Decision Analysis

Agenda

- Key Components of Decision Making
- Decision Making Criteria **With** and **Without** Probabilities
- Decision Trees

Expected Outcomes

- You will be able to:
  - **Analyse** situations & **Base** your decisions on a specific criterion
  - **Calculate** the expected value of perfect information
  - **Develop** skills to make decisions

Classification of Problems

	A single decision maker	More than two decision makers
Deterministic		
Stochastic		

Deterministic

Stochastic

Decision Analysis Overview

- Some parameters are not known (with certainty)
- Two categories of decision situations:
  - Probabilities can be assigned to future occurrences
  - Probabilities cannot be assigned to future occurrences

Problem: Lottery Ticket

- Choice A:

  A lottery ticket that wins £50

- Choice B:

  A lottery ticket that
  - wins £100, p= 0.50
  - loses (£0), p= 0.50

Steps of Decision Making

1. Define the problem
2. List **ALL** the possible alternatives
3. Identify all the possible outcomes for each alternative
4. Determine the payoff for each alternative and outcome combination
5. Use a decision modelling technique to make your decision

Choice A or B

A: win

B: win or lose

A -> £50

B -> £100 win

B -> £0 lose

Problem: Lottery Ticket 1

- Choice A:

  A lottery ticket that wins £50

- Choice B:

  A lottery ticket that
  - wins £100, p= 0.50
  - loses (£0), p= 0.50

Problem: Lottery Ticket 2

- Choice A:

  A lottery ticket that wins £5,000

- Choice B:

  A lottery ticket that { wins £10,000, p= 0.50

  loses (£0), p= 0.50

Problem: Lottery Ticket 3

- Choice A:

  A lottery ticket that wins £500,000

- Choice B:

  A lottery ticket that
  - wins £1,000,000, p= 0.50
  - loses (£0), p= 0.50

Human Behaviour

- Risk attitudes:
    - risk averse
    - risk neutral
    - risk prone

- Risk averse: Prefers the guaranteed payment
- Risk neutral: Indifferent between the two choices
- Risk prone: Prefers the bet

Key Components of a Decision Model

1. **Decisions**
2. **State of nature** is an actual event that may occur in the future
3. **Payoff table** is a means of organising a decision situation, presenting the payoffs from different decisions given the various states of nature

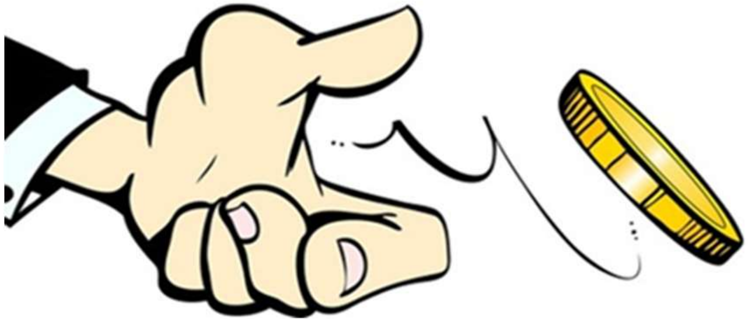
Payoff Table

an actual event that may occur in the future

	States of Nature	
Decision	s_1	s_2
x_1	a_{11}	a_{12}
x_2	a_{21}	a_{22}

the payoffs from different decisions given the various states of nature

Example: Flip a Coin



Bet A gives:
- £60, if you guess outcome correctly
- £20, if you don't

Bet B gives:
- £100, if you guess outcome correctly
- £0, if you don't

Key Components

- **Decisions: Bet A** or **Bet B**
- **States of nature: Guess outcome correctly** or **not**
- **Payoffs: £60**, **£20**, **£100**, **£0**

	States of Nature	
Decision	Guess correctly	Not guess correctly
Bet A	**£60**	**£20**
Bet B	**£100**	**£0**

Payoff Table

The components of a decision model are:

