Session 2: Inventory Management

Agenda

- Fundamentals of Inventory Management
- Pressures for **Low** and **High** inventories
- Inventory Models & Discounts

Expected outcomes

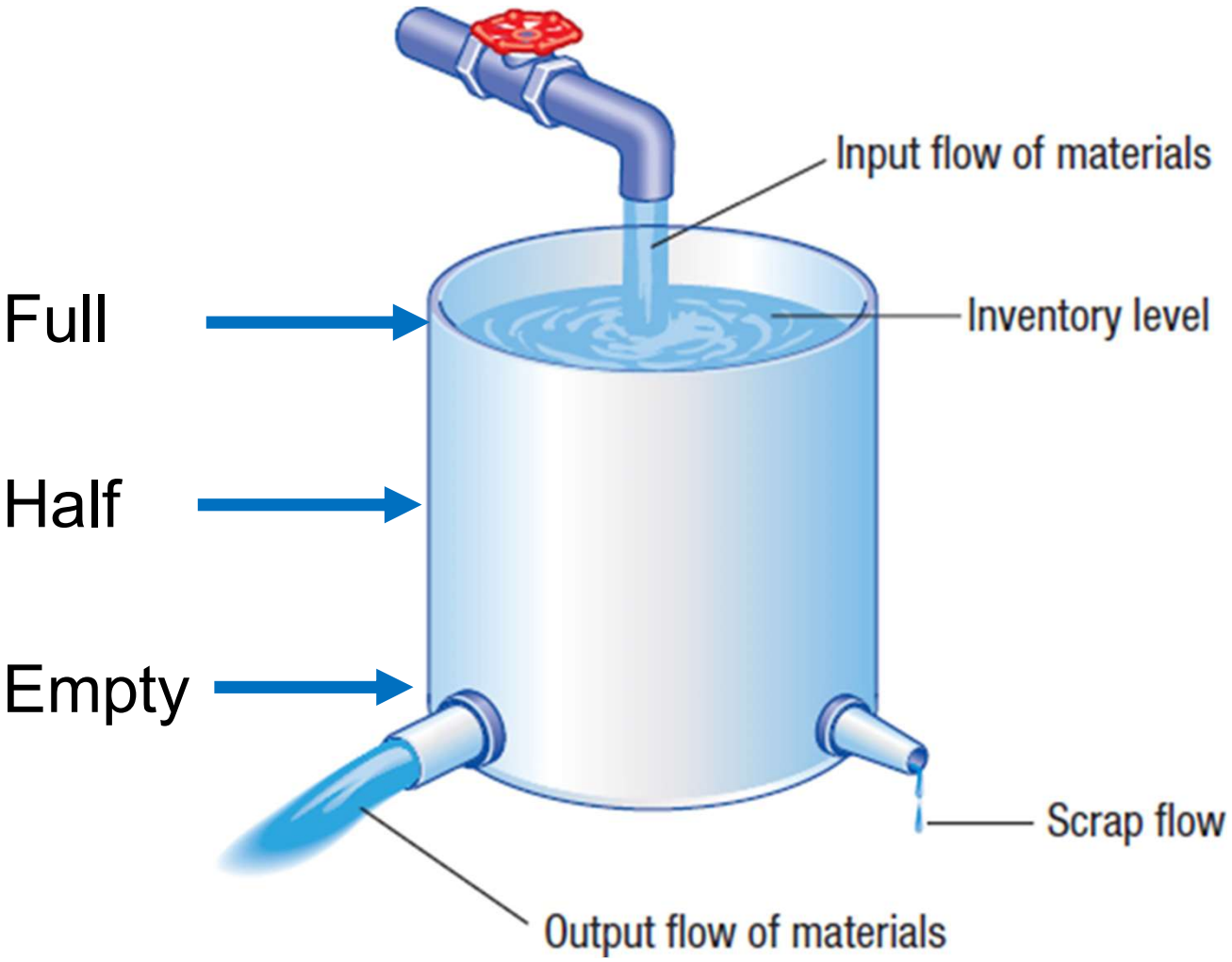
- You will be able to:
  - **Identify** pressures of high and low inventories
  - **Formulate** the total and operating (annual) cost as a function of the order quantity
  - **Calculate** the optimal order quantity
  - **Criticise** on different inventory models and **Select** the proper one

Definition

- What is inventory?
  - It is the term that refers to goods and materials that a business holds for sale in the near future

- Inventory is created when the receipt of materials, parts or finished goods exceeds their disbursement
- The role of inventory:
  - To **balance** supply and demand
  - To **buffer** uncertainty in demand and supply and mitigate the risk
  - To **enable** economies of scale and customization

