We can extend an already known example in order to see that if A is <u>not</u> d-totally bounded, then this does <u>not</u> imply the existence of one element balls.

Remember that when $X = B(\mathbb{N}, \mathbb{R})$ and $B = \left\{ (x_{n_i})_{n\in\mathbb{N}} \in X,\ x_{n_i} = \begin{cases} 0 & \text{if } n \neq i \\ 1 & \text{if } n = i \end{cases},\ i \in \mathbb{N} \right\}$ then we have concluded that B is not d_{sup}-totally bounded.

Notice that $B \subseteq O_{d_{sup}}(\mathbf{0}, \varepsilon)$, where $\mathbf{0} = (0, 0, \dots, 0, \dots)$ and $\varepsilon > 1$. This and the hereditarity properties we have already proven for total boundness imply that $O_{d_{sup}}(\mathbf{0}, \varepsilon)$ is also <u>not</u> d_{sup}-totally bounded for any $\varepsilon > 1$.
But if $x \in O_{d_{sup}}(\mathbf{0}, \varepsilon)$, then, $O_{d_{sup}}(x, \delta) \subseteq O_{d_{sup}}(\mathbf{0}, \varepsilon)$ for $\delta \leq \varepsilon - d_{sup}(\mathbf{0}, x)$, while $O_{d_{sup}}(x, \delta)$ does not only contain $x = (x_1, x_2, \dots, \dots)$ since it for example contains also $x^* := (x_1 + \delta/2, x_2 + \delta/2, \dots, \dots)$. $\square$