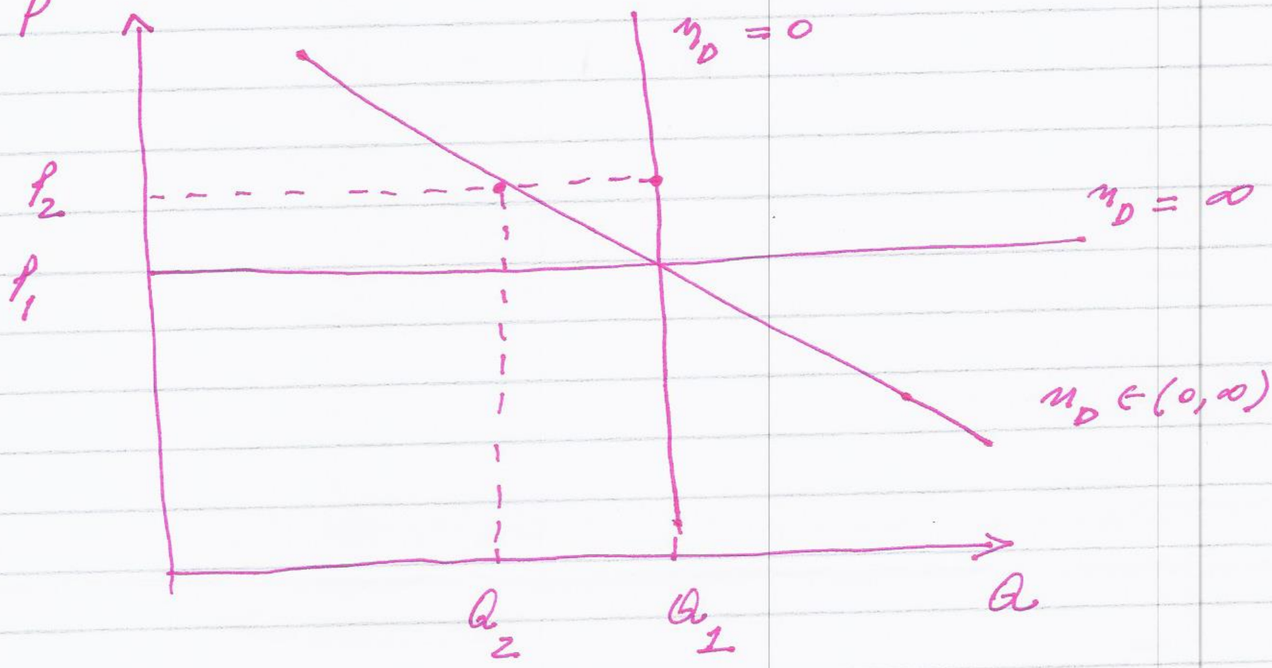
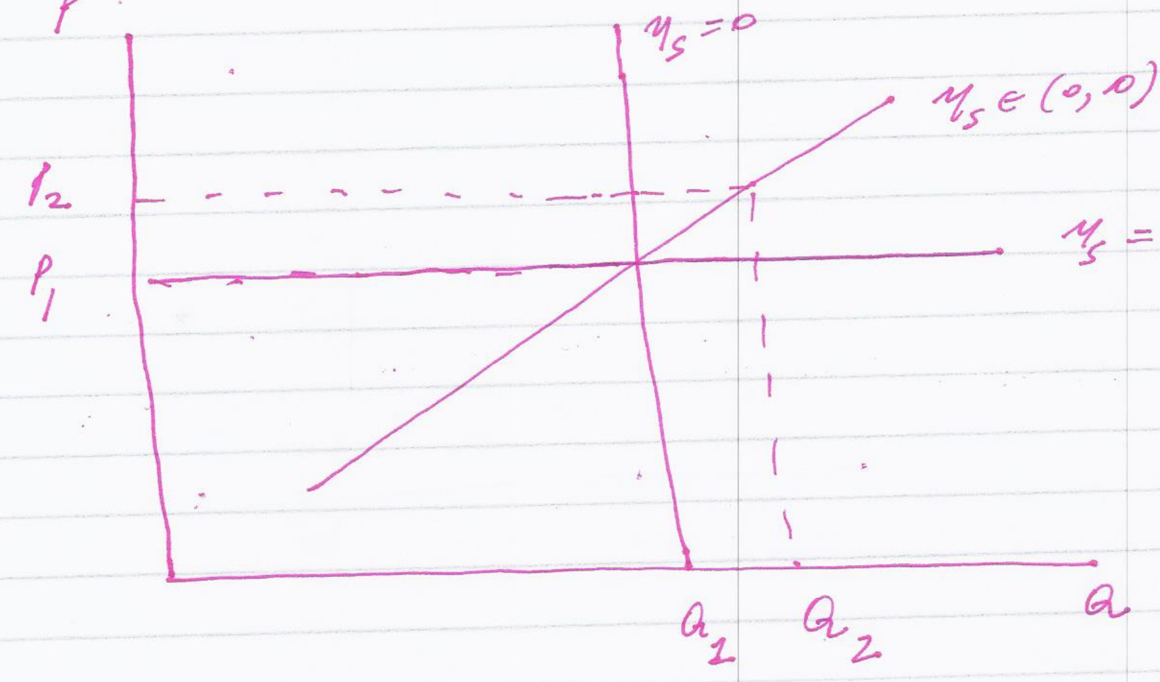
Tax Incidence + Elasticities

