# 1 STATIC MODELS OF OLIGOPOLY

## 1.1 Using the Cournot model to study network externalities

**Overview:** *Modeling demand side economies of scale. Entry as a substitute for quantity commitment.*

Katz and Shapiro (AER, 1985).

Each consumer buys one unit. The stand-alone value $r$ is uniformly distributed in $(-\infty, A)$.[1] The network value is $v(x^e)$, where $x^e$ is the consumer expectation about how many consumers will buy the product. The product price is $p$. If $r + v(x^e) \geq p$, the consumer buys, otherwise he does not.

### 1.1.1 A monopolist commits to selling $x$ units

The profits are,

$$\pi = x\left[A + v(x) - x\right].$$

The first order condition is,

$$A + v(x) - x + x\left[v'(x) - 1\right] = 0.$$

To simplify the analysis let's assume that $v(x) = v_0 + v_1 x$. The optimal $x$ is,

$$x^c = \frac{A + v_0}{2(1 - v_1)}.$$

### 1.1.2 A monopolist cannot commit

Suppose consumers still believe that the monopolist will produce $x^c$. The profits are,

$$\pi = x\left[A + v(x^c) - x\right].$$

The first order condition is,

$$A + v(x^c) - x - x = 0.$$

Solving with respect to $x$ we obtain,

$$x^n = \frac{A + v(x^c)}{2} < x^c.$$

The last inequality follows from the first order conditions. When the FOC becomes zero in the no commitment case, the FOC in the commitment case is positive, which implies that it will become zero at a higher $x$. This suggests that when the monopolist cannot commit he has incentives to produce less than in the commitment case.

[1] The assumption of no finite lower bound avoids corner solutions where all consumers enter the market.

However, if consumers are rational $x^n$ is not an equilibrium. In the self-fulfilling equilibrium, where $x = x^e$, we have,

$$x^e = \frac{A + v(x^e)}{2}.$$

Assuming that $v(x) = v_0 + v_1 x$ the optimal output is,

$$x^e = \frac{A + v_0}{2 - v_1}.$$

Observe that,

$$x^n < x^e < x^c.$$

The monopoly profits are,

$$\pi^m = \left(\frac{A + v_0}{2 - v_1}\right)^2.$$

### 1.1.3 Duopoly with network externalities and compatible products

The profit function of firm 1 is,

$$\pi_1 = x_1 \left[A + v(X^e) - (x_1 + x_2)\right].$$

The first order condition is,

$$A + v(X^e) - X - x_i = 0.$$

In a rational expectations symmetric equilibrium we have $X^e = X = 2x$ and $x_1 = x_2 = x$. This yields,

$$A + v(2x) = 3x.$$

If we assume that $v(X) = v_0 + v_1 X$ then,

$$x^d = \frac{A + v_0}{3 - 2v_1}.$$

The aggregate profit is,

$$\Pi^d = 2\left(x^d\right)^2 = 2\left(\frac{A + v_0}{3 - 2v_1}\right)^2.$$

Contrast $\Pi^d$ with the monopoly profit,

$$\Pi^m = \left(\frac{A + v_0}{2 - v_1}\right)^2.$$

It can be shown that $\Pi^d > \Pi^m \Leftrightarrow v_1 > .29$. If network externality is strong, then entry is profitable. Hence, it may be profitable for the monopolist to invite entry. It is not credible for the monopolist to say it will produce more. Entry solves the commitment problem. Nevertheless, profits of one firm, in the duopoly, are less than the monopoly profits. Hence, the incumbent must get compensated for inviting entry.

### 1.1.4 Duopoly with network externalities and incompatible products

The profit function of firm $i$ is,

$$\pi_i = x_i \left[A + v\left(x_i^e\right) - \left(x_1 + x_2\right)\right].$$

The first order condition is,

$$A + v\left(x_i^e\right) - x_j - 2x_i = 0.$$

In a rational expectations symmetric equilibrium we have,

$$A + v\left(x\right) = 3x.$$

If we assume that $v\left(X\right) = v_0 + v_1 x$ then,

$$x^d = \frac{A + v_0}{3 - v_1}.$$

Each firm in the symmetric equilibrium produces less than when the products are compatible. There are other equilibria. One is: $x_1 > 0$ and $x_2 = 0$. In this equilibrium,

$$A + v\left(x_1\right) - 2x_1 = 0.$$

Why would firm 2 choose $x_2 = 0$? Firm 2's first order condition is,

$$A + v\left(x_2\right) - 2x_2 - x_1 = \left(\text{when } x_2 = 0\right) = A - x_1.$$

If $x_1 > A$, then $x_2 = 0$. In turn, $x_1$ satisfies,

$$x_1 = \frac{A + v\left(x_1\right)}{2}.$$

If $v\left(A\right) > A$, then $x_1^d > A$, see figure **??**. This says that if network externalities are strong, then we have tipping where all consumers consume one product.