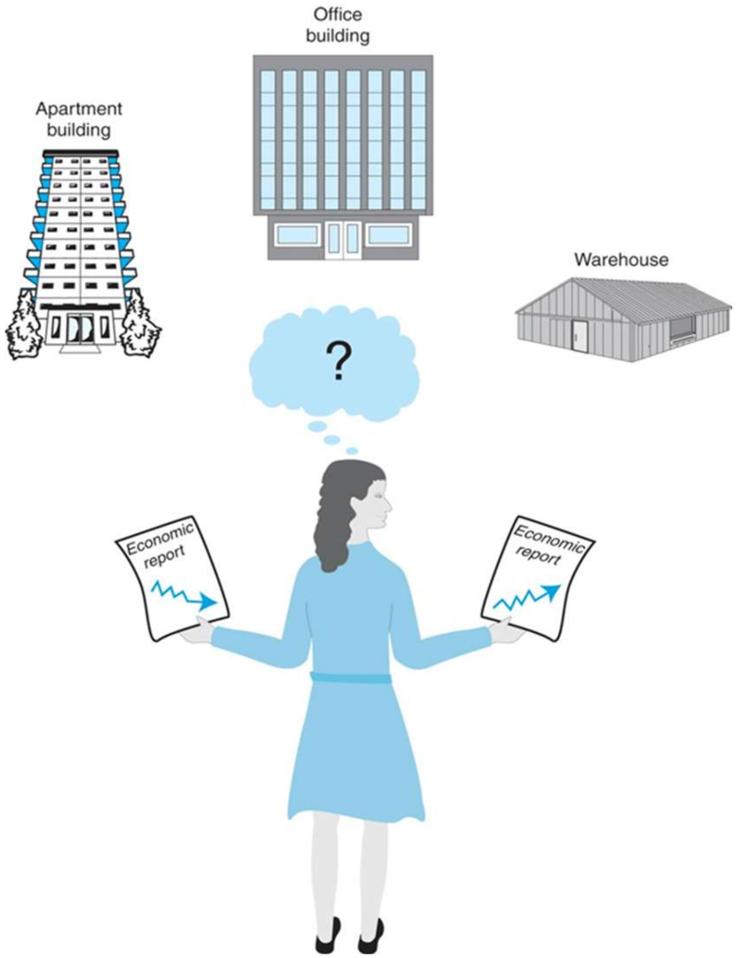
1) Decisions

2) States of nature

3) Payoffs

	States of Nature	
Decision	s_1	s_2
x_1	a_{11}	a_{12}
x_2	a_{21}	a_{22}

Real Estate Investment Decision



	States of Nature	
Decision (Purchase)	**Good Economic Conditions**	**Poor Economic Conditions**
Apartment building	$50,000	$30,000
Office building	100,000	−40,000
Warehouse	30,000	10,000

Decision-Making Criteria without Probabilities

1. Maximax Criterion
2. Maximin Criterion
3. Hurwicz Criterion
4. Equal Likelihood Criterion (or Laplace)

1. Maximax Criterion

The decision maker selects the decision that will result in the maximum of maximum payoffs (an optimistic criterion)

	States of Nature	
Decision (Purchase)	Good Economic Conditions	Poor Economic Conditions
Apartment building	$50,000	$30,000
Office building	100,000	−40,000
Warehouse	30,000	10,000

maximum

2. Maximin Criterion

The decision maker selects the decision that will reflect the maximum of the minimum payoffs (a pessimistic criterion)

	States of Nature	
Decision (Purchase)	**Good Economic Conditions**	**Poor Economic Conditions**
Apartment building	$50,000	$30,000
Office building	100,000	−40,000
Warehouse	30,000	10,000

maximum

3. Hurwicz Criterion

- The Hurwicz criterion is a compromise between the maximax and maximin criteria
- A coefficient of optimism, *a*, is a measure of the decision maker’s optimism
- The Hurwicz criterion **multiplies the best** payoff by *a* and **the worst** payoff by *1-a*, for each decision, and the best result is selected

Decision	**Values** (if *a = 0.40*)
Apartment building	50,000(0.4) + 30,000(0.6) = 38,000
Office building	100,000(0.4) - 40,000(0.6) = 16,000
Warehouse	30,000(0.4) + 10,000(0.6) = 18,000

4. Equal Likelihood Criterion

The equal likelihood (or Laplace) criterion multiplies the decision payoff **for each state of nature** by an equal weight, thus assuming that the states of nature are equally likely to occur

Decision	**Values**
Apartment building	50,000(0.5) + 30,000(0.5) = 40,000
Office building	100,000(0.5) - 40,000(0.5) = 30,000
Warehouse	30,000(0.5) + 10,000(0.5) = 20,000

Dominant Decision

A **dominant** decision is one that has a **better payoff** than another decision under each state of nature

Decision (Purchase)	States of Nature Good Economic Conditions	Poor Economic Conditions
Apartment building	$50,000	$30,000
Office building	100,000	−40,000
Warehouse	30,000	10,000

No reason at all to buy a warehouse

Decision-Making Criteria with Probabilities

Expected monetary value (EMV) is computed by multiplying each decision outcome under each state of nature by the probability of its occurrence

Number of states of nature

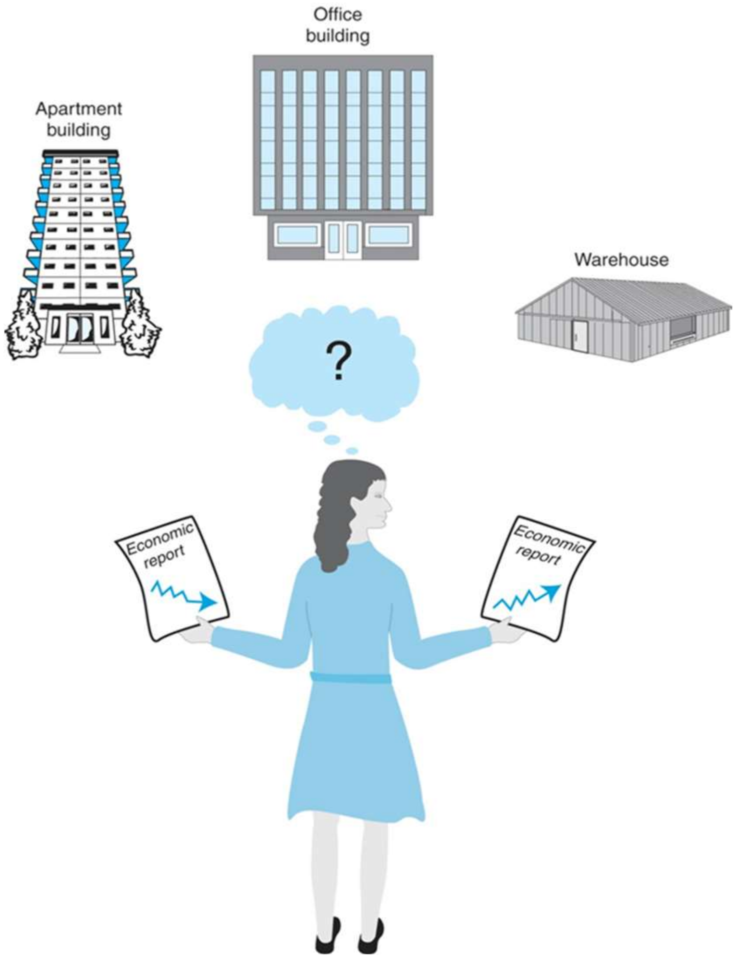
$$EMV(x_i) = \sum_{j=1}^{n} a_{ij} P(S = s_j)$$