Definitions

$$\eta_D = -\frac{\Delta Q}{\Delta P}\frac{P}{Q} = \frac{-(Q_2-Q_1)}{P_2-P_1}\frac{P_1}{Q_1}$$

$$\eta_S = \frac{\Delta Q}{\Delta P}\frac{P}{Q} = \frac{Q_2-Q_1}{P_2-P_1}\frac{P_1}{Q_1}$$



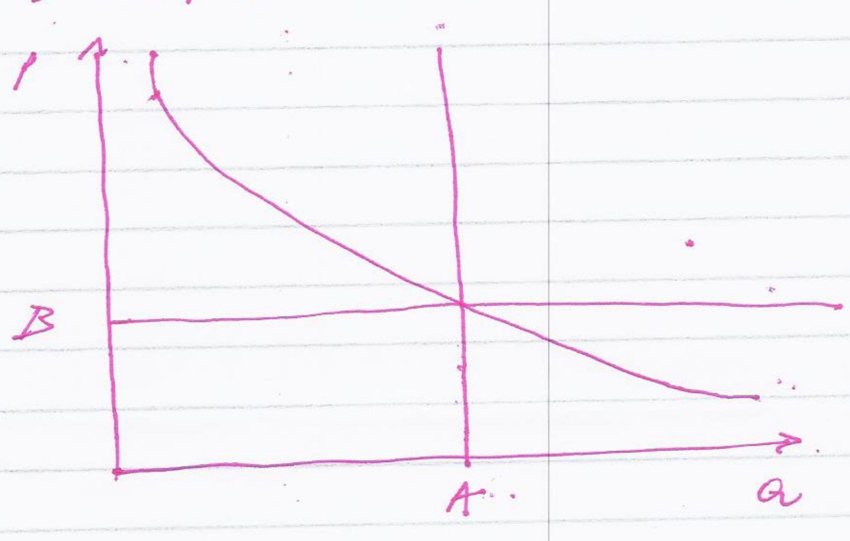


<u>Definitions</u> $\quad \eta_D = \lim_{\Delta P \to 0} \left(- \frac{\Delta Q}{\Delta P} \frac{P}{Q} \right) = - \frac{dQ}{dP} \frac{P}{Q}$

$$\eta_S = \lim_{\Delta P \to 0} \frac{\Delta Q}{\Delta P} \frac{P}{Q} = \frac{dQ}{dP} \frac{P}{Q}$$

<u>Example</u>: Constant elasticity of demand

Demand: $\quad Q = AP^{-\alpha} \quad \alpha \in (0, \infty)$



$$\eta_D = - \frac{dQ}{dP} \frac{P}{Q} = - A(-\alpha) P^{\alpha - 1} \frac{P}{Q}$$

