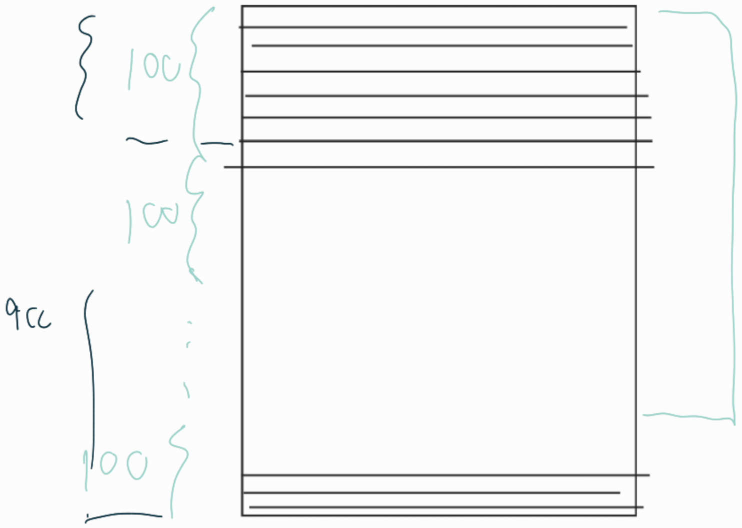
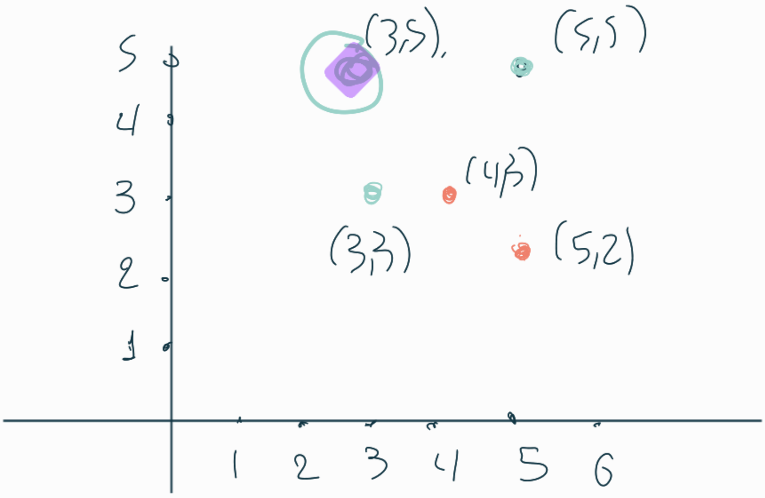


900 train || 100 test

KNN (K - Nearest Neighbor) Algorithm



(3,5)