Decision i

Probability of state of nature j

Payoff from decision i under state of nature j

Real Estate Investment Decision



	States of Nature	
Decision (Purchase)	Good Economic Conditions **(0.6)**	Poor Economic Conditions **(0.4)**
Apartment building	$50,000	$30,000
Office building	100,000	−40,000
Warehouse	30,000	10,000

Investor’s EMV

	States of Nature	
Decision (Purchase)	Good Economic Conditions **(0.6)**	Poor Economic Conditions **(0.4)**
Apartment building	$50,000	$30,000
Office building	100,000	−40,000
Warehouse	30,000	10,000

EMV(Apartment) = ?

Investor’s EMV

	States of Nature	
Decision (Purchase)	Good Economic Conditions **(0.6)**	Poor Economic Conditions **(0.4)**
Apartment building	$50,000	$30,000
Office building	100,000	−40,000
Warehouse	30,000	10,000

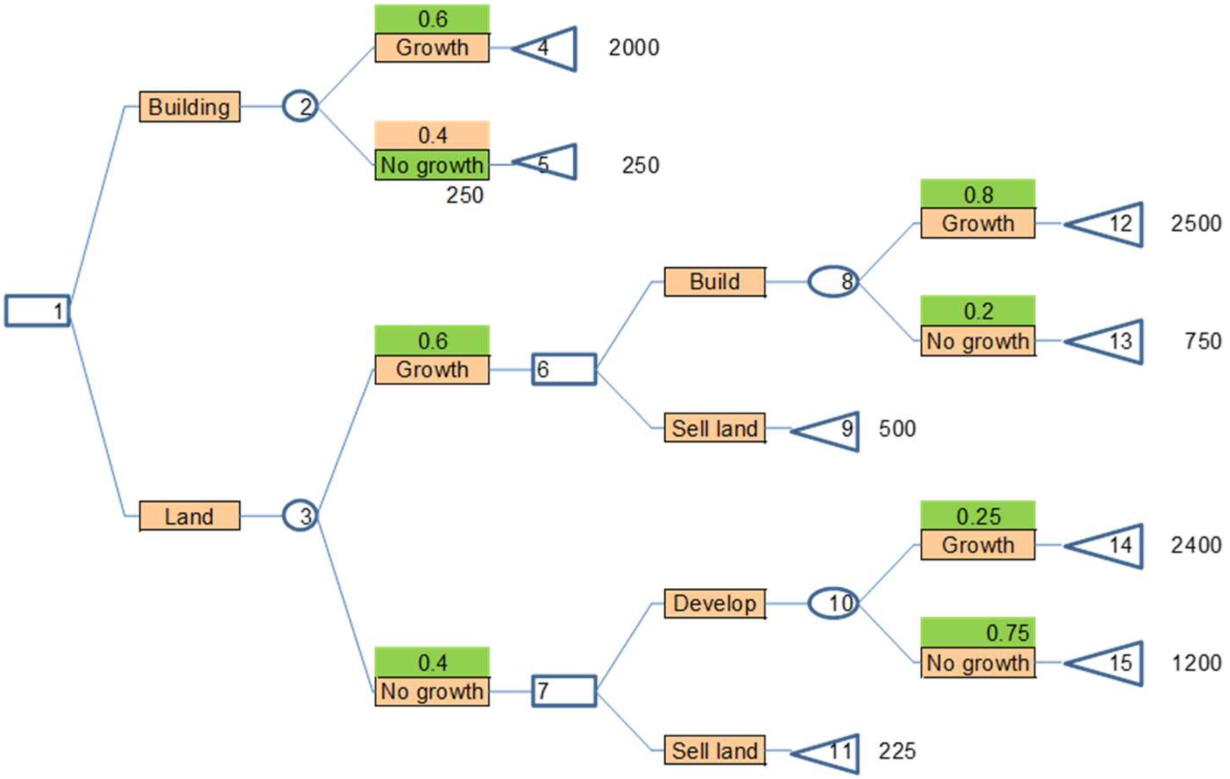
EMV(Apartment) = 50,000(0.6) + 30,000(0.4) = $42,000

EMV(Office) = 100,000(0.6) - 40,000(0.4) = $44,000

EMV(Warehouse) = 30,000(0.6) + 10,000(0.4) = $22,000

Decision Trees

- Simplify complex situations
- Visualise the problem
- Make the analysis easier
- Find the solution