Creation of Inventory



Types of Inventories

- **Accounting**:
  - **RM**: Raw materials
  - **WIP**: Work-in-process
  - **FG**: Finished goods (**SP**: spare parts)
- **Operations**:
  - Cycle Inventory
  - Safety Stock Inventory
  - Anticipation Inventory
  - Pipeline Inventory

- Reasons for holding inventories **vary** according the type of inventory
- Each type has different attributes and, thus, requires a **different management policy**

Types of Inventories - accounting

- **Accounting**:
  - **RM**: Raw materials (components, subassemblies, or materials purchased from suppliers)
  - **WIP**: Work-in-process (unfinished parts or products released in a production system)
  - **FG**: Finished goods (finished products not yet sold to customers)
  - **SP**: Spare parts (components used to repair or maintain equipment)





Types of Inventories - operations

- **Operations**:
  - **Cycle Inventory**: is the portion of total inventory that varies directly with the **order quantity**
  - **Safety Stock Inventory**: is the **buffer** inventory (to protect a company against uncertainties in demand, disasters, and/or late deliveries
  - **Anticipation Inventory:** Inventory used to **absorb uneven rates** of demand in supply (seasonal products)
  - **Pipeline Inventory:** Inventory that is created when an **order** for an item is issued but **not yet received**

Examples of Pipeline Inventory

Reasons for Holding Inventory

The reasons depend on the type of inventory

- **Raw materials**:
  - Batching (cycle stock)
  - Variability (safety stock / planned-safety lead time)
  - Obsolescence
- **Work-in-process**:
  - Queuing (cause: high utilisation - flow/process variability)
  - Processing
  - Waiting for batch (cause: large batch size and no lot splitting)
  - Moving
  - Waiting to match (cause: variability / no synch)

Reasons for Holding Inventory

- **Finished goods**:
  - Customer responsiveness (make-to-stock)
  - Batch production
  - Forecast errors
  - Production variability (compensate yield loss)
  - Seasonality (built-ahead inventory)
  - Factors are interrelated (seasonality-forecast)
- **Spare parts**:
  - Service (parts available for repairs)
  - Purchasing / production lead times
  - Batch replenishment
  - Similarities with FG (service level)

Measures of Inventory Performance (1)

https://www.youtube.com/watch?v=zERrqLFotSY

1. **Inventory turnover**: ratio between the average amount of inventory a firm holds and its level of sales

   = *Costs of goods sold / average inventory (in terms of cost)*

   - **High Inventory Turnover:** Indicates that a company is selling its inventory quickly. This can be a sign of strong sales or effective inventory management, but if too high, it may also indicate insufficient inventory levels, leading to stockouts and lost sales.
   - **Low Inventory Turnover:** Suggests that inventory is not selling quickly, which could be due to weak sales, overstocking, or ineffective inventory management. This may lead to higher holding costs and potential obsolescence.

Measures of Inventory Performance (2)

2. **Days of supply**: number of days of operations that can be supported with inventory on hand

   = *Current Inventory / Expected rate of daily demand*

3. **Service level**: is the probability of not losing sales (i.e., the probability of not hitting a stock-out)

Pressures for **Low** Inventories

Inventory Holding Cost

Components:

- **Cost of Capital**
  - To finance inventory, a firm must obtain a loan or forgo the opportunity of an investment promising an attractive return
- **Storage and Handling Costs**
  - Inventory takes up space (need for warehouse facilities) and must be moved into and out of storage
  - There is also an opportunity cost for storage when a firm could use storage space productively in some other way

Pressures for Low Inventories

- **Taxes and Insurance**
  - Taxes are related on the level of inventory
  - Insurance on assets are increased when there is more to insure
- **Shrinkage**
  - **Pilferage**: theft of inventory by customers or employees
  - **Obsolescence**: inventory can not used or sold at full value because of model changes, engineering modifications or unexpectedly low demand
  - **Deterioration**: physical spoilage or damage results in lost value

Pressures for **Low** Inventories

In summary:

- Inventory represents a **temporary monetary investment** in goods on which a firm must pay interest
- Inventory holding cost is the **variable** cost of keeping items on hand, including interest, storage and handling, taxes, insurance and shrinkage; it typically ranges from **20 to 40%** of the unit price

Why is it important to hold inventory?

