

Figure 1: Phase Diagram_Exercise 2.a

Homework Set 4

1. a)

$$\text{General Solution: } y_t = A_1 e^t + A_2 e^{0.5t} + \left(\frac{t^3}{3} - 2t^2 + 9t - 18\right)e^t$$

1. b)

$$\text{General Solution: } y_t = A_1 e^{-t} + A_2 t e^{-t} + t^2 - 4t + 6$$

2. a)

General Solution:

$$\begin{aligned} x_t &= A_1 e^{3t} + A_2 e^{-t} \\ y_t &= A_1 2e^{3t} + A_2 (-2)e^{-t} \end{aligned}$$

2.b)

General Solution:

$$\begin{aligned} x_t &= A_1 2e^{2t} + A_2 0.5e^{-t} \\ y_t &= A_1 e^{2t} + A_2 e^{-t} \end{aligned}$$

3)

The system of the two differential equations:

$$\begin{aligned} \dot{x} &= -0.1x + 5\lambda \\ \dot{\lambda} &= -\frac{20}{x} + 0.1\lambda \end{aligned}$$

The fixed point of the system:

$$\begin{aligned} x^* &= 100 \\ \lambda^* &= 2 \end{aligned}$$

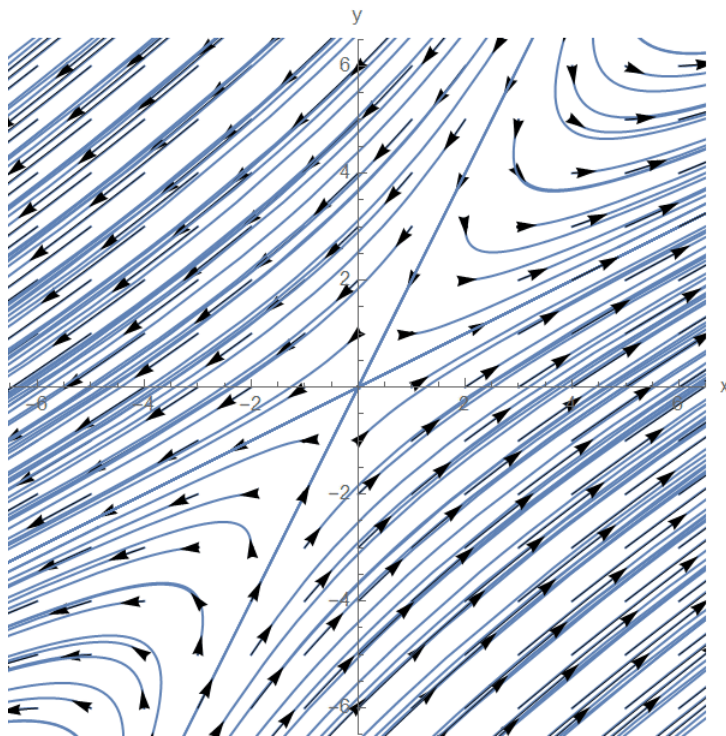


Figure 2: Phase Diagram_Exercise 2.b

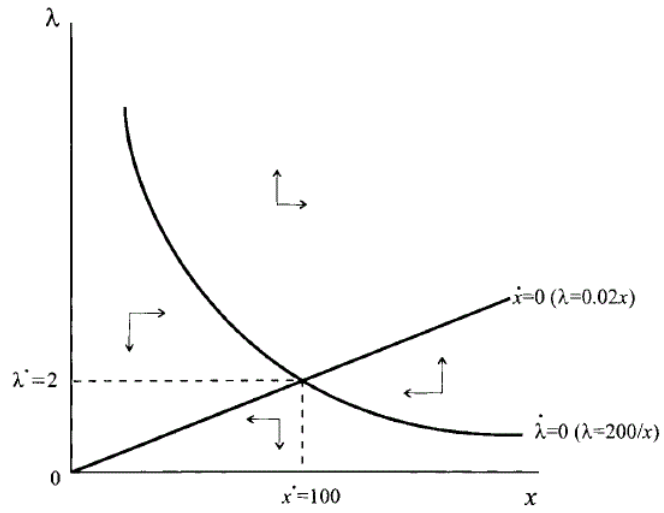


Figure 3: Phase Diagram_Exercise 3

The linearization of the system around the fixed point is:

$$\begin{aligned} \dot{x} &= -0.1(x - x^*) + 5(\lambda - \lambda^*) \\ \dot{\lambda} &= 0.002(x - x^*) + 0.1(\lambda - \lambda^*) \end{aligned}$$