Decision Trees

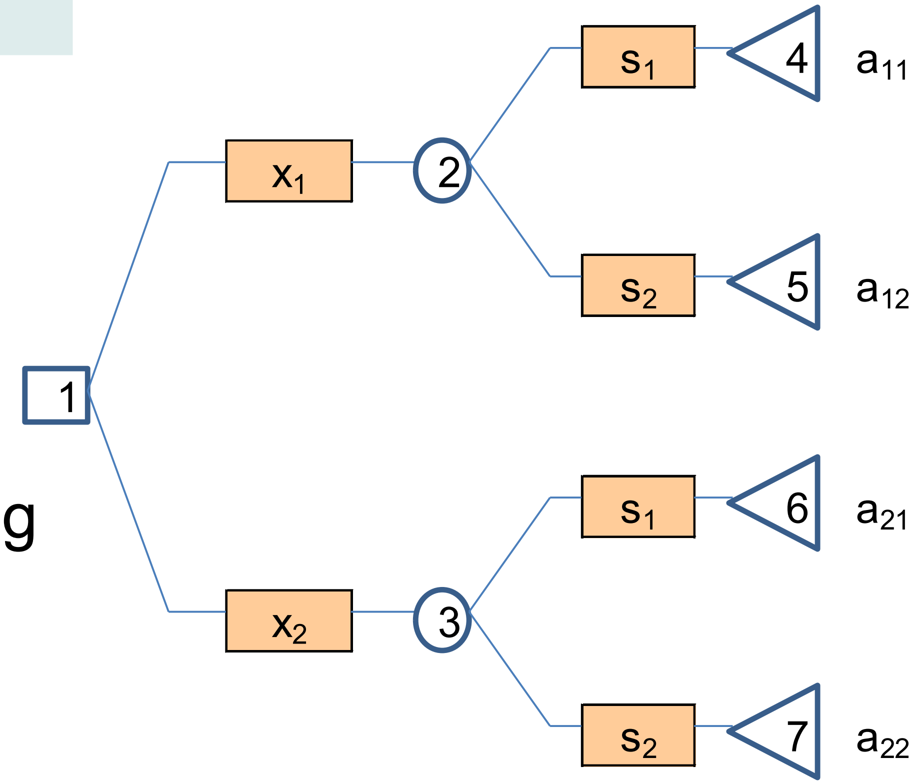
- A decision tree is a diagram consisting of the key components of decision-making models
- Recognise the key components:
  - Decisions,
  - States of nature,
  - Payoffs.
- Construct a decision tree
- Find the optimal decision

Construct a Decision Tree

Decision model	Decision tree	Symbol
Decisions	Decision nodes	□
States of nature	Probability nodes	○
Payoffs	End nodes	◁

	States of Nature	
Decision	s_1	s_2
x_1	a_{11}	a_{12}
x_2	a_{21}	a_{22}

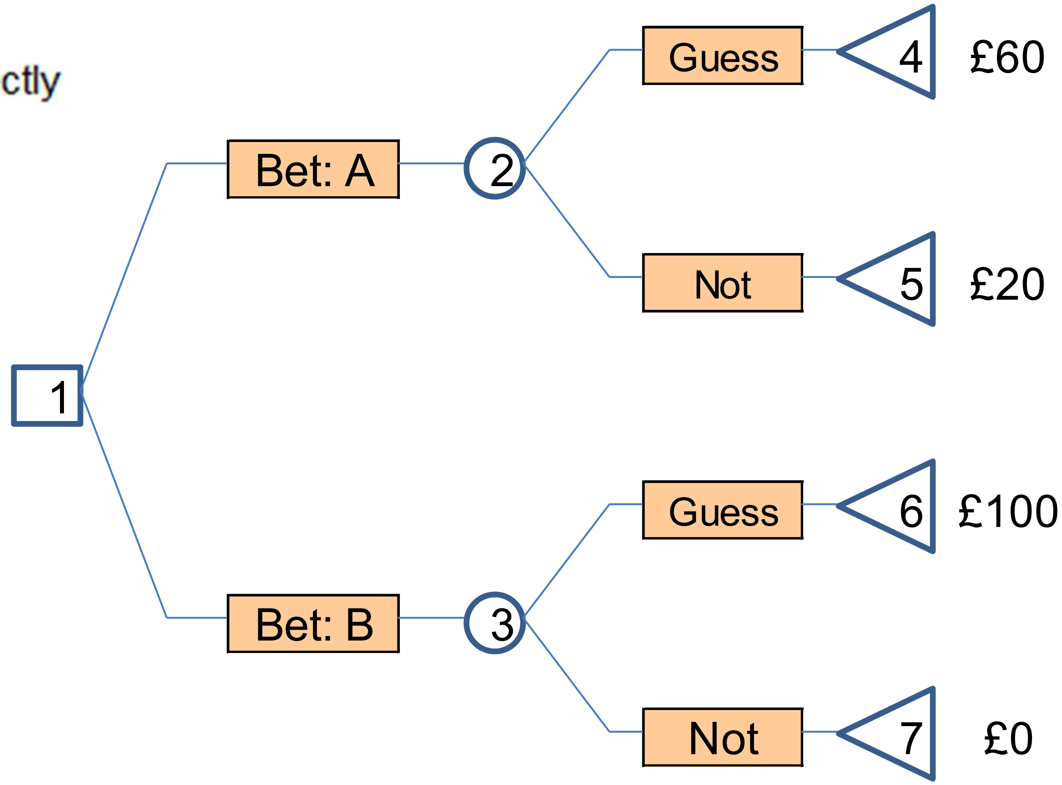
A decision tree is a diagram consisting of decision nodes, probability nodes, decision alternatives (branches), and payoffs



Tree Representation: Flip a Coin

Bet A gives:
£60, if you guess outcome correctly
£20, if you don't

Bet B gives:
£100, if you guess outcome correctly
£0, if you don't



1
Bet: A
2
Guess
4
£60
Not
5
£20
Bet: B
3
Guess
6
£100
Not
7
£0

Expected Monetary Value: Flip a Coin

$$EMV(x_i) = \sum_{j=1}^{n} a_{ij} P(S = s_j)$$

	States of Nature	
Decision	Guess correctly	Not guess correctly
Bet A	**£60**	**£20**
Bet B	**£100**	**£0**