$$= \alpha \, AP^{\alpha} \frac{1}{P} \frac{P}{Q}$$

$$= \alpha$$

If $\alpha \in (0,1)$, demand is inelastic $\eta_D \in (0,1)$

If $\alpha \in (1,\infty)$, demand is elastic $\eta_D \in (0,\infty)$

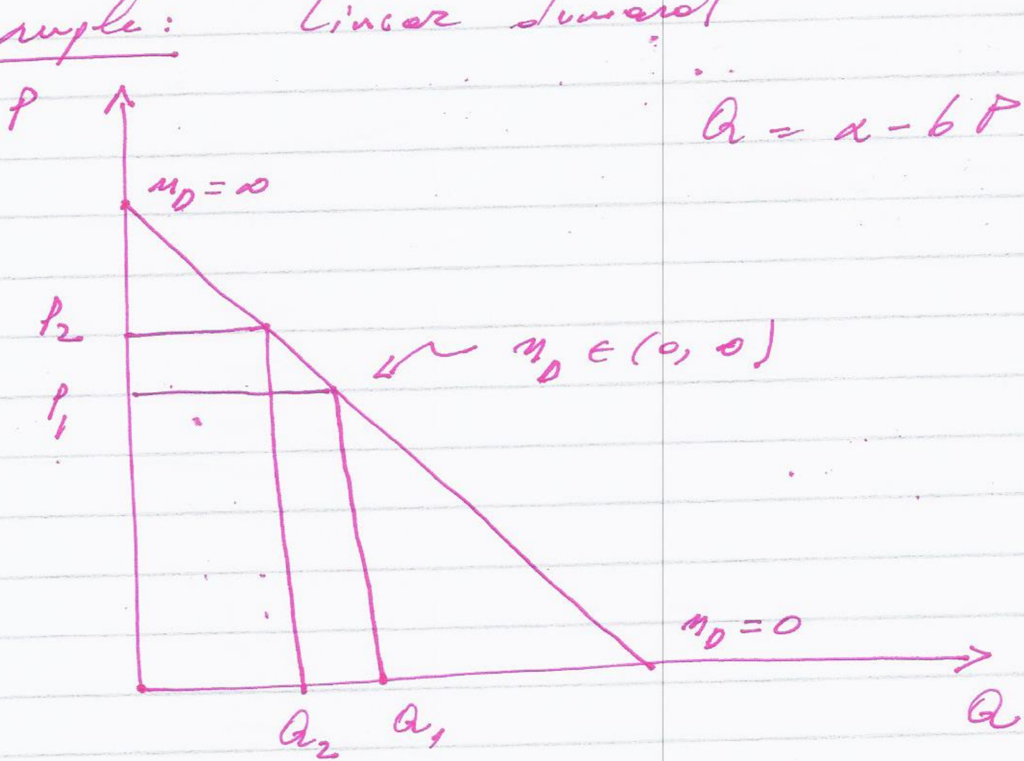
If $\alpha \to 0$, demand is perfectly inelastic, $\eta_D = 0$

