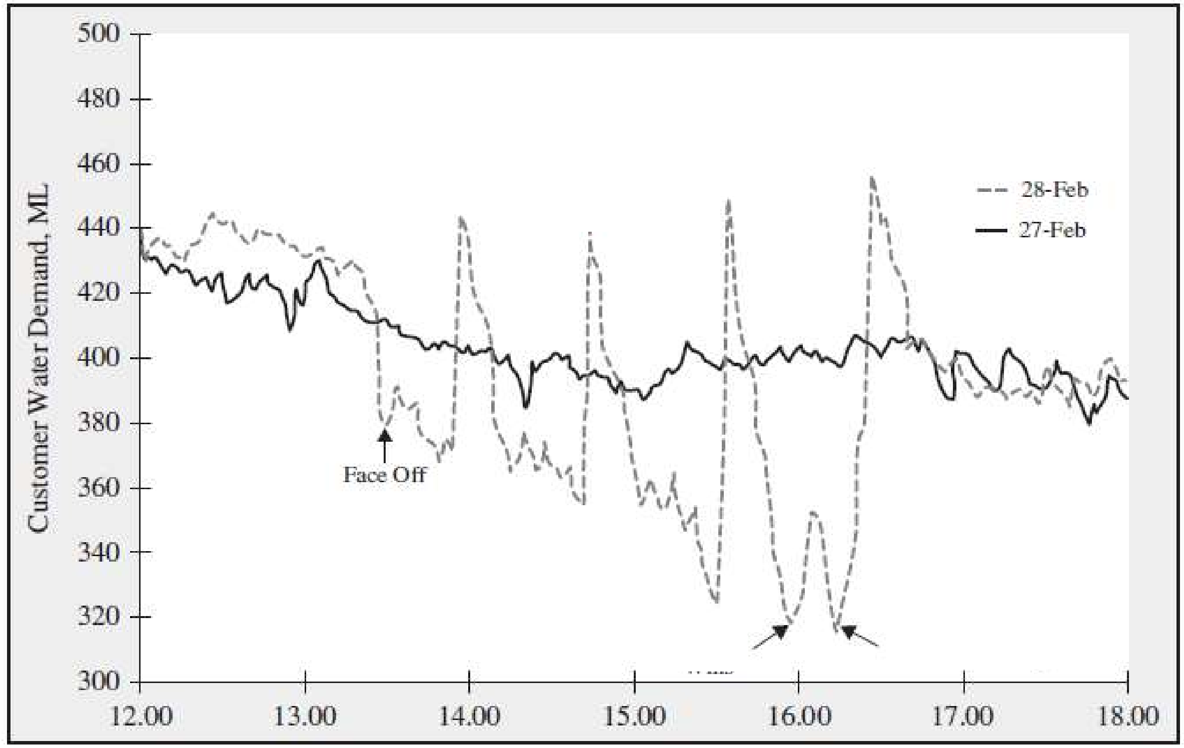
Session: Forecasting (Extra)

Forecasts

- Figure shows the **seasonality** in water consumption in a city in Canada.
- February 28, 2010, was a special day for the Canadians because their hockey team was playing for Olympic gold.
- Half of Canada was glued to their TV screens watching the live broadcast of the game.



Simple Exponential Smoothing

$$\hat{y}_{\mathrm{t+1}} = a \cdot y_t + (1 - a) \cdot \hat{y}_t$$

- Requires only three items of data
  - The last period's forecast
  - The actual data point for this period
  - A smoothing parameter, alpha (α), where $0 \leq \alpha \leq 1$

Simple Exponential Smoothing

Comments:

- Gives more weight to the recent observations and can be adjusted by changing the smoothing parameter
- α smoothing parameter controls the applied weights and the number of observations considered:
  - larger α values emphasize recent levels of data points and result in forecasts more responsive to changes in the underlying average
  - Smaller α values are analogous to increasing the value of n in the moving average method and giving greater weight to past demand

Simple Exponential Smoothing: Example

Reconsider the arrival data in the previous example. It is now the end of week 3 so the actual arrivals is known to be 411 ships.

Week	Arrivals
1	400
2	380
3	411

- Using $\alpha = 0.10$, calculate the exponential smoothing forecast for week 4
- What was the forecast error for week 4 if the actual demand turned out to be 415?
- What is the forecast for week 5?