Pressures for **High** Inventories

- **Customer Service**
  - Creating inventory can speed delivery and improve on-time delivery
  - Inventory reduces the potential for stock-outs and backorders

- **Stock-out:** it occurs when an item is not available to satisfy a demand the moment it occurs, resulting in loss of sale
- **Backorder:** it is a customer order that can not be filled when promised or demanded but is filled later (i.e. customers are willing to wait for receiving their orders, sometimes customers are given discounts for the inconvenience of waiting)

Pressures for **High** Inventories

- **Ordering Cost**
  - The cost of placing a new order (this can be the time that is spent on paperwork, follow-up and receiving an order)
- **Setup Cost**
  - The cost of changing a machine to produce a different component/item (it includes labour and time to make the changeover, cleaning and new tools or fixtures)
  - The cost of making a machine ready to be used

Pressures for **High** Inventories

- **Transportation Cost**
  - Having inventory on hand optimises fleet utilisation and also minimises the need to expedite shipments by more expensive modes of transportation
  - Combining orders of different items from same supplier may lead to rate discounts
- **Payments to suppliers**
  - To achieve economies of scale (e.g. quantity discounts)
  - To take advantage of information, for example if you learn that a key supplier is about to increase prices

Pressures for **High** Inventories

- **Labour and Equipment Utilisation**
  - By creating more inventory, management can increase workforce productivity and facility utilisation in three ways.
    1. To reduce the chance of costly rescheduling of production lines because the components needed to make the product are not in inventory
    2. To improve resource utilisation by stabilising the output rate for industries when demand is cyclical or seasonal and minimise the need for extra shifts, overtime, hiring and firing
    3. To reduce the number of setups, which add no value to a product or service

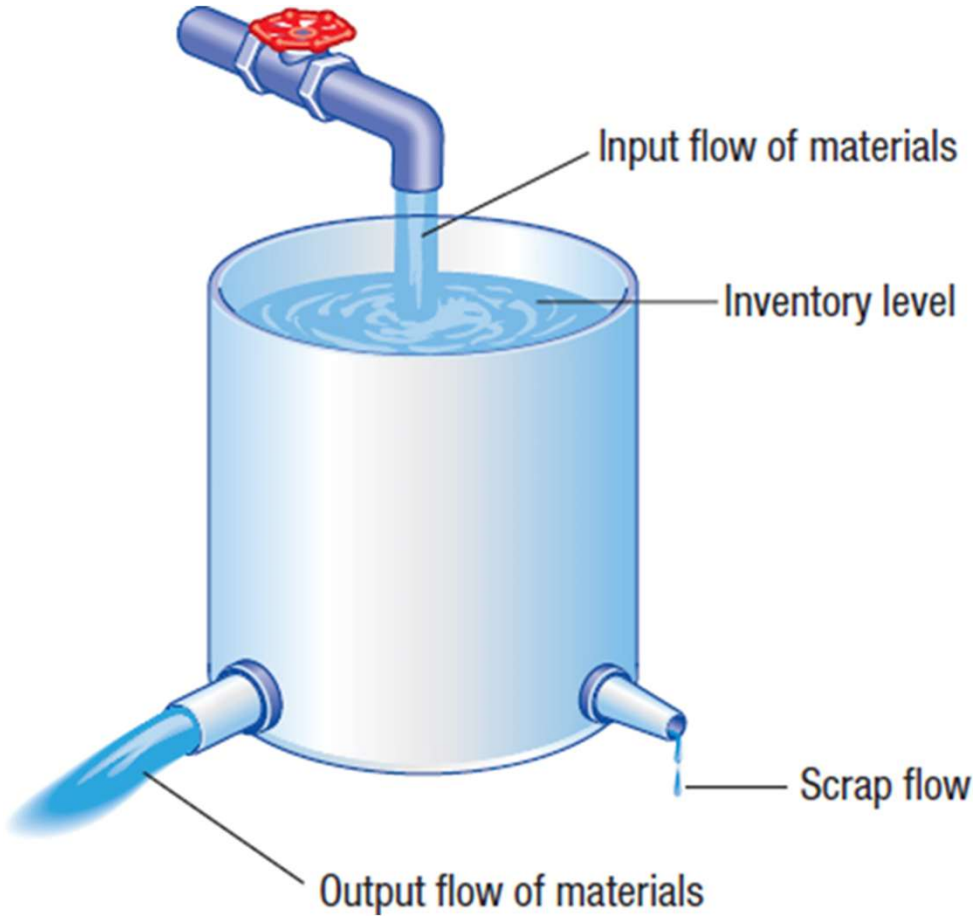
Pressures for **High** Inventories

In summary:

- Inventory can be used to increase customer satisfaction
- To reduce ordering, setup and transportation cost
- To achieve economies of scale
- To work more efficient (increase workforce productivity)

Inventory Trade-Offs

Creation of Inventory



Warehouse of the Future

Kiva Robots: https://www.youtube.com/watch?v=HYjc9h8oSsY

Ocado: https://www.youtube.com/watch?v=4DKrcpa8Z_E

Economic Order Quantity

- There are **conflicting pressures** to keep inventories **low** enough to avoid excess inventory holding costs but **high** enough to reduce the frequency of orders/setups.
- A good starting point for **balancing** these conflicting pressures and determining the best cycle-inventory level for an item is finding the **Economic Order Quantity** (EOQ).
- EOQ is defined as the lot size that **minimises** total annual **inventory holding** and **ordering (or setup)** costs.

