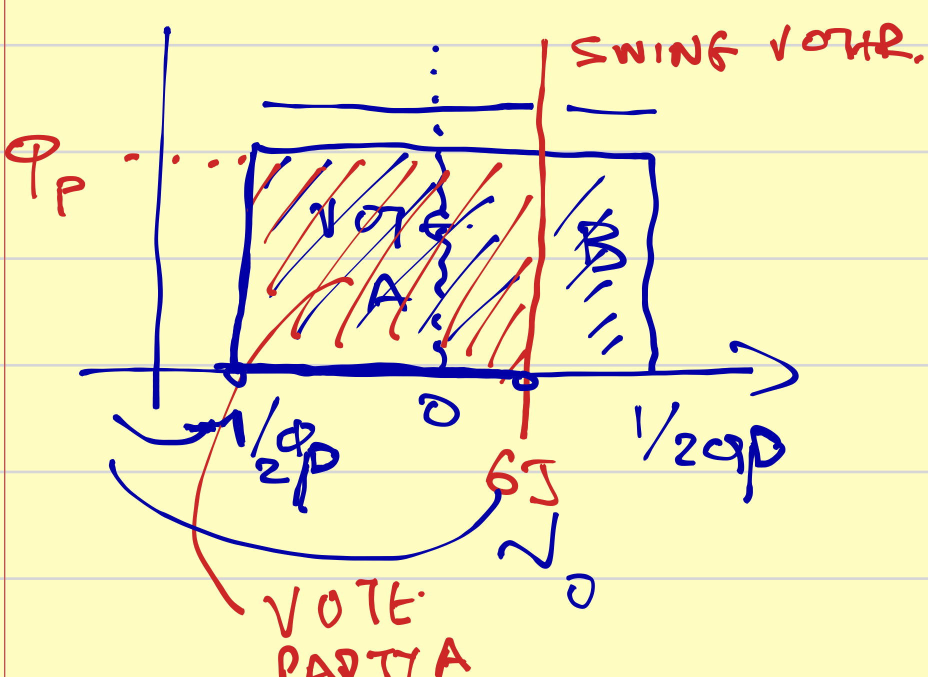
⇒ How is formed the majority in such a political context?

→ SWING VOTER. (οριακος ψηφοφορος)

There is a "swing voter" in each group J

$$\boxed{\sigma_J = U_J(g_A) - U_J(g_B)}$$

Let's assume that g_A ensures higher utility $U_J(g_A) > U_J(g_B)$ for group J.



The swing voter is this specific guy that his individual bias equals the difference in terms of utility from platform A (g_A)

Votes' share of PARTY A.

POOR

$$\text{Area} = \phi_P * \left(G_P - \left(-\frac{1}{2\phi_P}\right)\right) = \phi_P * \left(G_J + \frac{1}{2\phi_P}\right)$$

FRACTION OF POOR

$$= \frac{a_P * \phi_P * \left(G_P + \frac{1}{2\phi_P}\right)}{\text{FRACTION OF TOTAL POPULATION}} \quad (1)$$

→ Middle income.

$$= \frac{a_M * \phi_M * \left(G_M + \frac{1}{2\phi_M}\right)}{\text{FRACTION OF TOTAL POPULATION}} \quad (2)$$

→ Rich.

$$= \frac{a_R * \phi_R * \left(G_R + \frac{1}{2\phi_R}\right)}{\text{FRACTION OF TOTAL POPULATION}} \quad (3)$$

$$(1)-(3) \Rightarrow \boxed{\Pi_A = \sum_{J=1}^{3} a_J \phi_J \left(G_J + \frac{1}{2\phi_J}\right)}$$

$\Rightarrow$ What is the optimal strategy for each party in such a context.

The party wants to maximize the probability of win the elections.

$$\Pi_A > \frac{1}{2} \iff \sum_{J=1}^{3} a_J \phi_J \left[\sigma_J + \frac{1}{2\phi_J} \right] > \frac{1}{2} \iff$$

$$\iff \sum_{J=1}^{3} a_J \phi_J \cdot \sigma_J + \sum_{J=1}^{3} a_J \phi_J \frac{1}{2\phi_J} > \frac{1}{2}$$

$$\iff \sum_{J=1}^{3} a_J \phi_J \sigma_J + \frac{1}{2} \underbrace{\sum_{J=1}^{3} a_J}_{=1} > \frac{1}{2}$$

$$\iff \sum_{J=1}^{3} a_J \phi_J \sigma_J > 0$$

$$\underline{\sigma_J = U_J(g_A) - U_J(g_B)} \longrightarrow \sum_{i=1}^{3} a_J \phi_J \left[U_J(g_A) - U_J(g_B) \right]$$

$$\sum_{i=1}^{3} a_J \phi_J \left[U_J(g_A) - U_J(g_B) \right] > 0 \iff$$

$$\iff \sum_{i=1}^{3} a_J \phi_J U_J(g_A) > \sum_{i=1}^{3} a_J \phi_J U_J(g_B)$$

Social Welfare Function: $\sum_{j=1}^{3} \underbrace{a_J \phi_J}_{\text{WEIGHTS.}} U_J(g)$

a_J, ϕ_J

→ Olson (1965) "Logic of collective Action"