Simple Exponential Smoothing: Example

To obtain the forecast for week 4, using exponential smoothing with and the initial forecast of 390*, we calculate the forecast for week 4 as:

$$F4 = 0.10(411) + 0.90(390) = 392.1$$

The forecast error for week 4 is: E_4 = 415 − 392 = 23

The new forecast for week 5 would be:

$$F_5 = 0.10(415) + 0.90(392.1) = 394.4 \text{ or } 394 \text{ patients.}$$

Linear Regression

Dependent variable: The variable that one wants to forecast

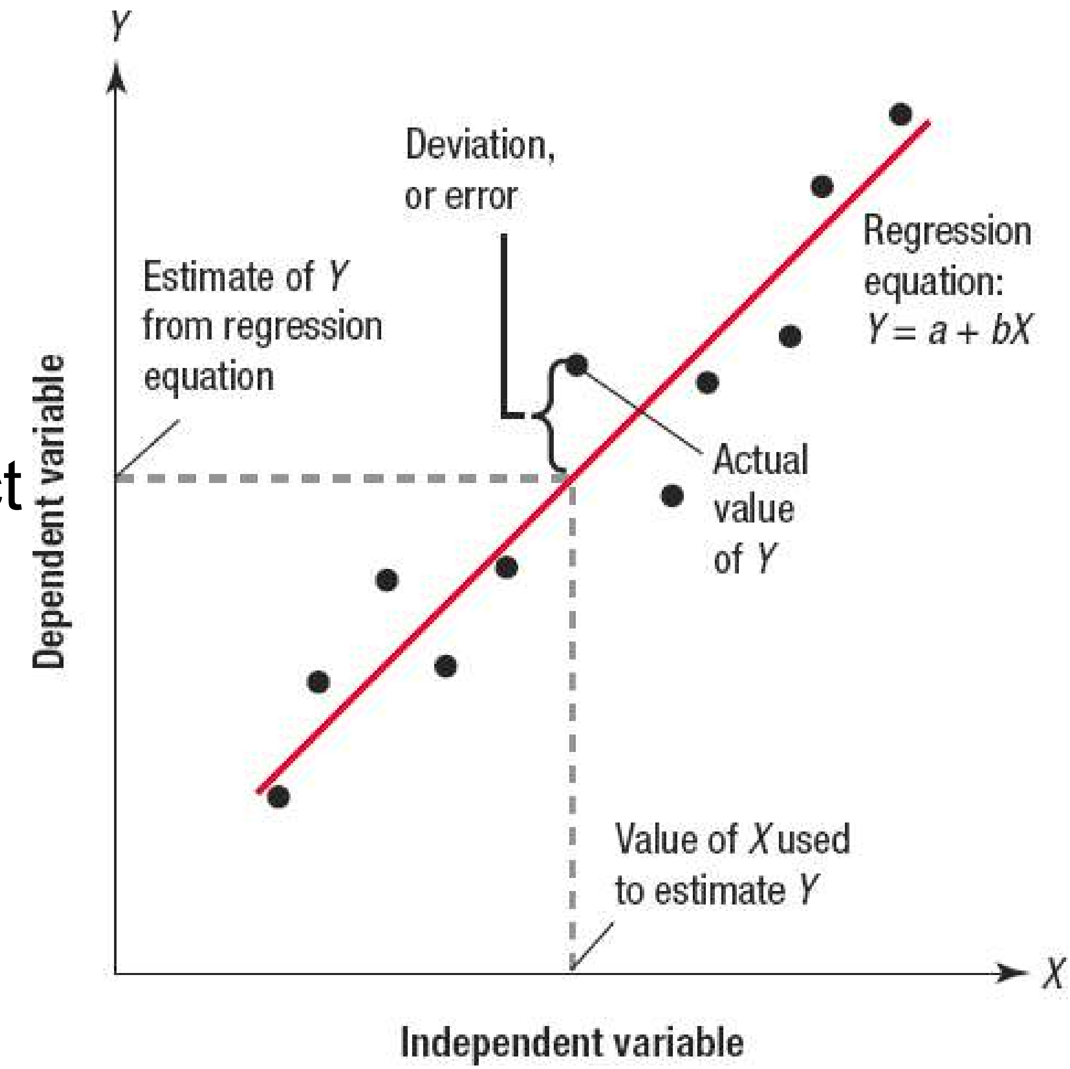
Independent variable: The variable that is assumed to affect the dependent variable and thereby "cause" the results observed in the past

