Athens University of Economics and Business
Department of Economics

Postgraduate Program - Master's in Economic Theory
Course: Econometrics II
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Introduction to Stochastic Processes

EXERCISES

1. Provide examples of your own of

- a weakly but not strictly stationary process
- a strictly but not weakly stationary process.

2. SP transformations and weak stationarity.

Consider an i.i.d process $(x_t)_{t\in\mathbb{Z}}$, $x_t \sim \text{Pareto type I}(a), x_{\min} = 1, \ \ \forall t \in \mathbb{Z}$.

The density of this distribution is

$$f_X(x) = \frac{a x_{\min}^a}{x^{1+a}}, \quad x \geq 1$$

For the Pareto distribution we know that for both the mean and variance to exist, it must be the case that its parameter is $a > 2$. If $a \leq 1$ not even the mean exists.

Examine what happens with weak stationarity in the following cases:

a. $a = 3$ **and the transformation** $y_t = x_t^3 = h(x_t)$

b. $a = 1$ **and the transformation** $y_t = \sigma \ln x_t = h(x_t), \ \ \sigma > 0$

Hint: *to obtain the density of* y_t *in both cases, apply the "change-of-variable" method for a strictly monotonic transformation.*

3. Provide an example of a "Long-memory" process, i.e. one whose autocovariance function tends to zero at the limit (the process is "regular"), but the corresponding autocovariance series is not absolutely summable.

Note: *A "Long-memory" process is an example where "square-summability" holds, but not "absolute summability".*

4. Suppose that $(u_t)_{t\in\mathbb{Z}}$ is a stationary and ergodic white noise process, and z is a random variable with $E(z)=0,\ 0<E(z^2)<+\infty$.

- Show that the process $(\varepsilon_t)_{t\in\mathbb{Z}}$, $\varepsilon_t := u_t + z,\ \forall t\in\mathbb{Z}$ is strictly and weakly stationary.
- Taking into account some variant of a Law of Large Numbers, what is an alternative way to show that the process $(\varepsilon_t)_{t\in\mathbb{Z}}$, is ***not*** ergodic? (by "alternative" it is meant "don't replicate the method used in the similar example in the Notes". Hint: use *reductio ad absurdum*).

5. For $(\varepsilon_t)_{t\in\mathbb{Z}}$ as in Exercise 4, and $(x_t)_{t\in\mathbb{Z}}$ a strictly stationary and ergodic process, with $0<E(x^2)<+\infty$, consider the process

$$(y_t)_{t\in\mathbb{Z}},\ \ y_t := \beta x_t + \varepsilon_t,\quad \forall t\in\mathbb{Z},\quad \beta\in\mathbb{R}$$

Show that the Ordinary Least Squares Estimator (OLSE) for β is inconsistent.

6. In the framework of Exercise 5, assume also that ε_t is independent of $f_s := \sigma(x_{s-i}, i\geq 0),\ \forall t,s\in\mathbb{Z}$. Show that the OLSE for β is unbiased.

7. Suppose that $(z_t)_{t\in\mathbb{Z}}$ is an i.i.d. process, with

$$E(z_t)=0,\ E(z_t^2)=1,\ E(z_t^4)<+\infty,\quad \forall t\in\mathbb{Z}$$

For $m>0$ consider the process $(\varepsilon_t)_{t\in\mathbb{Z}}$, $\varepsilon_t := \prod_{j=0}^{m} z_{t-j}$.

For $f_t := \sigma(z_{t-i}, i\geq 0) = \sigma(z_t, z_{t-1}, z_{t-2}, \ldots)$

- Find the process $(h_t)_{t\in\mathbb{Z}}$, $h_t := \operatorname{Var}(\varepsilon_t \mid f_{t-1})$.
- Show that it is strictly stationary, weakly stationary, and ergodic.
- Find the *P-a.s* limit of $\frac{1}{T}\sum_{t=1}^{T} h_t$ as $T\to+\infty$.

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