£40

EMV(A)= $60 * \frac{1}{2} + 20 * \frac{1}{2}$

EMV(B)= $100 * \frac{1}{2} + 0 * \frac{1}{2}$

£50

Probability(“Guess correctly”) = 1/2

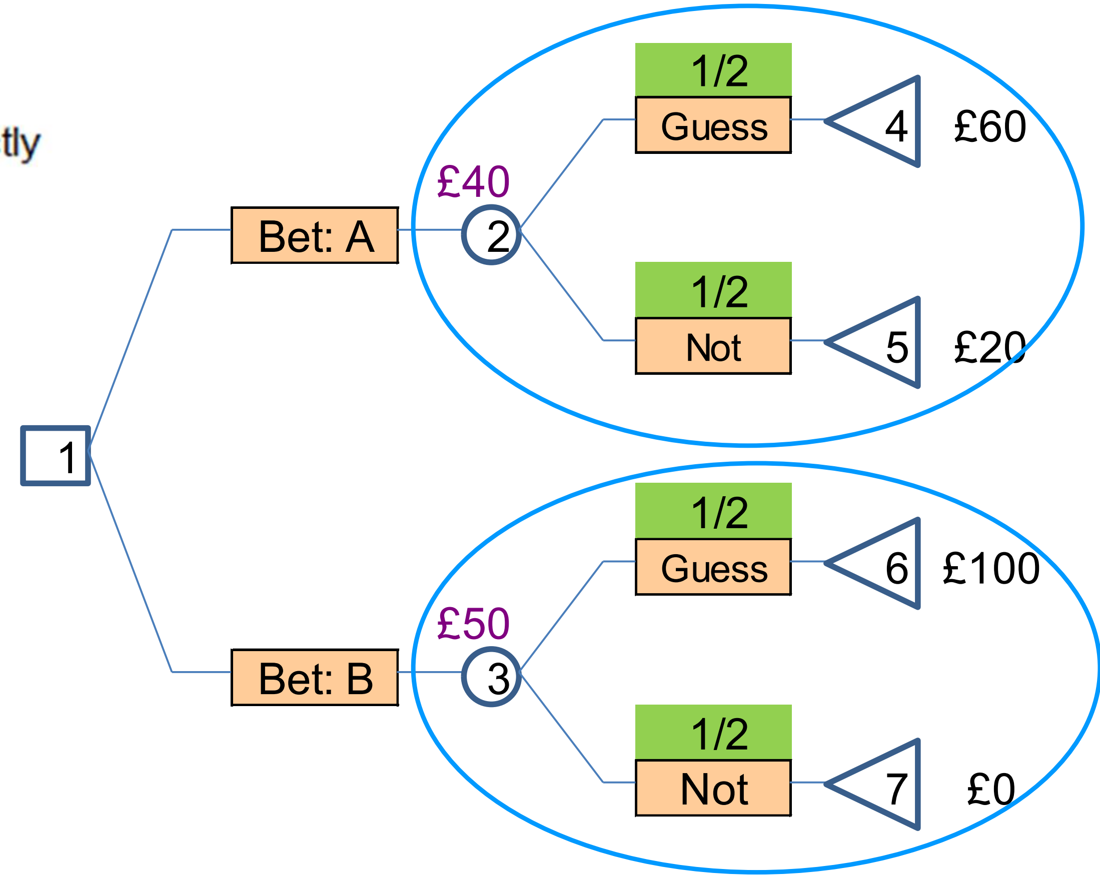
Probability(“Not guess correctly”) = 1/2

Solution: Flip a Coin

Bet A gives:
£60, if you guess outcome correctly
£20, if you don't

Bet B gives:
£100, if you guess outcome correctly
£0, if you don't

Criterion: Expected Monetary Value (EMV)