(5,5) : $(5-3)^2 + 0 = 4$ ✓

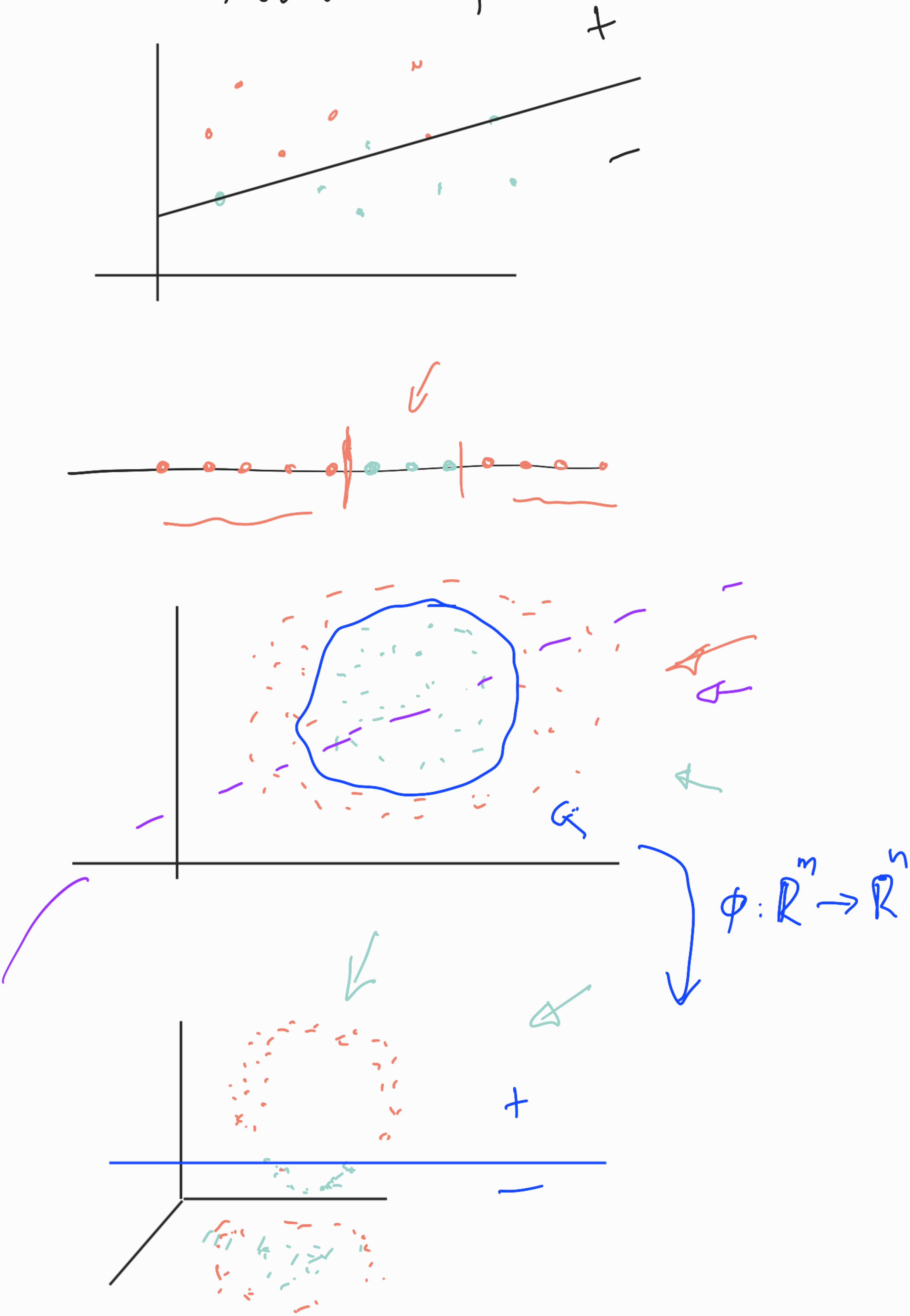
(3,3) : $(0)^2 + 2^2 = 4$ ✓

(4,3) : $1^2 + 2^2 = 5$ ✓

~~(5,2) : $2^2 + 3^2 = 13$~~

3-NN : {(5,5), {3,3}, {4,3}}

Μέθοδοι πυρήνα



$$\phi : x \mapsto \phi(x) \qquad \mathbb{R}^n \to \mathbb{R}^m$$

$$\{x_1, x_2, \dots x_n\}$$ τα δεδομένα

$$\downarrow \phi$$

$$\{\phi(x_1), \phi(x_2), \dots \phi(x_n)\}$$

$$K :\equiv K(x_i, x_j) = \langle \phi(x_i), \phi(x_j) \rangle = \phi(x_i)^T \phi(x_j)$$

στοιχείο του $K(i,j)$

$$K = \begin{pmatrix} \|\phi(x_1)\|^2 & \phi(x_1)^T \cdot \phi(x_2) & \dots & \phi(x_1)^T \phi(x_n) \\ \phi(x_2)^T \phi(x_1) & \|\phi(x_2)\|^2 & & \\ & & \ddots & \\ & & & \end{pmatrix}$$

Παράδειγμα

$$\phi : \mathbb{R}^2 \to \mathbb{R}^3 \qquad \| \qquad \phi(x) = (x_1^2, x_2^2, \sqrt{2}\, x_1 x_2)$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \end{pmatrix} \to (36, 9, \sqrt{2}\, 18)$$

$$K(x,y) = \phi(x)^T \phi(y) = (x_1, x_2, \sqrt{2}\, x_1 x_2) \cdot \begin{pmatrix} y_1 \\ y_2 \\ \sqrt{2}\, y_1 y_2 \end{pmatrix}$$

$$= x_1^2 y_1^2 + x_2^2 y_2^2 + 2 x_1 y_1 x_2 y_2 = (x_1 y_1 + x_2 y_2)^2 =$$

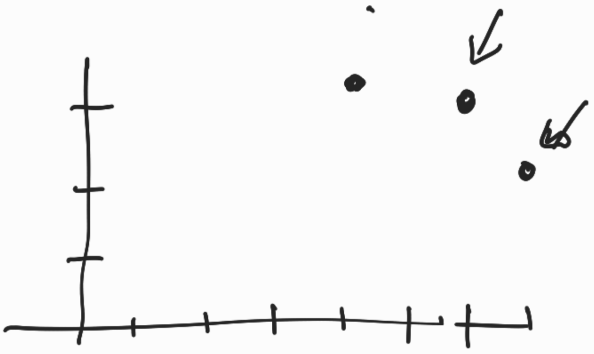
$$(x^T y)^2 \to K : \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

$$\phi(x)^T \phi(y) = \underline{x^T y} \qquad (\Pi\alpha\rho\acute{\alpha}\delta\varepsilon\iota\gamma\mu\alpha\ 1)$$

$$\phi(x) \to x$$

$$\{x_1, x_2, x_3\} = \left\{ \begin{pmatrix} 6 \\ 3 \end{pmatrix}, \begin{pmatrix} 7 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ 3 \end{pmatrix} \right\}$$

$$K(x_1, x_2) = x_1^T x_2 = 42 + 6 = 48$$



$$\longrightarrow K \begin{pmatrix} \underline{45} & \underline{48} & \times \\ 48 & 53 & \times \\ \times & \times & 25 \end{pmatrix}$$

