Problem 1

Consider a firm that produces goods A and B using good X as an input, with technology described by the production set

$$Y=\left\{[A,B,-X]\in\mathbb{R}^3 : A\geq 0\ , B\geq 0,\ X\geq A^2+AB+B^2\right\} \quad (1)$$

Let $p=[\alpha,\beta,1]\in\mathbb{R}^3_{++}$ be a price vector, where $\alpha=\text{price of good A},\beta=\text{price of good B},1\text{=price of good X}$ .The profit function of this firm is

$$\Pi=\alpha A+\beta B-X \quad (2)$$

The vector $y=\left[A,B,-X\right]$ is the net supply vector of the firm.

1.derive the firm's net supply curves.

2.using your answer to question 1, derive the firm's net supply vectors y^1,y^2 at the price vectors $p^1=[1,4,1], p^2=[2,6,1]$

3.derive the largest production set YO consistent with the dataset $D=\{[p^t,y^t],t=1,2\}$

4.derive the net supply of the firm Y at prices $p=[4,6,1]$,using your answer to question 1

5.derive the net supply of the firm YO at prices $p=[4,6,1]$,using your answer to question 4

6.Compare your answers to questions 4 and 5

Answers to problem 1

1.solve the following maximization problem

Objective function $\Pi=\alpha A+\beta B-X$

Constraints $A\geq 0, B\geq 0,\ X\geq A^2+AB+B^2$

variables A,B,X

parameters α,β

conditions on parameters $\alpha > 0, \beta > 0$

net supply curves of Y

$$[A,B,\Pi] = \begin{cases} [0, \frac{\beta}{2}, \frac{\beta^2}{4}] & \alpha \le \frac{\beta}{2} \\ [\frac{2\alpha-\beta}{3}, \frac{2\beta-\alpha}{3}, \frac{1}{3}\alpha^2 - \frac{1}{3}\alpha\beta + \frac{1}{3}\beta^2] & \frac{\beta}{2} \le \alpha \le 2\beta \\ [\frac{\alpha}{2}, 0, \frac{\alpha^2}{4}] & \alpha \ge 2\beta \end{cases} \tag{3}$$

$X = A^2 + AB + B^2$

2.Use (3)

$$\begin{aligned} p^1 &= [1,4,1], y^1 = [0,2,-4] \\ p^2 &= [2,6,1], y^2 = [0,3,-9] \end{aligned} \tag{4}$$

3.

$$YO = \{[A,B,-X] \in \mathbb{R}^3 : A \ge 0, B \ge 0, X \ge 0, X \ge A + 4B - 4, X \ge 2A + 6B - 9\} \tag{5}$$

4.use (3)

$$y_Y([4,6,1]) = [\frac{2}{3}, \frac{8}{3}, -\frac{28}{3}] \tag{6}$$

5. solve the following maximization problem

Objective function $\Pi = 4A + 6B - X$

Constraints $A \ge 0, B \ge 0, X \ge 0, X \ge A + 4B - 4, X \ge 2A + 6B - 9$

variables A, B, X

The problem has no global maximum, because the points $Q_A = [A, B = 0, X = 2A - 9]$ are feasible for all $A \ge 5$ and $\Pi(Q_A) = 2A + 9 \to +\infty$ as $A \to +\infty$ .Hence

$$y_{YO}([4,6,1]) = \text{NONE} \tag{7}$$

6.Compare (7) to (6)

Problem 2

Consider an economy with one consumer, one firm, and three goods.

- Goods: 1,2,3
- Preferences

$$u(x) = -\frac{1}{2}x_1^2 - \frac{17}{2}x_2^2 + x_1 + 2x_2 + x_3 - 4x_1x_2 \quad (8)$$

- Endowment $e = \overset{1,2,3}{[0,0,1]}$
- Consumption set $\mathbb{R}_+^3$
- The consumer owns the firm.
- The firm produces goods 1 and 2 using good 3 as an input, with a technology described by the production set

$$Y = \left\{[y_1, y_2, -y_3] \in \mathbb{R}^3 : y_1 \geq 0, y_2 \geq 0, y_3 \geq y_1^2 + y_1y_2 + y_2^2 \right\} \quad (9)$$

- The firm pays a tax δ per unit of revenue from good 1. Assume that $0 \leq \delta < \frac{9}{11}$
- The consumer receives the tax proceeds as a lump-sum transfer.