If $\alpha \to +\infty$, demand is perfectly elastic, $\eta_D = \infty$

Likewise for supply

Example: Linear demand

$Q = \alpha - bP$

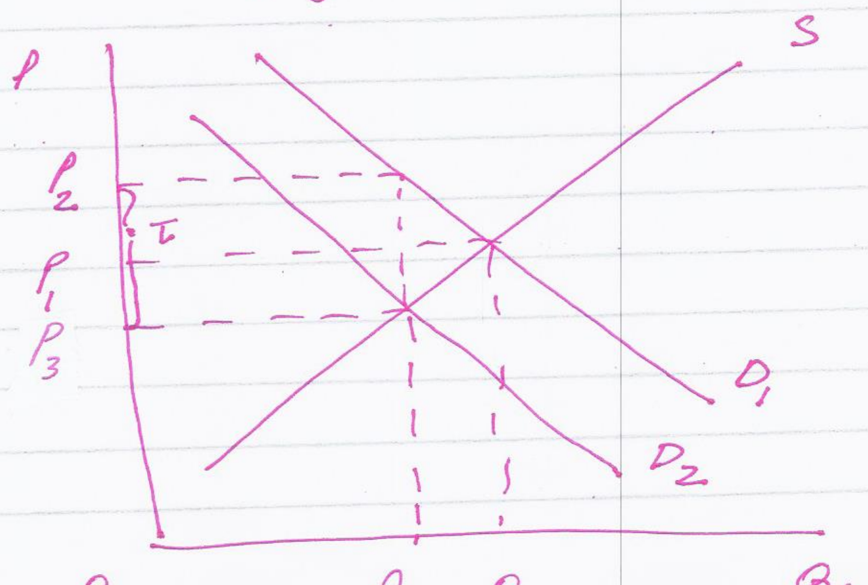


$$\eta_D = -\frac{dQ}{dP}\frac{P}{Q} = -(-b)\frac{P}{Q} = b\frac{P}{Q}$$

Remark: $\eta_D \downarrow$ with Q along the demand curve

Tax Incidence

- Excise tax, τ
- Consumers pay the tax



$$\eta_D = \frac{-(Q_2 - Q_1)}{P_2 - P_1} \cdot \frac{P_1}{Q_1}$$

$$\eta_S = \frac{(Q_2 - Q_1)}{P_3 - P_1} \cdot \frac{P_1}{Q_1}$$

$$\Rightarrow \eta_D = \frac{-(Q_2 - Q_1)}{P_3 + \tau - P_1} \cdot \frac{P_1}{Q_1}$$

$$\Rightarrow \eta_D = \frac{-(Q_2 - Q_1)}{P_3 - P_1 + \tau} \cdot \frac{P_1}{Q_1}$$

In general,

$$\eta_D = \frac{-\Delta Q}{\Delta P + \tau} \cdot \frac{P}{Q} \quad (1)$$

$$\eta_S = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q} \quad (2)$$

$$(1) \Rightarrow -\frac{\Delta Q}{Q} = \frac{\eta_D (\Delta P + \tau)}{P} \quad (3)$$

