



Statistics for Business

Fall Semester 2017–2018

Assignment 2

(Release Date: 05/09/2017)

Answer the following questions and hand in your answers by **Thursday, September 14, 2017**. These should either be typed and sent electronically or handwritten, scanned and sent electronically (via E-mail at pkonstantinou@aueb.gr). Please make sure that your files do not exceed 5MB in size, otherwise the server at AUEB might block them!

Discrete Random Variables

1. A technician services mailing machines at companies in the Athens area. Depending on the type of malfunction, the service call can take one, two, three, or four hours. The different types of malfunctions occur at about the same frequency.
 - (a) Develop a probability distribution for the duration of a service call.
 - (b) Draw a graph of the probability distribution.
 - (c) Show that your probability distribution satisfies the conditions required for a discrete probability function.
 - (d) What is the probability that a service call will take three hours?
 - (e) A service call has just come in, but the type of malfunction is unknown. It is 3:00 P.M. and service technicians usually get off work at 5:00 P.M. What is the probability that the service technician will have to work overtime to fix the machine today?

2. The number of students taking the IELTS test has risen greatly. Students are allowed to repeat the test in hopes of improving the score that is sent to university admission offices. The number of times the IELTS test was taken and the number of students are as follows:

<u>Nr. of Times</u>	<u>Nr. of Students</u>
1	721, 769
2	601, 325
3	166, 736
4	22, 299
5	6, 730

- (a) Let x be a random variable indicating the number of times a student takes IELTS. Show the probability distribution for this random variable.
- (b) What is the probability that a student takes IELTS more than one time?
- (c) What is the probability that a student takes IELTS three or more times?

- (d) What is the expected value of the number of times IELTS is taken? What is your interpretation of the expected value?
- (e) What is the variance and standard deviation for the number of times IELTS is taken?
3. The following probability distributions of job satisfaction scores for a sample of senior executives and middle managers range from a low of 1 (very dissatisfied) to a high of 5 (very satisfied).

Table 1: Job Satisfaction Probabilities

Job Satisfaction Score	Probability	
	Senior Executive	Middle Manager
1	0.05	0.04
2	0.09	0.10
3	0.03	0.12
4	0.42	0.46
5	0.41	0.28

- (a) What is the expected value of the job satisfaction score for senior executives?
- (b) What is the expected value of the job satisfaction score for middle managers?
- (c) Calculate the variance of job satisfaction scores for executives and middle managers.
- (d) Calculate the standard deviation of job satisfaction scores for both probability distributions.
- (e) Compare the overall job satisfaction of senior executives and middle managers.
4. Consider a binomial experiment with $n = 20$ and $p = 0.70$.
- (a) Compute $\Pr(x = 12)$.
- (b) Compute $\Pr(x = 16)$.
- (c) Compute $\Pr(x \geq 16)$.
- (d) Compute $\Pr(x \leq 15)$.
- (e) Compute $E(x)$.
- (f) Compute $\text{Var}(x)$ and σ .
5. A survey reported that 25% of employees said their company is loyal to them. Suppose 10 employees are selected randomly and will be interviewed about company loyalty.
- (a) Is the selection of 10 employees a binomial experiment? Explain.
- (b) What is the probability that none of the 10 employees will say their company is loyal to them?

- (c) What is the probability that 4 of the 10 employees will say their company is loyal to them?
 - (d) What is the probability that at least 2 of the 10 employees will say their company is loyal to them?
6. AUEB found that 20% of its students withdraw without completing the introductory economics course. Assume that 20 students registered for the course.
- (a) Calculate the probability that two or fewer will withdraw.
 - (b) Calculate the probability that exactly four will withdraw.
 - (c) Calculate the probability that more than three will withdraw.
 - (d) Calculate the expected number of withdrawals.
7. When a new machine is functioning properly, only 3% of the items produced are defective. Assume that we will randomly select two parts produced on the machine and that we are interested in the number of defective parts found.
- (a) Describe the conditions under which this situation would be a binomial experiment.
 - (b) Draw a tree diagram showing this problem as a two-trial experiment.
 - (c) How many experimental outcomes result in exactly one defect being found?
 - (d) Calculate the probabilities associated with finding no defects, exactly one defect, and two defects.