EOQ Assumptions

- When the following **assumptions** hold, the optimal EOQ can be calculated analytically:
  - The model can only be used for "independent" demand and instantaneous production.
  - The demand rate is constant and known in advance.
  - There are no constraints in terms of lot size.
  - The only relevant costs are the inventory holding cost and fixed (ordering or setup) cost.
  - There is no uncertainty in lead-time or supply (immediate).
- In reality, very few situations are so simple and well behaved.
- EOQ is a **reasonable first approximation** of average lot sizes.

Calculating EOQ

- To calculate the Economic Order Quantity:
  1. **Formulate** the total annual cost for any quantity.
  2. **Derive** EOQ, which is the lot size (i.e. quantity) that minimises total cost.

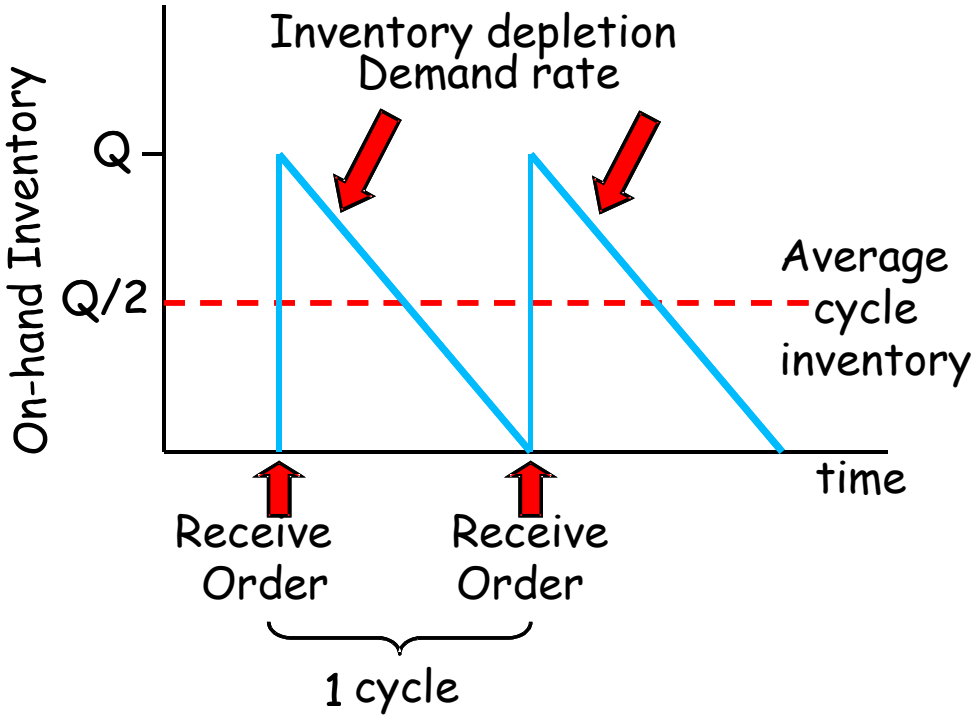
- Given the assumption of constant and known demand rate, we are able to convert EOQ into the elapsed time between orders (TBO).

Notation

- Parameters
  - S: Ordering (or setup) cost
  - H: Holding cost (it is often calculated as a proportion of the item's value/price)
  - D: Annual demand
  - P: Unit value/price (it is **NOT** the retail price)
- Variables - Decisions
  - Q: Lot size (order quantity)