$$(2) \Rightarrow -\frac{\Delta Q}{Q} = -\frac{\eta_S \Delta P}{P} \quad (4)$$

$$(3) \,\&\, (4) \Rightarrow \frac{\eta_D (\Delta P + \tau)}{\cancel{P}} = \frac{-\eta_S \Delta P}{\cancel{P}} \quad (5)$$

$$\Rightarrow -\Delta P = \frac{\eta_D}{\eta_S + \eta_D} \tau \quad (6)$$

Recall that the tax burden to consumers is:

$$P_2 - P_1 =$$

$$P_3 + \tau - P_1 =$$

$$P_3 - P_1 + \tau$$

Or, in general,

$$\Delta P + \tau$$

Hence, in view of (6) the tax burden to consumers (TBC) is

$$TBC = -\frac{\eta_D}{\eta_S + \eta_D}\tau + \tau =$$

$$\left(1 - \frac{\eta_D}{\eta_S + \eta_D}\right)\tau \qquad (7)$$

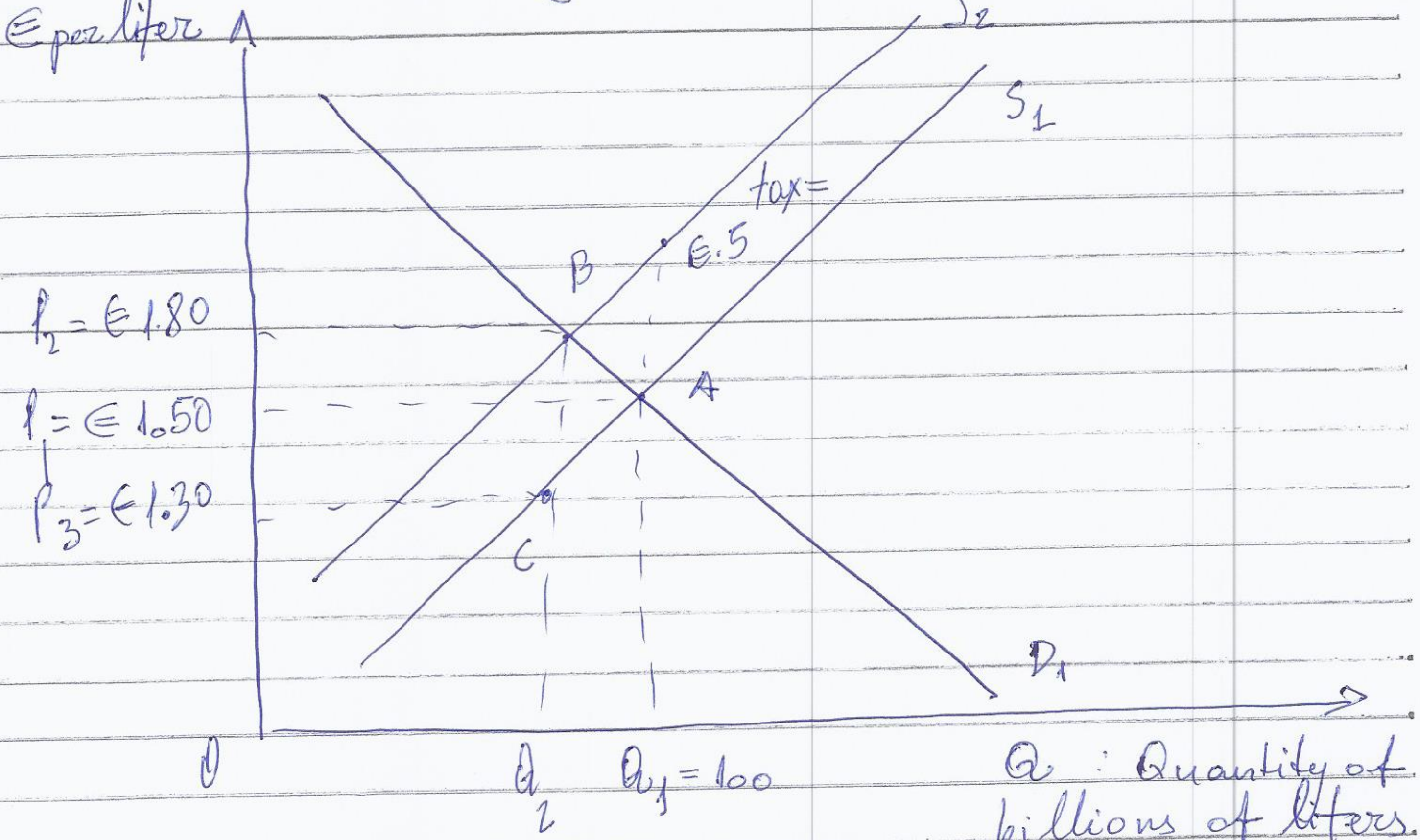
__Remark__: If demand is perfectly inelastic (i.e., $\eta_D = 0$), the tax burden to consumers is the full tax. If demand is perfectly elastic (i.e., $\eta_D = \infty$) the tax burden to consumers is 0. And, therefore producers are levied the full burden of the tax. In general, from (7)