Continuous Random Variables

8. Using “TABLE 3 Areas under the Normal Curve” (Appendix I Tables) find the following probabilities for a standard normal random variable Z :
- (a) $\Pr(0 \leq Z \leq 0.83)$
 - (b) $\Pr(-1.57 \leq Z \leq 0)$
 - (c) $\Pr(Z \geq 0.44)$
 - (d) $\Pr(Z \geq -0.23)$
 - (e) $\Pr(Z \leq 1.20)$
 - (f) $\Pr(Z \leq -0.71)$
 - (g) $\Pr(-1.98 \leq Z \leq 0.49)$
 - (h) $\Pr(0.52 \leq Z \leq 1.22)$
 - (i) $\Pr(-1.75 \leq Z \leq -1.04)$

9. In an article about the cost of health care, it was reported that a visit to a hospital emergency room for something as simple as a sore throat has a mean cost of €328. Assume that the cost for this type of hospital emergency room visit is normally distributed with a standard deviation of €92. Answer the following questions about the cost of a hospital emergency room visit for this medical service.
- (a) What is the probability that the cost will be more than €500?
 - (b) What is the probability that the cost will be less than €250?
 - (c) What is the probability that the cost will be between €300 and €400?
 - (d) If the cost to a patient is in the lower 8% of charges for this medical service, what was the cost of this patient's emergency room visit?
10. For a person to qualify for membership at SISA (a special society) he/she must score in the upper 2% of the population on an a special skills test. If test scores are normally distributed with a mean of 100 and a standard deviation of 15, what score must a person have to qualify SISA?
11. The average stock price for companies making up the FTSE-100 is £41, and the standard deviation is £9.75. Assume the stock prices are normally distributed.
- (a) What is the probability a company will have a stock price of at least £45?
 - (b) What is the probability a company will have a stock price no higher than £23?
 - (c) How high does a stock price have to be to put a company in the top 5%?
12. In May 2012, the Public sector worker spent an average of 98 hours logged on to the Internet while at work. Assume the population mean is 98 hours, the times are normally distributed, and that the standard deviation is 25 hours.
- (a) a. What is the probability that in May 2012 a randomly selected public sector worker spent fewer than 60 hours logged on to the Internet?
 - (b) What percentage of workers spent more than 120 hours in May 2012 logged on to the Internet?
 - (c) A person is classified as a heavy user if he or she is in the upper 20% of usage. In May 2012, how many hours did a worker have to be logged on to the Internet to be considered a heavy user?

Multivariate Probability Distributions

13. After analyzing monthly ales data, the marketing director of KOTSOVOLOS (Patisserie Branch) produced the following joint probability distribution of the number of refrigerators and TVs sold daily.

Table 2: Joint Probability Distribution of Sales

TVs	Refrigerators		
	0	1	2
0	0.08	0.14	0.12
1	0.09	0.17	0.13
2	0.05	0.18	0.04

- Find the marginal probability distribution of the number of refrigerators sold daily.
- Find the marginal probability distribution of the number of TVs sold daily.
- Compute the mean and variance of the number of refrigerators sold daily.
- Compute the mean and variance of the number of stoves sold daily.
- Compute the covariance and the coefficient of correlation.
- Calculate $\Pr(1 \text{ refrigerator} | 0 \text{ TVs})$.
- Calculate $\Pr(0 \text{ TVs} | 1 \text{ refrigerator})$.
- Calculate $\Pr(2 \text{ refrigerators} | 2 \text{ TVs})$.

14. A portfolio is composed of two stocks. The proportion of each stock, their expected values, and standard deviations are listed in the following table. For each of the follow-

Table 3: Two Stock Portfolio

Stock	1	2
Proportion of portfolio	0.30	0.70
Mean	0.12	0.25
Standard deviation	0.02	0.15

ing coefficients of correlation, calculate the expected value and standard deviation of the portfolio:

- $\rho = 0.5$
- $\rho = 0.2$
- $\rho = 0$

15. The joint probability distribution of X and Y is shown in the following table:

y	x		
	1	2	3
1	0.42	0.12	0.06
2	0.28	0.08	0.04

- Determine the marginal distributions of X and Y .

- (b) Compute the covariance and coefficient of correlation between X and Y .
- (c) Develop the probability distribution of $X + Y$.