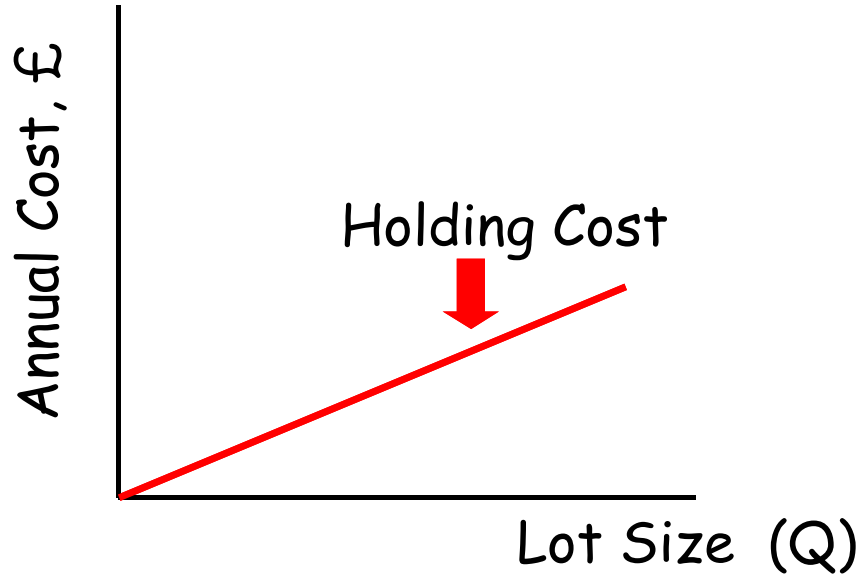
Calculating EOQ

- When the EOQ assumptions are satisfied, cycle inventory behaves as shown.
- A cycle begins with *Q* units held in inventory, which happens when a new order is received.
- During the cycle, on-hand inventory is used at a constant rate.
- Since demand is known with certainty and the lead time is zero, a new order will be placed when inventory falls to 0.



Calculating EOQ

- Inventory varies uniformly between *Q* and 0, so the average cycle inventory equals half the lot size, *Q*.
- The annual cost of holding inventory increases linearly with *Q*.

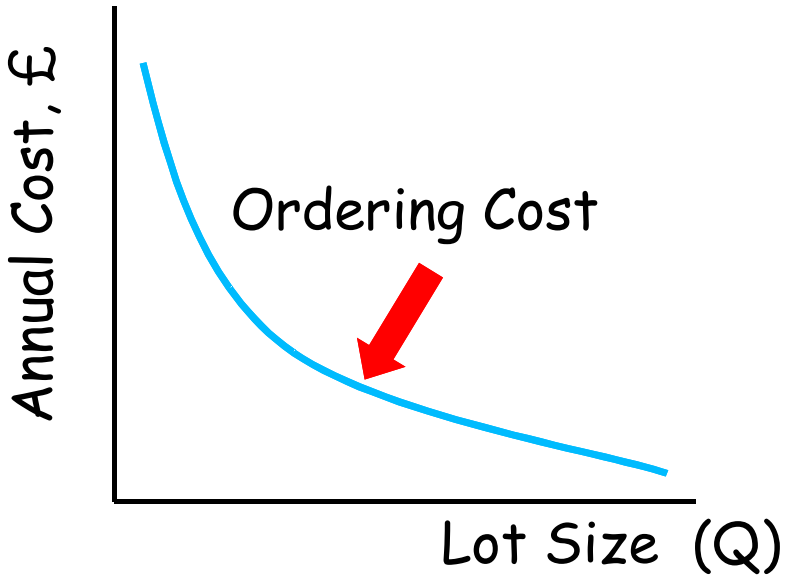


Annual holding cost = (Average cycle inventory) x (Unit holding cost)

$$AHC = \frac{Q}{2} \times H$$

Calculating EOQ

- The number of orders per year equals annual demand *D* divided by *Q*.
- The total ordering or setup cost decreases non-linearly as *Q* increases, because fewer orders are placed.



Annual ordering or setup cost =

(Number of orders / year) x (Ordering or Setup fixed cost)

$$AOC = \frac{D}{Q} \times S$$

Calculating EOQ

Total annual cost incorporates except for the inventory holding cost and ordering (or setup) cost the purchase cost of the materials studied

$$C = \boxed{\frac{Q}{2}H + \frac{D}{Q}S} + PD$$

Operating cost

Where:

C = total cost per year

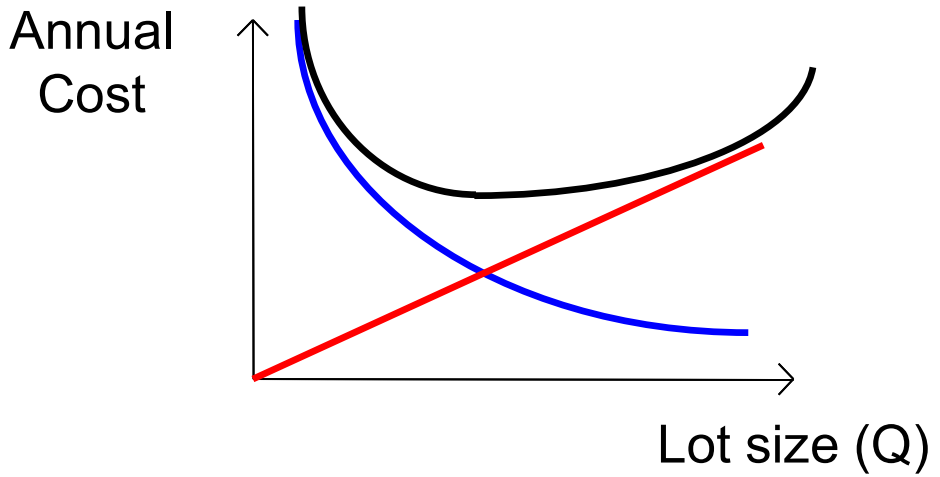
Q = lot size in units

H = cost of holding per unit of inventory for a year

D = demand in units per year

S = fixed cost of ordering or setting up one lot, per lot

P = unit price (fixed)