$$TBC = \left(1 - \frac{1}{1 + \frac{\eta_S}{\eta_D}}\right)\tau$$

So TBC increases with $\frac{\eta_S}{\eta_D}$. That is, the greater is η_S and the lower is η_D.

Taxation and Economic Efficiency

P: The deadweight (welfare) loss of a tax



Before Tax Market Equ : (Q_1, P_1)

Tax : € .5 per liter

After Tax Market Equ : (Q_2, P_2)

Loss of consumer surplus : $P_2 B A P_1$

Loss of producer surplus : $P_1 A C P_3$

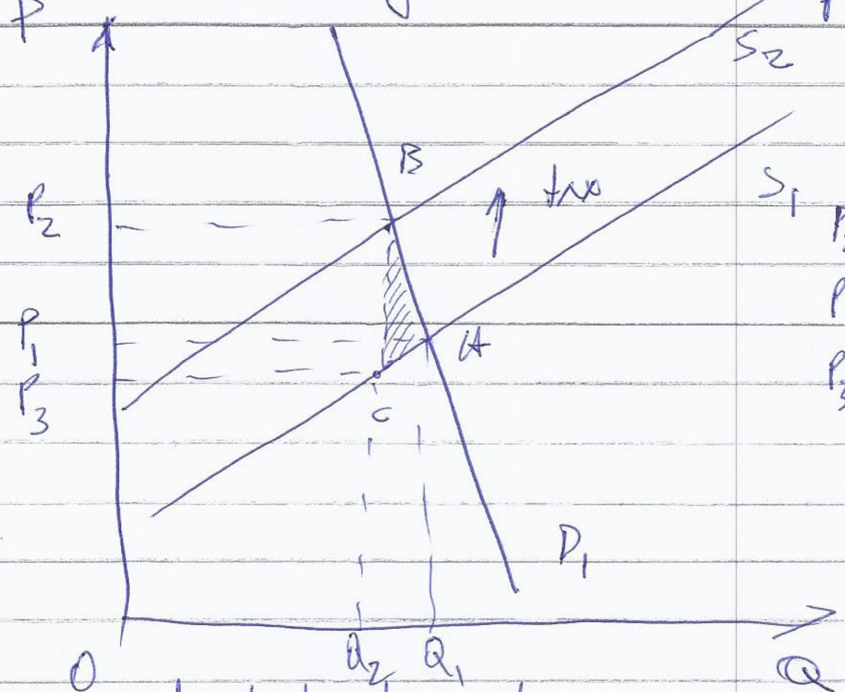
Government revenues : $P_2 B C P_3$

Deadweight loss : BAC

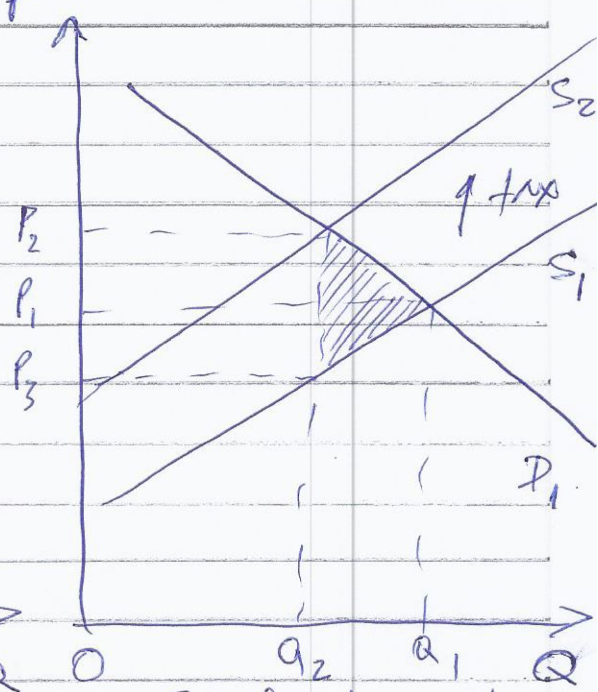
Hamberger triangle

II. Deadweight loss of a tax and elasticities

Graphically:



Inelastic demand



Elastic demand

Algebraically (Linear Demand + Supply)

$$DWL = -\frac{1}{2} \cdot \Delta Q \cdot \tau$$

ΔQ: change in quantity; τ: tax

From the tax burden formulas:

$$\left.\begin{aligned} \frac{\Delta Q}{Q} &= \eta_S \frac{\Delta P}{P} \text{ and} \\ \Delta P &= \frac{\eta_d}{\eta_S - \eta_d} \times \tau \end{aligned}\right\} \Longrightarrow$$

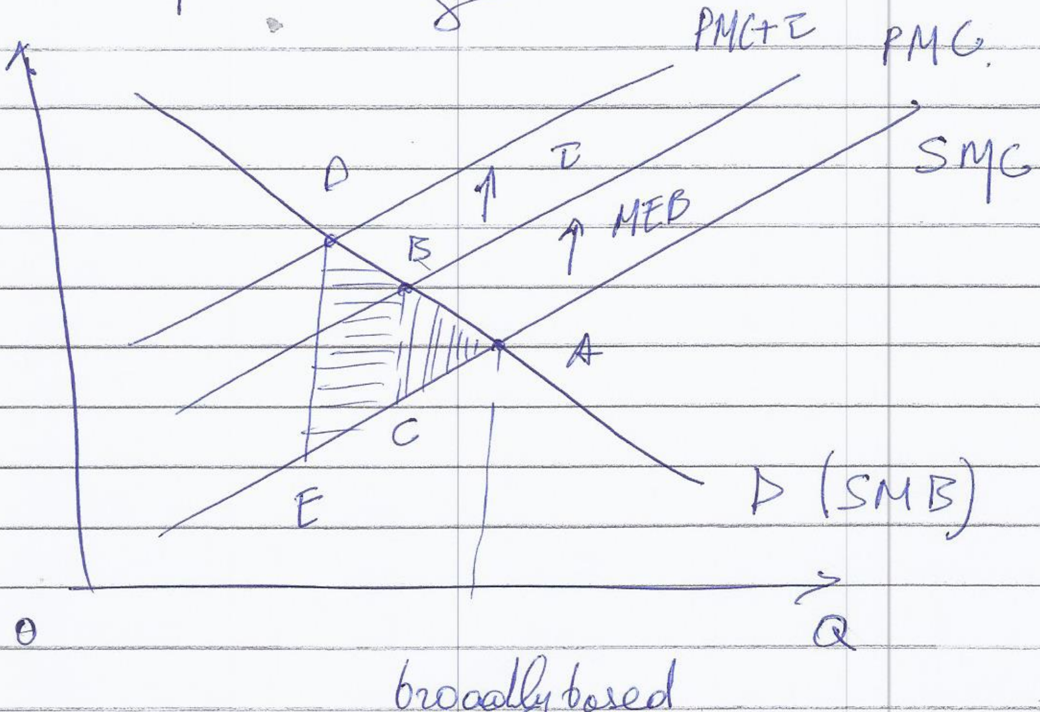
$$\Delta Q = \frac{\eta_S \eta_d}{\eta_S - \eta_d} \tau \frac{Q}{P}$$