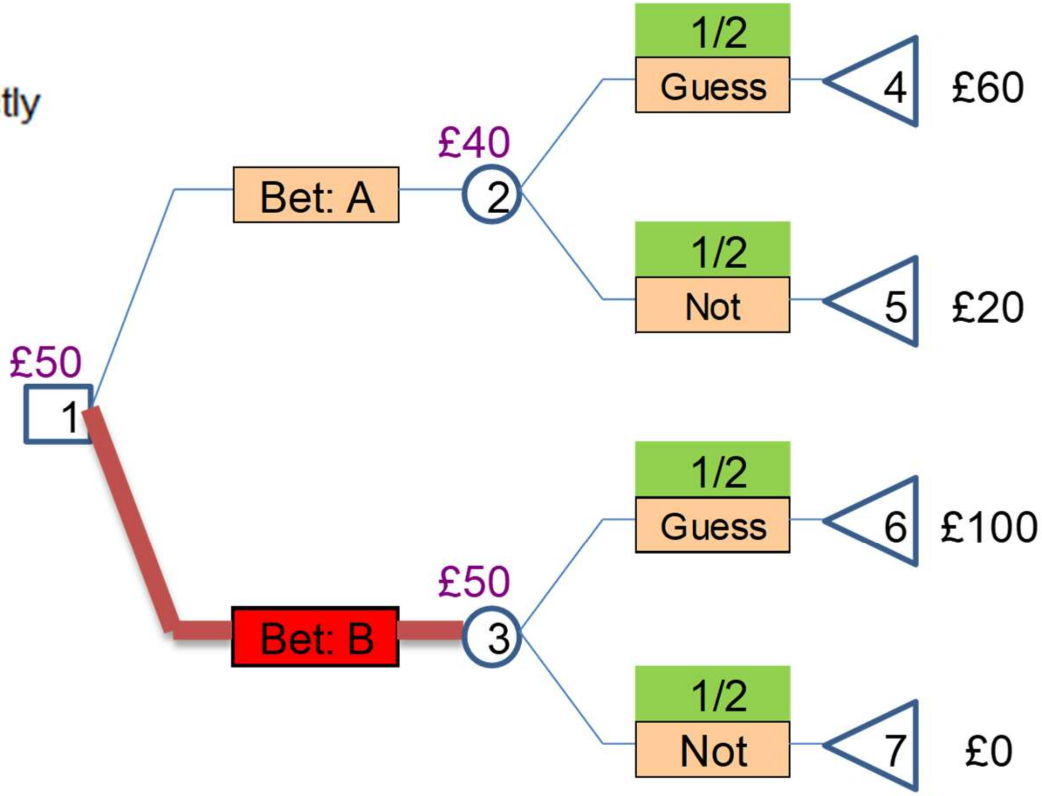


Solution: Flip a Coin

$$\text{Bet A gives: } \begin{cases} \text{£60, if you guess outcome correctly} \\ \text{£20, if you don't} \end{cases}$$

$$\text{Bet B gives: } \begin{cases} \text{£100, if you guess outcome correctly} \\ \text{£0, if you don't} \end{cases}$$

Criterion: Expected Monetary Value (EMV)



Homework

Thompson Lumber Company

- John Thompson is the founder and president of Thompson Lumber Company, a profitable firm located in Norwich.
- The problem that John Thompson identifies is whether to expand his product line by manufacturing and marketing a new product, backyard storage sheds.
- Assuming that John decides that his alternatives are as follows:
  - build a large plant to manufacture the storage sheds,
  - build a small plant to manufacture the storage sheds, or
  - build no plant at all.

Thompson Lumber Company

- Demand for sheds will be:
  - High,
  - Moderate,
  - Low.
- Payoff table:

Decisions	**States of Nature**		
	High Demand	Moderate Demand	Low Demand
Build Large Plant	£200,000	£100,000	-£120,000
Build Small Plant	£90,000	£50,000	-£20,000
Do Nothing	0	0	0

Thompson Lumber Company

What will you suggest John to do based on:

1. Maximax Criterion
2. Maximin Criterion
3. Hurwicz Criterion (for $a = 0.45$)
4. Equal Likelihood Criterion (or Laplace)

Maximax Criterion

Decisions	States of Nature			
	High Demand	Moderate Demand	Low Demand	Max
Build Large Plant	£200,000	£100,000	-£120,000	£200,000
Build Small Plant	£90,000	£50,000	-£20,000	£90,000
Do Nothing	0	0	0	0

Maximin Criterion

Decisions	States of Nature			
	High Demand	Moderate Demand	Low Demand	Min
Build Large Plant	£200,000	£100,000	-£120,000	-£120,000
Build Small Plant	£90,000	£50,000	-£20,000	-£20,000
Do Nothing	0	0	0	0

Hurwicz Criterion (for a = 0.45)

Decisions	States of Nature			
	High Demand	Moderate Demand	Low Demand	Value
Build Large Plant	£200,000	£100,000	-£120,000	£24,000
Build Small Plant	£90,000	£50,000	-£20,000	£29,500
Do Nothing	0	0	0	0

Build Large Plant: $0.45 \times 200{,}000 + 0.55 \times (-120{,}000) = 24{,}000$

Build Small Plant: $0.45 \times 90{,}000 + 0.55 \times (-20{,}000) = 29{,}500$

Do Nothing: $0.45 \times 0 + 0.55 \times 0 = 0$

Equal Likelihood Criterion

Decisions	States of Nature			
	High Demand	Moderate Demand	Low Demand	Value
Build Large Plant	£200,000	£100,000	-£120,000	£60,000
Build Small Plant	£90,000	£50,000	-£20,000	£40,000
Do Nothing	0	0	0	0

Build Large Plant: $(200{,}000 + 100{,}000\ - 120{,}000)/3\ = 60{,}000$

Build Small Plant: $(90{,}000 + 50{,}000\ - 20{,}000)/3 = 40{,}000$

Do Nothing: $(0 + 0 + 0)/3 = 0$

Homework (2)

Could you eliminate any of the decisions A, B, C, D, E?