Calculating EOQ

$$C = \frac{Q}{2} \times H + \frac{D}{Q} \times S + PxD \Rightarrow \frac{dC}{dQ} = 0$$

$$\frac{d(\frac{Q}{2}H + \frac{D}{Q}S)}{dQ} = 0 \Rightarrow -\frac{SD}{Q^2} + \frac{H}{2} = 0$$

$$EOQ = \sqrt{\frac{2DS}{H}}$$

$$\frac{d^2TC}{dQ^2} = \frac{d(-\frac{SD}{Q^2} + \frac{H}{2})}{Q} = \sqrt{\frac{2SD}{H}} > 0$$

Time Between Orders (TBO)

- Under the model assumptions, the order quantity is related to the frequency of orders (setups) and the time between two successive orders (cycle length).
- $\frac{D}{Q}$: indicates the number of orders that should be placed to satisfy the annual demand.
- Time Between Orders (TBO) equals to the time period of aggregate demand divided by the number of orders

In our case: TBO = 1 year / (D/Q) = (Q/D) x (1 year)

Sensitivity Analysis

Formula: $EOQ = \sqrt{\frac{2DS}{H}}$

Parameter	Parameter change	EOQ change	Comments
Demand (D)	↑		
Ordering cost (S)	↑		
Holding cost (H)	↑		

Sensitivity Analysis

Formula: $EOQ = \sqrt{\frac{2DS}{H}}$

Parameter	Parameter change	EOQ change	Comments
Demand (D)	↑	↑	Increase in lot size is in proportion to the square root of D
Ordering cost (S)	↑	↑	• Pressure for high inventories • Larger lots are justified when ordering cost increases
Holding cost (H)	↑	↓	• Pressure for low inventories • Smaller lots are justified when holding cost decreases

EOQ Example

- A company produces air-conditioners, the air-conditioning units it requires to this purpose are handled as independent demand
- The production is stable across the year (Make-To-Stock policy)
- The demand rate for air-conditioners is 500 units per week
- The air-conditioning units supplier charges £0.3 per unit, while the cost of placing a purchase order is £10
- The annual cost of holding an air-conditioning unit in inventory is 25% of its value
- If the company orders 5,000 units when its inventory is exhausted, what is the annual operating cost?
- How many air-conditioning units and how often should the company order to minimise its annual operating cost?

Solution

- The total annual demand for air-conditioning units is:

  D = (500 units / week) x (52 weeks / year) = 26,000 units / year

- The annual inventory holding cost for one air-conditioning unit is:

  H = 0.25 (£0.3 / unit) = £0.075

- The annual operating cost for ordering 5,000 units is:

$$C = \frac{Q}{2} \times H + \frac{D}{Q} \times S = \frac{5000}{2} \times 0.075 + \frac{26000}{5000} \times 10 = 239.5 \text{ in pounds (£)}$$

- The Economic Order Quantity becomes:

$$EOQ = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times 26000 \times 10}{0{,}075}} \cong 2633 \text{ units}$$

Solution

- The annual operating cost for ordering EOQ (2633 units) becomes:

$$C = \frac{Q}{2} \times H + \frac{D}{Q} \times S = \frac{2633}{2} \times 0.075 + \frac{26000}{2633} \times 10 = 197.48$$

- In case that we would like to calculate the total annual cost, we need to add to the operating cost the purchasing cost as well. The purchasing cost equals: *26,000 x 0.3 = 7,800* pounds (£)

- The time between successive orders (expressed in weeks) is:

$$TBO_{EOQ} = \frac{EOQ}{D}(52 weeks / year) = \frac{2633}{26000} \times 52 \cong$$ 5.26 weeks

Summary of EOQ Model

- The EOQ model **balances** two pressures for high and low inventories
- **Holding cost** creates a need for **smaller** lot sizes
- **Ordering or setup cost** creates a need for **larger** lot sizes
- Based on the EOQ formula we **calculate** the **optimal lot size** (order quantity) and **determine** the **frequency of orders**
- The unit price is **fixed** and **independent** of the lot size

Models with Discounts

It is common in practice, suppliers/wholesalers/retailers to offer discounts if an order exceeds a threshold

Discounts offerings with quantity constitute motives for ordering higher quantities

Why to offer Discounts?