1. Using good 3 as a numeraire, compute all competitive equilibria **only** for those values of the parameters that generate **STRICTLY POSITIVE** demand and supply functions for all goods.

2. Plot the equilibrium prices of goods 1 and 2 as functions of the tax rate δ

3.Plot equilibrium GDP as functions of the tax rate δ

4.Plot equilibrium utility as a function of the tax rate δ

Answers to problem 2

1. NAME the price of each good

p_i = price of good i.Normalize $p_3 = 1$

2.DEFINE consumer income

$$M = 1 + \Pi + T \quad (10)$$

2. SOLVE the optimization problem of the firm

Objective function $\Pi = p_1(1-\delta)y_1 + p_2y_2 - y_3$

Constraints $y_1 \geq 0, y_2 \geq 0, \ y_3 \geq y_1^2 + y_1y_2 + y_2^2$

variables y_1, y_2, y_3

parameters p_1, p_2, δ

conditions on parameters $p_1 > 0, \ p_2 > 0, 0 \leq \delta < \frac{9}{11}$

By (3) we obtain

$$\text{net supply curves of } Y \text{ when } \frac{p_2}{2(1-\delta)} \leq p_1 \leq \frac{2p_2}{(1-\delta)}$$

$$\begin{aligned} y_1 &= \frac{2(1-\delta)p_1 - p_2}{3} \\ y_2 &= \frac{2p_2 - (1-\delta)p_1}{3} \\ y_3 &= y_1^2 + y_1y_2 + y_2^2 \\ \Pi &= \frac{1}{3}(1-\delta)^2 p_1^2 - \frac{1}{3}(1-\delta)p_1p_2 + \frac{1}{3}p_2^2 \end{aligned} \tag{11}$$

3. SOLVE the optimization problem of the consumer

Objective function $u = -\frac{1}{2}x_1^2 - \frac{17}{2}x_2^2 + x_1 + 2x_2 + x_3 - 4x_1x_2$

Constraints $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, \ p_1x_1 + p_2x_2 + x_3 \leq M$

variables x_1, x_2, x_3

parameters p_1, p_2, M

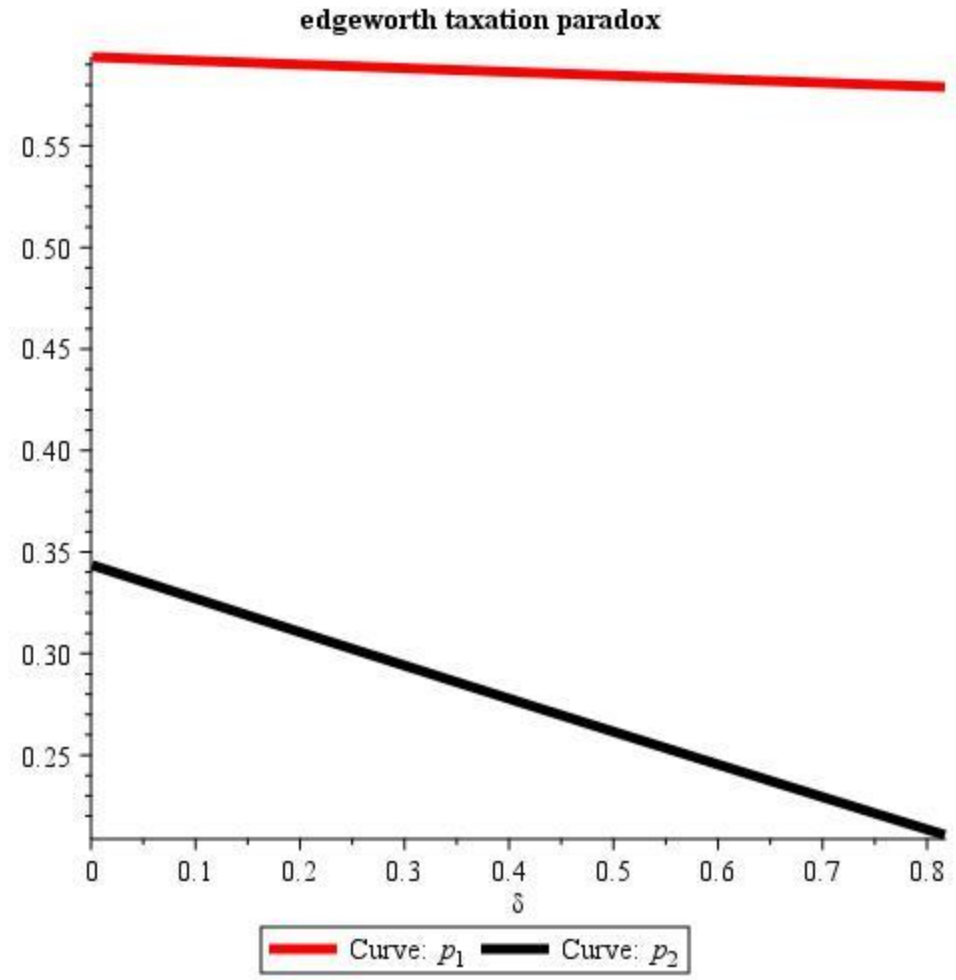
conditions on parameters $p_1 > 0, \ p_2 > 0, M > 0$