Simple linear regression model:

Y* = *a* + *bX

where

- *Y* = dependent variable
- *X* = independent variable
- *a* = Y-intercept of the line
- *b* = slope of the line



Linear Regression

- To calculate the parameters *a*, *b* to minimise the sum of squared differences between the true data and the forecasted:

$$min \sum (y - \hat{y})^2$$

$$\hat{\beta} = \frac{v \sum_{i=1}^{v} X_i Y_i - \left(\sum_{i=1}^{v} X_i \right) \left(\sum_{i=1}^{v} Y_i \right)}{v \left(\sum_{i=1}^{v} X_i^2 \right) - \left(\sum_{i=1}^{v} X_i \right)^2}$$

$$\hat{a} = \bar{Y} - \hat{\beta} \bar{X}$$

$$\bar{Y} = \frac{1}{v} \left(\sum_{i=1}^{v} Y_i \right) \qquad \bar{X} = \frac{1}{v} \left(\sum_{i=1}^{v} Y_i \right)$$

$$r = \frac{\sum_{i=1}^{v} \left(X_i - \bar{X} \right) \left(Y_i - \bar{Y} \right)}{\sqrt{\sum_{i=1}^{v} \left(X_i - \bar{X} \right)^2} \sqrt{\sum_{i=1}^{v} \left(Y_i - \bar{Y} \right)^2}}$$

Linear Regression

The sample correlation coefficient, r

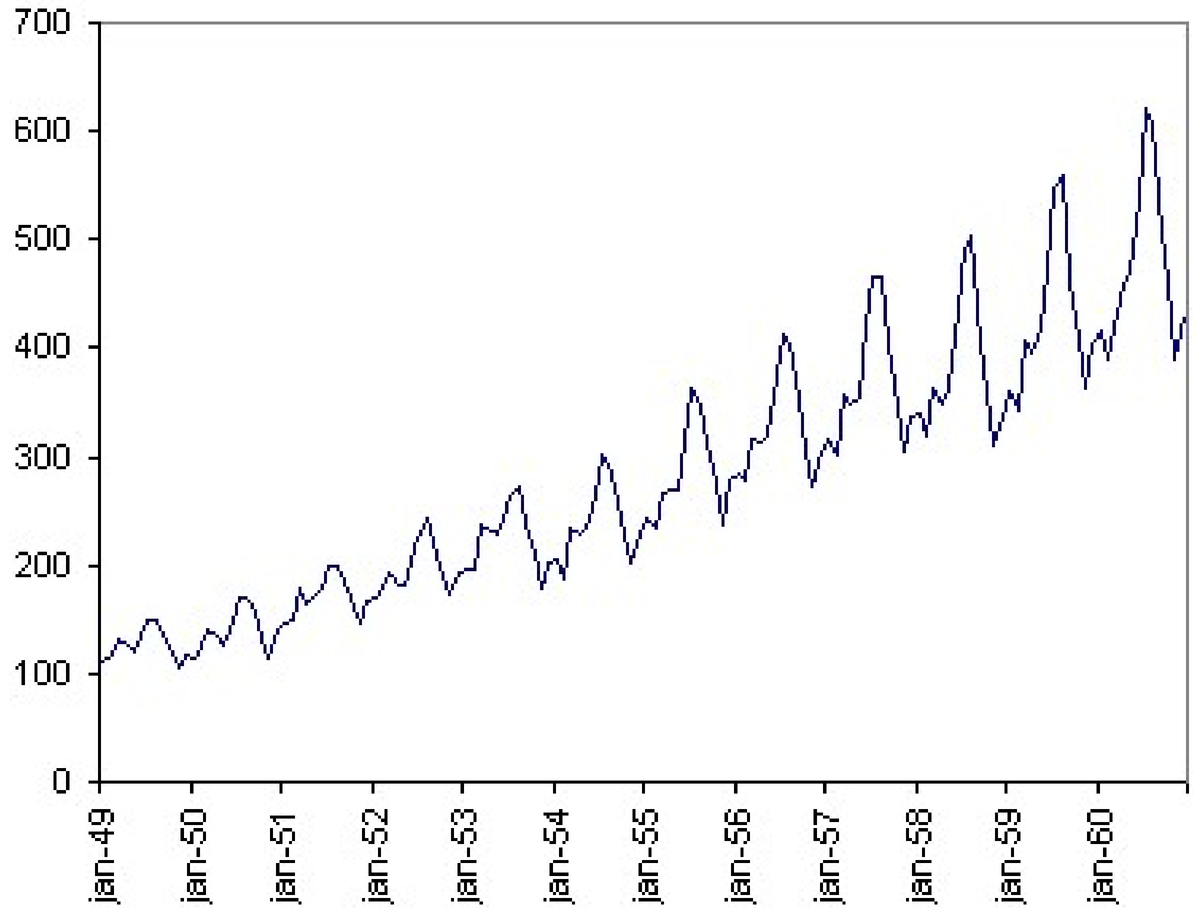
- Measures the direction and strength of the relationship between the independent variable and the dependent variable
- The value of r can range from $-1.00 \leq r \leq 1.00$