- Because, we can:
  - Reduce operating cost
  - Achieve sales goals/targets
  - Attract new and repeat customers
  - Boost reputation
  - Free up room
  - Achieve economies of scale and work more efficient
  - . . . .

Quantity Discounts

- There are different type of discounts
- We focus on **Quantity Discounts**
  - All-units discounts: If an order exceeds a given quantity threshold, a price reduction is applied to all units purchased
  - Incremental discounts: If an order exceeds a given quantity threshold, the reduced price is granted for only the units purchased beyond the threshold

Quantity Discounts Model

- The unit price (P) does not remain constant, but varies along with the ordering quantity (Q)
- $$P(Q) = \begin{cases} \$4, & if\ Q < 100 \\ \$3.5, & if\ 100 \leq Q < 200 \\ \$3, & if\ Q \geq 200. \end{cases}$$
- There are incentives for ordering higher quantities
- Therefore, it becomes crucial to find the trade-off between price discount and cost increase due to keeping larger inventory
- It is necessary to consider the total annual cost which incorporates the purchase cost

Quantity Discounts Model

- The total annual cost (C) is:

$$C = \frac{Q}{2} \times H + \frac{D}{Q} \times S + P \times D$$

Where:

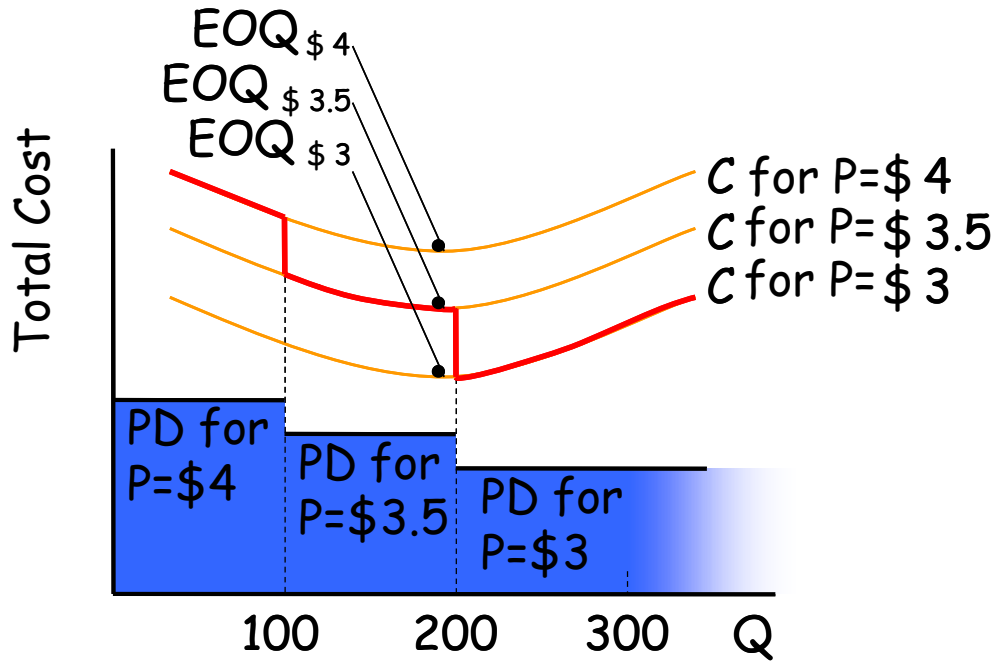
Q = lot size in units

H = cost of holding per unit of inventory for a year

D = annual demand in units

S = fixed cost of ordering or setting up one lot, per lot

P = unit price



Calculating Lot Size

- To determine the optimum lot size in the case of discounts proportional to the quantity ordered, the following approach should be followed:
  1. Calculate EOQ, starting from the lowest price to the next higher price
  2. Check for feasibility (EOQ within the volume range from the respective price)
  3. If the first feasible EOQ is found for the lowest price, it is optimal
  4. Otherwise, calculate total cost for the first feasible EOQ and the larger price break quantity at each lower price level

Quantity Discounts Example

- The retailer of Heat International is negotiating the air-conditioning units' respective purchase cost along with the quantity ordered
- Heat International offers the discounts summarized in the table provided below:

Quantity	Price /Unit
0 - 299	£60
300-499	£58,8
500-	£57

- The annual demand for air-conditioning units is 6,500 units
- Ordering cost is £50 and the annual inventory holding cost is estimated 25% of the unit's price
- How many air-conditioning units should the retailer order to minimise its total annual cost?