$$\underline{\mu_\phi} = \frac{1}{3} \underline{\sum_{i=1}^{3} \phi(x_i)} = \frac{1}{3} \sum x_i = \frac{1}{3} \begin{pmatrix} 17 \\ 8 \end{pmatrix} = \begin{pmatrix} \frac{17}{3} \\ \frac{8}{3} \end{pmatrix}$$

$$\|\mu_\phi\|^2 = \mu_\phi^T \mu_\phi = \frac{17^2}{9} + \frac{64}{9} = \underline{c}$$

$$\frac{1}{9} \left(\sum_{i=1}^{3} \sum_{j=1}^{3} K(x_i, x_j) \right) = \frac{1}{9} \left(45 + 2 \cdot 48 + 53 + 25 + 2 * + 2 \times \right)$$

$$= \underline{c}$$

Διανυσματικοί πυρήνες

• Πολυωνυμικές πυρήνας (ομογενής, μη ομογενής)

$$K_r(x,y) = \phi(x)^T\phi(y) = (x^Ty)^r$$

(r=1) παράδειγμα 1

(r=2)
$$K_2(x,y) = (x^Ty)^2 \quad \text{(ομογενής)}$$
$$\tilde{K}_2(x,y) = (c+x^Ty)^2 \quad \text{(μη ομογενής)}$$

Έστω $c=1$ στον μη ομογενή πυρήνα βαθμού r. Τότε

$$(1+x^Ty)^r = 1 + r\,x^Ty + \binom{r}{2}(x^Ty)^2 + \dots + r(x^Ty)^{r-1} + (x^Ty)^r$$

$\bar{a}$ $\tilde{a}$

Σε οποιονδήποτε πολυωνυμικό πυρήνα μπορώ να βρω την ϕ

$$K_r(x,y) = (c+x^Ty)^r = \left(c+\sum_{k=1}^{d} x_k y_k\right)^r = (c + x_1y_1 + x_2y_2 + \dots + x_dy_d)^r$$

$n = (n_0, n_1, \dots, n_d)$ - $|n| = \sum_{i=0}^{d} n_i$

$$= \sum_{|n|=r} \binom{r}{n} c^{n_0} (x_1y_1)^{n_1}(x_2y_2)^{n_2}\cdots(x_dy_d)^{n_d} =$$

$$= \sum_{|n|=r} \underbrace{\binom{r}{n} c_0^{n_0}}_{a_n} (x_1^{n_1}x_2^{n_2}\cdots x_d^{n_d})(y_1^{n_1}y_2^{n_2}\cdots y_d^{n_d}) =$$

$$= \sum_{|n|=r} \left(\sqrt{a}\cdot\prod_{i=1}^{d} x_i^{n_i}\right)\left(\sqrt{a}\prod_{i=1}^{d} y_i^{n_i}\right) = \phi(x)^T\cdot\phi(y)$$

Άρα $\underline{\phi(x)} = (\ldots, a_n x^n, \ldots)^T$

Άσκηση

$$K(x,y) = (1+x^T y)^2 = (1 + x_1 y_1 + x_2 y_2)^{2 = r}$$

Ποια είναι η $\phi(x)$; Να αποδειχθεί ότι η διάσταση του χώρου χαρακτηριστ. δίνεται $m = \binom{d+r}{r}$

$\phi : \mathbb{R}^2 \to$

$\phi(x)^T \cdot \phi(y)$

Αν $r = 1$ $\quad (1 + x_1 y_1 + x_2 y_2) = (1, x_1, x_2)(1, y_1, y_2)$

$$(1 + x_1^2 x_1^2 + x_2^2 y_2^2 + 2x_1 y_1 + 2x_2 y_2 + 2 x_1 y_1 \cdot x_2 y_2)$$

$\downarrow$

Πυρήνας Gauss (Radial basis function)

$$K(x,y) = e^{-\left\{ \frac{\|x-y\|^2}{c} \right\}} \quad \boxed{c = 2\sigma^2}$$

$K(x,x) = 1$

$$e^x = \sum_{i=0}^{\infty} \frac{x^i}{i!} = 1 + x + \frac{x^2}{2} + \ldots\ldots$$

$$K(x,y) = e^{-\frac{\|x-y\|^2}{c}} = e^{\left(-\frac{\|x\|}{c}\right)} \cdot e^{-\frac{\|y\|_2}{c}} \cdot e^{\left(\frac{2x^T y}{c}\right)}$$

$$\|x-y\|_2^2 = (x-y)^T (x-y) = x^T x + y^T y - 2 x^T y$$

$$e^{\left(\frac{2x^T y}{c}\right)} = \sum_{i=0}^{\infty} \frac{1}{i!} \left(\frac{2}{c}\right)^i \left(x^T y\right)^i = 1 + \frac{2}{c} x^T y + \frac{4}{c^2} \frac{1}{2!} \left(x^T y\right)^2 \cdots$$