$$\boxed{DWL = -\frac{1}{2} \frac{\eta_S \eta_d}{\eta_S - \eta_d} \tau^2 \frac{Q}{P}}$$

	DWL	$\frac{\Delta(DWL)}{\Delta\tau}$
η_S	+	
η_D	+	
τ (τ^2)	+	(+)

Caveats : compensated elasticities matter (avoids income effects)

preexisting distortions



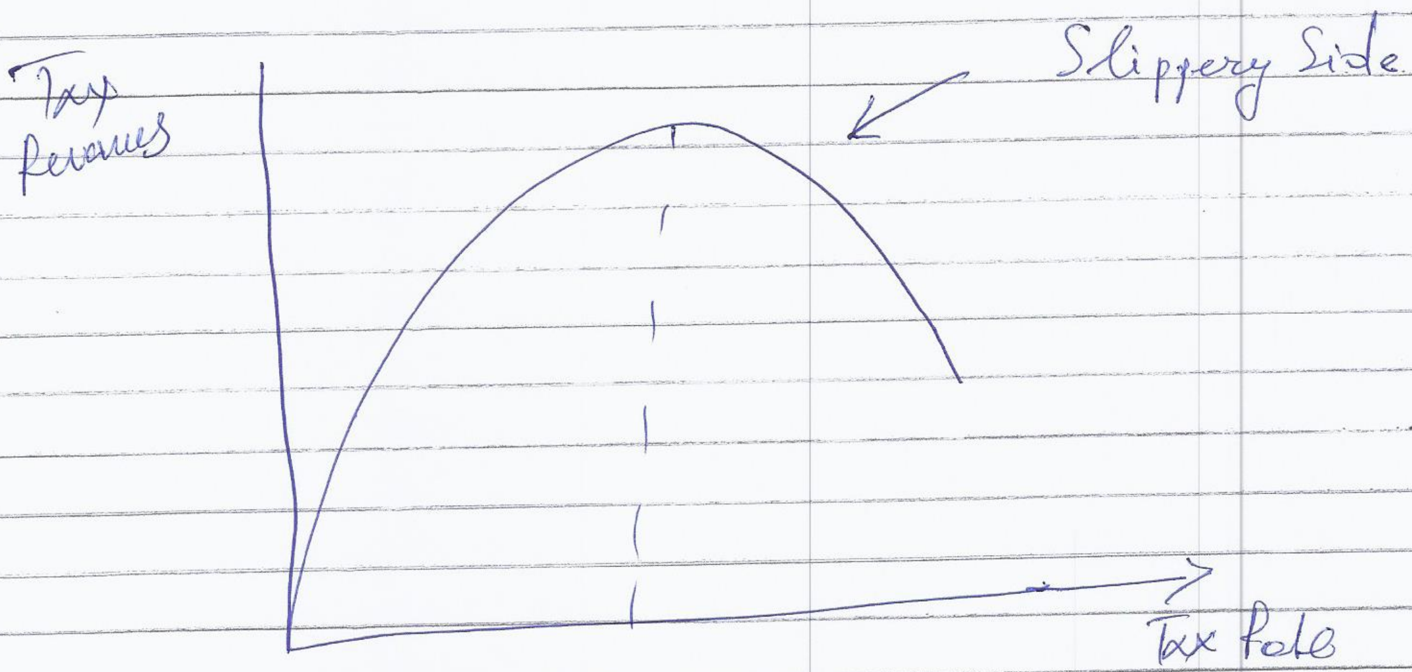
broadly based
low rates vs progressive rates
narrowly based

Commodity Taxes

Ramsey Rule (Minimize ΣDWL given $\Sigma T_i q_i = T$)

$$T_i \ni \frac{\left[\Delta(DWL_i)/\Delta T_i\right]}{\left(\Delta R_i/\Delta T_i\right)} = \lambda$$

Income Taxes



Laffer Curve