PROBLEM 1

Let $\mathbb{R}^n \xrightarrow{T} \mathbb{R}^m$ be an injective linear map and A a closed set in $\mathbb{R}^n$.

1.Show that $T(A)$ is a closed set.

2.Provide an example of a linear map T and a closed set A such that $T(A)$ is not closed

3.Generalize to injective affine maps

PROBLEM 2

Let A be the line in $\mathbb{R}^n$ through the points [1,0,0], [0,1,0]. Derive an explicit formula for the nearest point map $\mathbb{R}^n \xrightarrow{\Pi} A$ onto A

PROBLEM 3

Let C be a convex set of dimension r in $\mathbb{R}^n$, b a vector in C .Show that there exists a simplex σ_r of dimension r such that

$$b \in \sigma_r \subseteq C \qquad (1)$$

PROBLEM 4

Let A be a set in $\mathbb{R}^n$ .Show that

$$cl(A^c) = \left(\text{int}(A)\right)^c \qquad (2)$$

PROBLEM 5

The perspective map $\mathbb{R}^n \times \mathbb{R}_{++} \xrightarrow{P} \mathbb{R}^n$ is defined by

$$P(x,t) = \frac{x}{t} \qquad (3)$$

Show that

1.if A is a convex set in $\mathbb{R}^n \times \mathbb{R}_{++}$,then $P(A)$ is a convex set in $\mathbb{R}^n$

2.if B is a convex set in $\mathbb{R}^n$,then $P^{-1}(B)$ is a convex set in $\mathbb{R}^{n+1}$

PROBLEM 6

Let A, B be nonempty convex subsets of $\mathbb{R}^n$ .Show that the following are equivalent:

1.A and B are strongly separated, i.e. there exist a nonzero vector $p \in \mathbb{R}^n$,a real number θ and a positive number ε such that

$$\sup_{\alpha \in A}(p\alpha) < \theta - \varepsilon < \theta + \varepsilon < \inf_{\beta \in B}(p\beta)$$

2. $0 \notin \text{closure}(A - B)$