ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ
ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS
ΣΧΟΛΗ ΟΙΚΟΝΟΜΙΚΩΝ ΕΠΙΣΤΗΜΩΝ
SCHOOL OF ECONOMIC SCIENCES
ΤΜΗΜΑ ΟΙΚΟΝΟΜΙΚΗΣ ΕΠΙΣΤΗΜΗΣ
DEPARTMENT OF ECONOMICS

Πατησίων 76, 104 34 Αθήνα. Τηλ.: 210 82033437 / Fax: 210 8203301
76, Patision Street, Athens 104 34 Greece. Tel.: (+30) 210 8203422/ Fax: (+30) 210 8203301
Αναπληρωτής Καθηγητής Στυλιανός Αρβανίτης. Associate Professor Stelios Arvanitis. E-mail: stelios@aueb.gr

Syllabus of the Course: "Mathematical Economics"

General Information:
Office address: Derigni Wing, 4th floor.
Office Hours (12/2/2019-23/5/2019): Wednesdays, 11:30-13:30.[1]
e-Class: https://eclass.aueb.gr/courses/OIK231/.[2]
Tutorial Information: tba.

Course Description
The course is an introduction to notions of mathematical analysis appearing in the theory of metric spaces with applications in economic theory and/or econometrics. We examine topological notions enabled in general metric spaces. Examples are the notion of *convergence* of sequences of elements, or the *continuity* of functions defined between them, finitary notions such as compactness, etc. We are also occupied with non-topological notions, such as uniformities and completeness, and their interplay with the topological ones. In this respect we construct a vocabulary which initially enables us to address the issue of approximation of optimisation problems and possibly consider relevant applications. Furthermore the aforementioned construction enables us to state and prove a variety of fixed point theorems. We use them in order to establish *existence (and occasionally uniqueness and/or approximability)* of solutions of general systems of equations. We apply those notions to problems appearing in dynamic optimisation, game theory, etc. The combination of the aforementioned applications enables the unified consideration of both the existence of solutions in problems appearing in economic theory as well as the approximation of those (potentially not easily tractable) solutions with ones that are possibly easier to derive.

Outline
The following consists of a synopsis of the course material. It is understood that any partial modification, rearrangement, etc, is in the instructor's facility.

A.1 Sets, cardinality, sequences, structures and morphisms. Metric and metrizable spaces, metrics, open and closed balls. Topological notions enabled by the existence of open balls: open and closed subsets, convergence and separation properties, continuity and characterization by convergent sequences, continu-

[1] The course's e-class must be consulted in all cases for possible changes.

[2] The course's e-class contains the course's blog, notes, exercises, further readings and information concerning the lectures, corrections, announcements, etc. The relevant material could be updated during the course. The students must consult the e-class systematically and are strongly encouraged to upload questions, answers, comments, etc.

ous mapping theorem. Compactness and connectedness. Non topological notions: bounded and totally bounded metric spaces, metric entropy, metric completeness, Cauchy sequences, Lipchitz and uniform continuity. Topological comparison and general comparison between metrics defined on the same set. Examples: discrete spaces, Euclidean spaces, spaces of bounded and continuous functions, etc. Self-maps, contractions and the Banach fixed point theorem. Approximation of the unique fixed point. Generalisations and the fixed point theorem of Matkowski. Retractions, the Lemma of Borsuk and Ulam, the Brower's fixed point theorem and generalisations.
A.2 Applications: Convergence of sequences of minimisers and approximation of optimisation problems. Parametric optimisation. Existence and uniqueness of solutions to functional equations such as the Bellman equation in dynamic programming, or the Fredholm integral equation. Differential equations and Picard's Theorem. Existence of Nash equilibria in games. Sequences of games, notions of limit games and convergence of sequences of Nash equilibria.
B. (If time permits): Probability theory on metric spaces. Borel algebras and measures. Measurability and random elements with values on metric spaces. Polish spaces, measurability of suprema, the argmax theorem and consistency of M-estimators. Examples.

Indicative Readings

The following references are merely indicative. During the lectures this catalogue can be enriched with further readings. In any case the students are *strongly advised to study from more available sources and try to solve plethora of exercises*.

1. Aliprantis Ch., and K.C. Border. *Infinite Dimensional Analysis*, Springer, 2005.
2. Ok Efe. *Real Analysis with Economic Applications.* Princeton University Press, 2007.
3. Corbae D., Stinchcombe M, and J. Zeman. *An Introduction to Mathematical Analysis for Economic Theory and Econometrics.* Princeton U.P., 2009.
4. O'Searcoid, M. *Metric Spaces*. Springer Science & Business Media, 2006.
5. Sutherland, Wilson Alexander. *Introduction to metric and topological spaces*. Oxford University Press, 1975.
6. Border, K. C. *Fixed Point Theorems with Applications to Economics and Game Theory*. *Cambridge Books,* 1990.
7. Ambrosio, Luigi, and Paolo Tilli. *Topics on analysis in metric spaces*. Vol. 25. Oxford University Press on Demand, 2004.