Solution

- For the lowest price:

$$EOQ_{57} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times 6500 \times 50}{0.25 \times 57}} = 213.57$$

Not feasible

- For the next price:

$$EOQ_{58.8} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times 6500 \times 50}{0.25 \times 58.8}} = 210.28$$

Not feasible

- For the last price:

$$EOQ_{60} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times 6500 \times 50}{0{,}25 \times 60}} = 208{,}16$$

Feasible

Solution

- The total annual cost for the feasible EOQ (Q = 208) is:

$$C_{208} = \frac{Q}{2}H + \frac{D}{Q}S + PD = \frac{208}{2}0.25 \times 60 + \frac{6500}{208}50 + 60 \times 6500 = 393{,}122.5$$

- The total annual cost when ordering 300 units:

$$C_{300} = \frac{300}{2}0.25 \times 58.8 + \frac{6500}{300}50 + 58.8 \times 6500 = 385{,}488.3$$

- The total annual cost when ordering 500 units:

$$C_{500} = \frac{500}{2}0.25 \times 57 + \frac{6500}{500}50 + 57 \times 6500 = 374{,}712.5$$

- **Since £374,712.5 is the minimum total annual cost, the optimal ordering quantity is 500 units**

Homework (1)

- A candy wholesaler faces steady daily demand of 2 tons
- The value of one ton of candies is £1,000
- The holding cost is 20% of the value of the candies
- The ordering cost is £400 per ordering
- The wholesaler operates 200 days per year
- Determine:
  1. the annual operating cost if the lot size is equal to 100 tons
  2. the lot size that minimises the total annual cost
  3. the total annual setup cost for the optimal lot size
  4. the time between orders (cycle length) if the wholesaler orders under the EOQ

Solution Homework (1)

- Annual demand D is 2*200 = 400 tons
- Holding cost H is 20%*1000 = £200
- Ordering cost S is £400

1. If Q = 100, then the annual operating cost is:
   400*(400/100) + ½* 200*100 = £11,600
2. $EOQ = \sqrt{\frac{2DS}{H}}$ = 40 tons
3. If Q = 40, the total annual setup cost is 400*(400/40) = £4,000
4. Since Q = 40 and the daily demand is 2 tons, this means the wholesaler places an order every 20 days

Homework (2)

- The value of one ton of candies is depends on the order quantity as follows:

Order Quantity	Price per ton
0-19	1000
20-39	900
40-59	800
60-	700

- Determine the lot size that minimises the total annual cost

Solution Homework (2)

- Calculate EOQ, starting from the lowest price to the next higher price

$$EOQ = \sqrt{\frac{2DS}{H}}$$

Order Quantity	Price per ton	Optimal order quantity
0-19	1000	No reason to calculate that price
20-39	900	No reason to calculate that price
40-59	800	**44.72 tons, feasible**
60-	700	**47.81 tons, not feasible**

The total annual cost when ordering 44.72 tons is:

$$\frac{44.72}{2} * 0.20 * 800 + \frac{400}{44.72} * 400 + 400 * 800 = 327{,}155$$

The total annual cost when ordering 60 tons is:

$$\frac{60}{2} * 0.20 * 700 + \frac{400}{60} * 400 + 400 * 700 = 286{,}867$$

Homework (3)

- The John Equipment Company estimates its holding cost at 15% and its ordering cost at £9 per order. The estimated annual requirement is 48,000 units at a price of £4 per unit.

- Questions:
  1. What is the most economical number of units to order?
  2. How many orders should be placed in a year?
  3. How often should an order be placed?

Solution Homework (3)

1. $EOQ = \sqrt{\frac{2DS}{H}}$ = 1200 units
2. Annual requirement / EOQ = 48,000/1,200 = 40 orders
3. Frequency of orders = 360 days / 40 orders = 9 days

Homework (4)

- To date, Raymond Bro. has been purchasing an item in lots of 900 units. This equates to a three-month supply. The cost per unit is £12, the order cost is £16 per order, and the holding cost is 25%.

- **Question**: How much can Raymond Bro. save per year by purchasing the item in the most economical quantities?

Solution Homework (4)

- Operating cost (Q=900) = 4*16 + ½*900*(0.25*12) = £1,414
- $EOQ = \sqrt{\frac{2DS}{H}}$ = 196 units
- Operating cost (Q=196) = (3,600/196)*16 + ½*196*(0.25*12) = £588
- Savings: Operating cost (Q=900) - Operating cost (Q=196)

  1,414 – 588 = £826

Homework (5)

- A manufacturing company places a semi-annual order of 24,000 units at a price of £20 per unit. The holding cost is 15% and the order cost is £12 per order.

- Questions:
  1. What is the most economical order quantity?
  2. How many orders need to be placed?

Solution Homework (5)

1. $EOQ = \sqrt{\frac{2DS}{H}}$ = 620 units

2. Number of orders per year =

   Annual requirement / EOQ = 48,000/620 = 77.42 => 78 orders