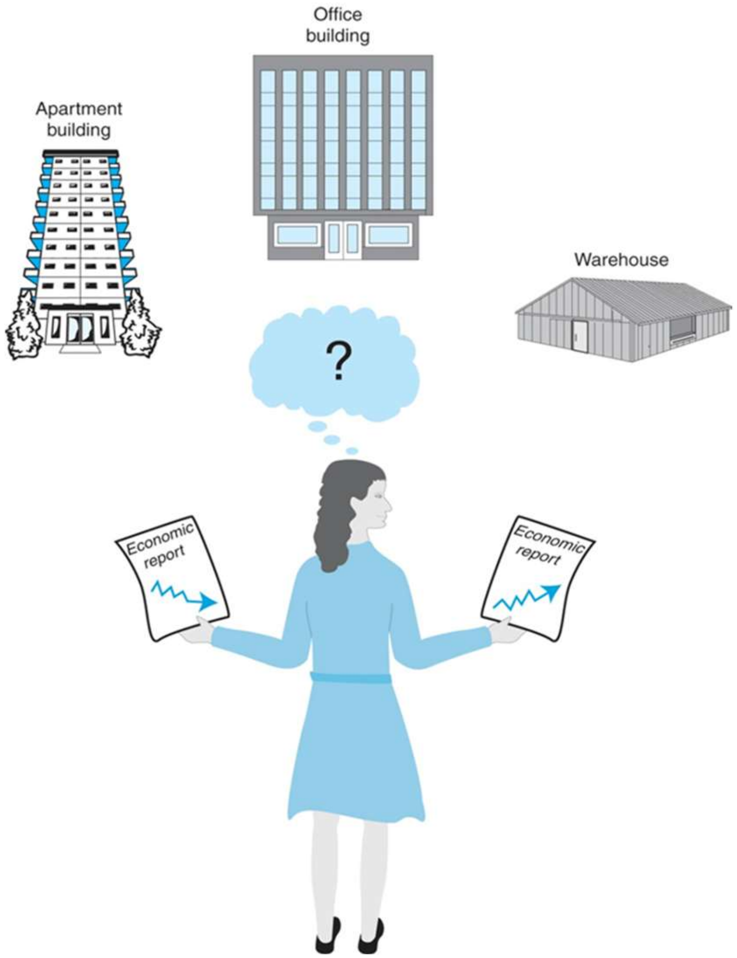
	States of Nature		
Decision	**Good**	**Fair**	**Poor**
A	10	6	3
B	20	5	-1
C	15	5	0
D	12	7	5
E	13	8	4

Homework (2) - Solution

Could you eliminate any of the decisions A, B, C, D, E?

	States of Nature		
Decision	**Good**	**Fair**	**Poor**
B	20	5	-1
C	15	5	0
D	12	7	5
E	13	8	4

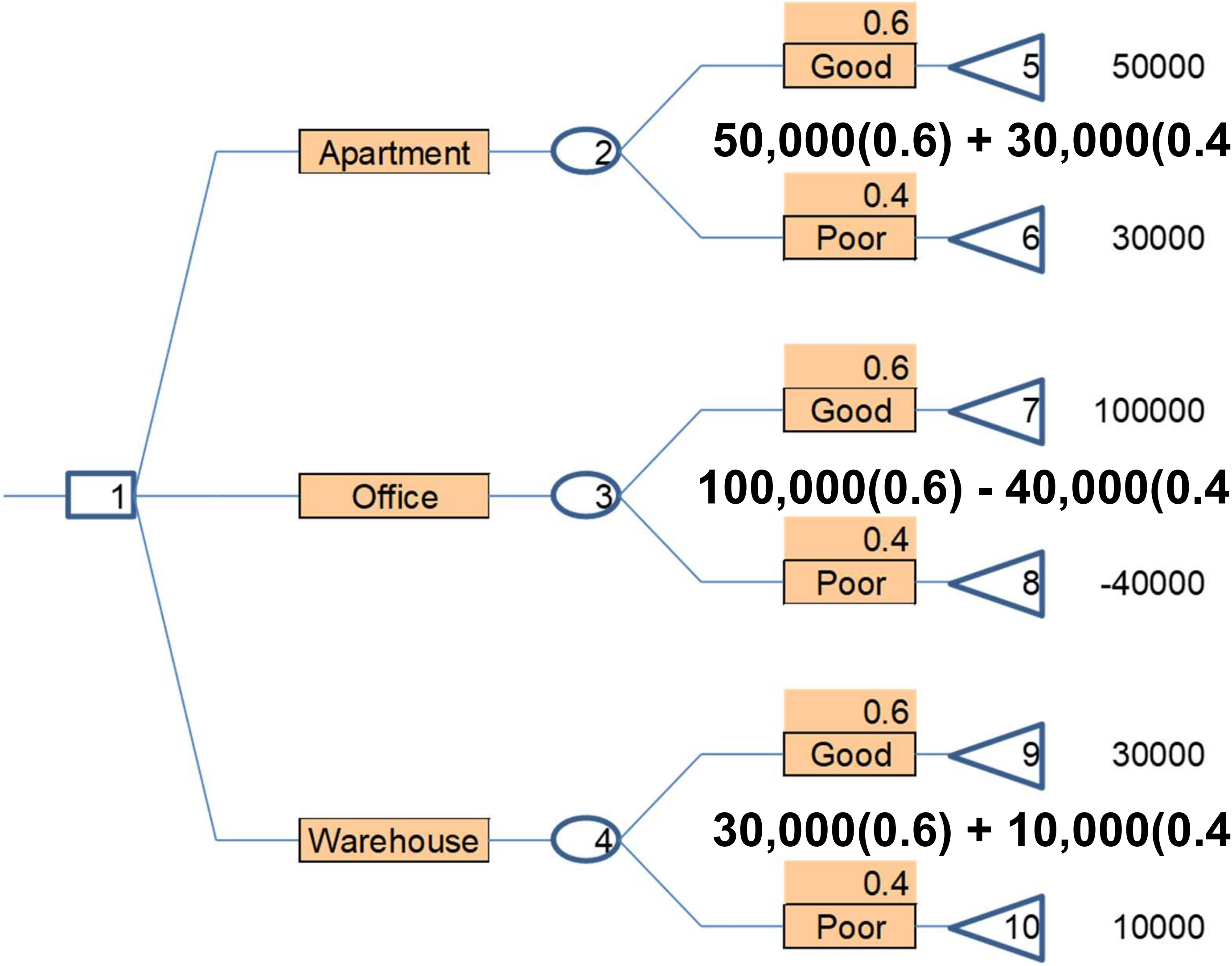
Homework (3) Real Estate Investment Decision - Tree Representation