$$\begin{array}{|l|}\hline \text{demand curves when} \\ \frac{p_2}{4}+\frac{1}{2}\le p_1 \le \frac{9}{17}+\frac{4p_2}{17},\ p_2<2 \\ -17p_1^2+8p_1p_2-p_2^2+9p_1-2p_2<M \\ \hline x_1=-17p_1+4p_2+9 \\ x_2=4p_1-p_2-2 \\ x_3=17p_1^2-8p_1p_2+p_2^2+M-9p_1+2p_2 \\ \hline \end{array} \tag{12}$$

5. SOLVE the equilibrium conditions

$$x_1=y_1, x_2=y_2, x_3+y_3=1 \tag{13}$$

$$\begin{array}{|l|}\hline \text{equilibria} \\ p_1=\frac{19}{\delta+32},\ p_2=\frac{11-5\delta}{\delta+32} \\ x_1=y_1=\frac{9-11\delta}{\delta+32},\ x_2=y_2=\frac{3\delta+1}{\delta+32}, \\ x_3=\frac{-96\delta^2+240\delta+933}{(\delta+32)^2},\ y_3=\frac{97\delta^2-176\delta+91}{(\delta+32)^2} \\ \Pi=\frac{97\delta^2-176\delta+91}{(\delta+32)^2},\ M=\frac{-111\delta^2+59\delta+1115}{(\delta+32)^2} \\ T=\frac{171\delta-209\delta^2}{(\delta+32)^2},\ GDP=\frac{182-15\delta^2-181\delta}{(\delta+32)^2} \\ \hline \end{array} \tag{14}$$


