Differential Equations

Mathematics for Economists, Fall 2025-26

Homework Exercises Set 4

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1 Find the solution of the following differential equations, and determine the arbitrary constants:

a) $2y''(t) - 3y'(t) + y = (t^2 + 1)e^t$, $y(0) = 5$, $y'(0) = 14$

b) $y''(t) + 2y'(t) + y = t^2$, $y(0) = 0$, $y'(0) = 1$

2 Solve the following linear autonomous differential equations systems and construct their phase diagrams:

a) $\dot{x} = x + y$, $\dot{y} = 4x + y$

b) $\dot{x} = 3x - 2y$, $\dot{y} = 2x - 2y$

3 Solve the equilibrium for the following control problem. Linearise the system about the equilibrium and establish its stability properties:

$$\max_{\{u\}} J = \int_0^\infty (20 lnx - 0.1u^2)dt$$

$$s.t. \quad \dot{x} = u - 0.1x$$