Syllabus: Mathematical Economics

Academic year: 2025–2026

General information & contact

- **Instructor:** Stelios Arvanitis
- **Email:** stelios@aueb.gr
- **Office:** Derigni Wing, 4th fl.
- **Office hours:** Tuesdays 16:00-18:00
- **e-Class:** https://eclass.aueb.gr/courses/OIK231/
- **Tel.:** (+30) 210 8203437

Tutorials

- **Tutor:** Pantelis Argyropoulos
- **Tutor office:** Doma, Derigni Wing, 5th fl.
- **Tutor office hours:** By arrangement
- **Tutor email:** pantelisarg00@gmail.com

Course description

The course is essentially an introduction to notions of mathematical analysis appearing in the theory of metric spaces with applications in economic theory and/or econometrics.
A metric space is a non-empty set paired with a distance function. This structure enables the generalization of analytical notions that are available in the real line like boundedness and convergence into more general frameworks—e.g. in sets of functions. Thus, topological notions like sequential convergence and function continuity, as well as non-topological notions like (total) boundedness, uniformities and completeness, as well as their interplay, are examined.
Via the vocabulary constructed several applications are considered. Examples involve the issue of approximation of optimization problems and its subsequent applications in economics and/or econometrics. Some fixed-point theorems are also derived. They are used to establish existence, and occasionally uniqueness and approximability, of solutions of general systems of equations. They are applied to problems in a wide scope of areas like dynamic macroeconomics, game theory, general equilibrium, etc.

Remark (e-Class): The course's e-class contains the course's blog, notes, exercises, further readings, and information concerning the lectures, corrections, announcements, etc. The relevant material could be updated during the course. Students must consult the e-class systematically and are strongly encouraged to upload questions, answers, comments, etc.

Outline

The following consists of a synopsis of the course material. It is understood that any partial modification, rearrangement, etc., is in the instructor's facility.

- **A.1** Sets, cardinality, sequences, structures, and morphisms. Metric and metrizable spaces, metrics, open and closed balls. Topological notions enabled by the existence of open balls: open and closed subsets, convergence and separation properties, continuity and characterization by convergent sequences, continuous mapping theorem. Compactness and connectedness. Non-topological notions: bounded and totally bounded metric spaces, metric entropy, metric completeness, Cauchy sequences, Lipschitz, and uniform continuity. Topological comparison and general comparison between metrics defined on the same set. Examples: discrete spaces, Euclidean spaces, spaces of bounded and continuous functions, etc. Self-maps, contractions and the Banach fixed-point theorem. Approximation of the unique fixed point. Generalizations and the fixed point theorem of Matkowski. Retractions, the Lemma of Borsuk and Ulam, Brouwer's fixed-point theorem and generalizations.
- **A.2** Applications: Convergence of sequences of minimizers and approximation of optimization problems. Parametric optimization. Existence and uniqueness of solutions to functional equations such as the Bellman equation in dynamic programming, or the Fredholm integral equation. Differential equations and Picard's Theorem. Existence of Nash equilibria in games. Sequences of games, notions of limit games and convergence of sequences of Nash equilibria.
- **B (if time permits)** Probability theory on metric spaces. Borel algebras and measures. Measurability and random elements with values on metric spaces. Polish spaces, measurability of suprema, the argmax theorem and consistency of M-estimators. Examples. Wasserstein distances on spaces of probability measures with applications in economics.

Indicative readings

The following references are merely indicative. During the lectures this catalogue can be enriched with further readings. In any case the students are strongly advised to study from more available sources and try to solve a plethora of exercises.

1. Aliprantis Ch., and K.C. Border. *Infinite Dimensional Analysis*. Springer, 2005.
2. Ok, Efe. *Real Analysis with Economic Applications*. Princeton University Press, 2007.
3. Corbae D., Stinchcombe M., and J. Zeman. *An Introduction to Mathematical Analysis for Economic Theory and Econometrics*. Princeton University Press, 2009.
4. O'Searcoid, M. *Metric Spaces*. Springer Science & Business Media, 2006.
5. Sutherland, Wilson Alexander. *Introduction to Metric and Topological Spaces*. Oxford University Press, 1975.
6. Border, K.C. *Fixed Point Theorems with Applications to Economics and Game Theory*. Cambridge University Press, 1990.
7. Ambrosio, Luigi, and Paolo Tilli. *Topics on Analysis in Metric Spaces*. Oxford University Press, 2004.
8. Subrahmanyam, P.V. *Elementary Fixed-Point Theorems*. Springer, 2018.
9. Galichon, A. *Optimal Transport Methods in Economics*. Princeton University Press, 2016.