edgeworth taxation paradox
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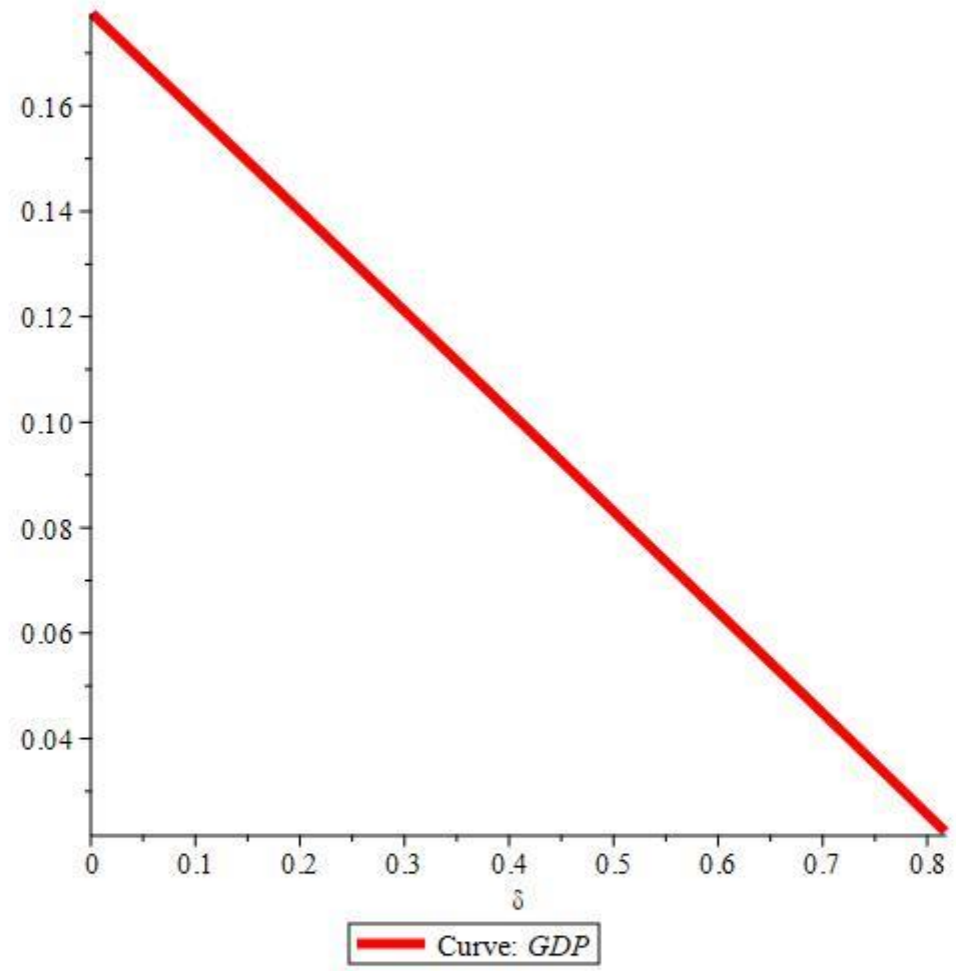
δ
Curve: p1
Curve: p2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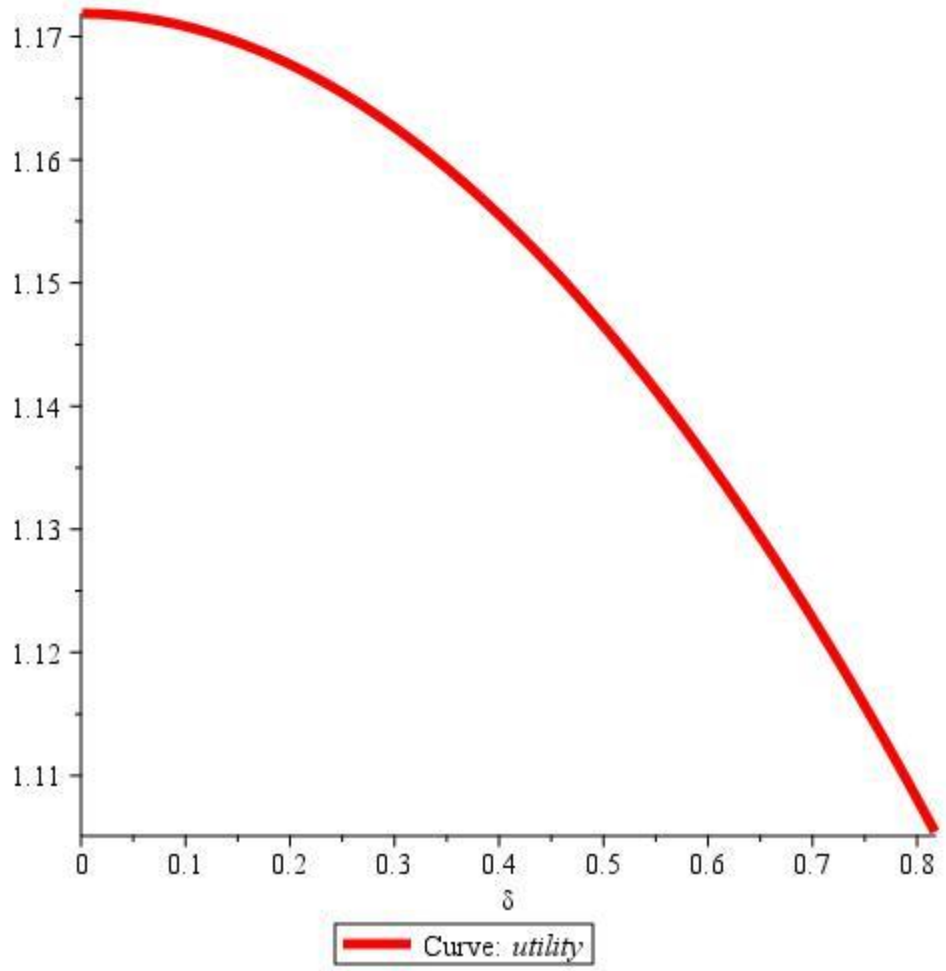
δ
Curve: GDP



In this example, GDP is a correct index of welfare. Prices are not.