	States of Nature	
Decision (Purchase)	**Good Economic Conditions (0.6)**	**Poor Economic Conditions (0.4)**
Apartment building	$50,000	$30,000
Office building	100,000	−40,000
Warehouse	30,000	10,000

Criterion: Expected Monetary Value (EMV)

Tree Representation

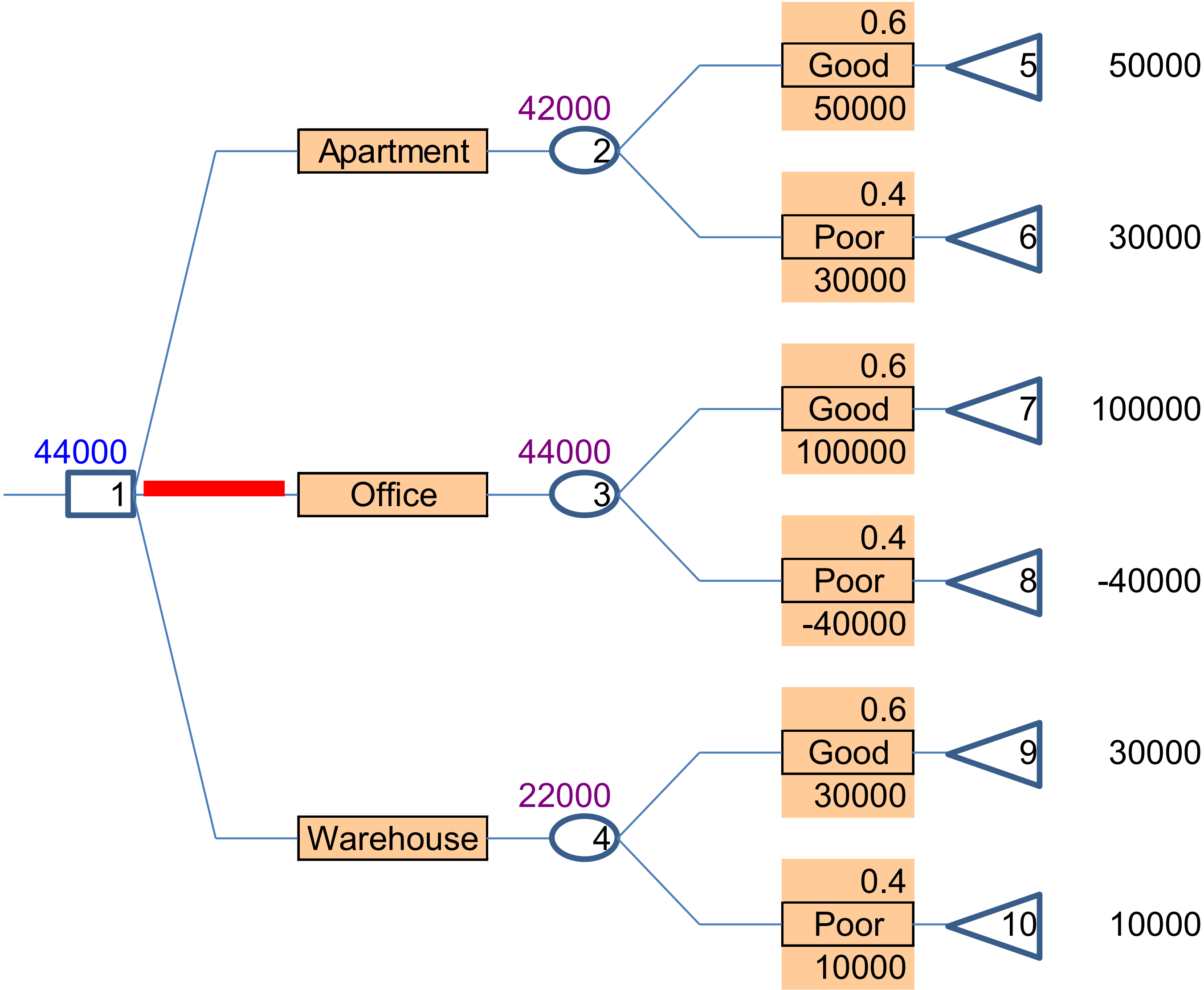


50,000(0.6) + 30,000(0.4) = 42,000

100,000(0.6) - 40,000(0.4) = 44,000

30,000(0.6) + 10,000(0.4) = 22,000

Solution: EMV



Homework (4)

A corporate raider contemplates the future of a recent acquisition. Three alternatives are being considered in two states of nature. The payoff table is below

Decision	State of Nature Good Conditions (0.7)	State of Nature Poor Conditions (0.3)
Expand	$800,000	$500,000
Maintain status quo	1,300,000	−150,000
Sell now	320,000	320,000

a) Determine the best decision by using the expected value criterion.

b) Compute expected value of perfect information.

c) Develop a decision tree with expected value at the nodes.

Homework (4) - Solution

Expected monetary value decision: **Maintain status quo**

Expand	800,000(0.7) + 500,000(0.3) = $710,000
Status quo	1,300,000(0.7) - 150,000(0.3) = $865,000
Sell	320,000(0.7) + 320,000(0.3) = $320,000

Homework - Solution

Compute EVPI

Decision without perfect information:

EMV(Maintain status quo) = $865,000

Decision with perfect information:

1,300,000(0.7) + 500,000(0.3) = $1,060,000

EVPI: 1,060,000 - 865,000 = $195,000

Homework - Solution