The sample coefficient of determination, r^2

- Measures the amount of variation in the dependent variable about its mean that is explained by the regression line
- The value of r^2 can range from $0.00 \leq r^2 \leq 1.00$

Multiplicative Seasonal Method

A method whereby seasonal factors are multiplied by an estimate of average demand to arrive at a seasonal forecast



Multiplicative Seasonal Method

Steps:

1. For each year, calculate the **average demand** for each season by dividing annual demand by the number of seasons per year
2. For each year, divide the **actual demand** for each season by the **average demand** per season, resulting in a **seasonal factor** for each season
3. Calculate the average seasonal factor for each season using the results from Step 2
4. Calculate each season's forecast for next year

Multiplicative Seasonal Method: Example

- A carpet cleaning company needs a quarterly forecast of the number of customers expected next year. The carpet cleaning business is seasonal, with a peak in the third quarter and a low in the first quarter.
- The manager wants to forecast customer demand for each quarter of year 5, based on an estimate of total year 5 demand of 2,600 customers.

Demand for the last 4 years

Quarter	Year 1	Year 2	Year 3	Year 4
1	45	70	100	100
2	335	370	585	725
3	520	590	830	1160
4	100	170	285	215
Total	1000	1200	1800	2200

Multiplicative Seasonal Method

Step 1: For each year, calculate the average demand for each season by dividing annual demand by the number of seasons per year

Year 1: 1000/4 = 250
Year 2: 1200/4 = 300
Year 3: 1800/4 = 450
Year 4: 2200/4 = 550

Multiplicative Seasonal Method

Step 2: For each year, divide the actual demand for each season by the average demand per season, resulting in a seasonal factor for each season

Quarter	Year 1	Year 2	Year 3	Year 4
1	45/250=0.18	70/300=0.23	100/450=0.22	100/550=0.18
2	335/250=1.34	370/300=1.23	585/450=1.30	725/550=1.32
3	520/250=2.08	590/300=1.97	830/450=1.84	1160/550=2.11
4	100/250=0.40	170/300=0.57	285/450=0.63	215/550=0.39
Total	1000	1200	1800	2200

Multiplicative Seasonal Method

Step 3: Calculate the average seasonal factor for each season using the results from Step 2

Quarter	Average Seasonal Index
1	(0.18+0.23+0.22+0.18)/4=0.20
2	(1.34+1.23+1.30+1.32)/4=1.30
3	(2.08+1.97+1.84+2.11)/4=2.00
4	(0.40+0.57+0.63+0.39)/4=0.50

Multiplicative Seasonal Method

Step 4: Calculate each season's forecast for next year

- Estimation of total year 5 demand of 2,600 customers
- Average demand for each season: 2,600/4 = 650
- Forecast for each season:
  - Q1: 650 * 0.20 = 130
  - Q2: 650 * 1.30 = 845
  - Q3: 650 * 2.00 = 1,300
  - Q4: 650 * 0.50 = 325