ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ — ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS

Statistics for Business **Fall Semester 2017–2018**

Assignment 1

(Release Date: 30/08/2017)

Answer the following questions and hand in your answers by **Thursday, September 7, 2017**. These should either be typed and sent electronically or handwritten, scanned and sent electronically (via E-mail at pkonstantinou@aueb.gr). Please make sure that your files do not exceed 5MB in size, otherwise the server at AUEB might block them!

Calculating Probabilities

1. The TZOKER lotte1y is played twice each week. To play TZOKER a participant must select five numbers from the digits 1 through 45 and a TZOKER number from the digits 1 trough 20. To determine the winning numbers for each game, lottery officials draw five balls out of a drum with 45 balls numbered from 1 to 45 (white balls), and one ball out of a drum with 20 balls numbered from 1 to 20 (red balls). To win the jackpot a participant's numbers must match the numbers on the five white balls in any order and the number on the red ball. Five friends in Thessaloniki claimed the record €8 million jackpot about two years ago. A variety of other cash prizes are awarded each time the game is played. Suppose that a prize of €100,000 is paid if the participant's five numbers match the numbers on the five white balls.

   (a) Compute the number of ways the first five numbers can be selected.

   (b) What is the probability of winning a prize of €100,000 by matching the numbers on the five white balls?

   (c) What is the probability of winning the TZOKER jack-pot?

2. Suppose there are six items and use the letters A, B, C, D, E, and F to identify them.

   (a) How many ways can three items be selected from the above group of six items? List each of the different combinations.

   (b) How many permutations of three items can be selected from the above group of six? List each of the permutations of items B, D, and F.

3. Consider the experiment of rolling a pair of dice. Suppose that we are interested in the sum of the face values showing on the dice.

   (a) How many sample points are possible?

   (b) List the sample points.

(c) What is the probability of obtaining a value of 7?

(d) What is the probability of obtaining a value of 9 or greater?

(e) Because each roll has six possible even values (2, 4, 6, 8, 10, and 12) and only five possible odd values (3, 5, 7, 9, and 11). the dice should show even values more often than odd values. Do you agree with this statement? Explain.

4. Do you think the government protects property rights? This question was part of an online survey of individuals aged less than 65 living in Greece and Germany. The numbers of individuals from Greece and Germany who answered Yes, No, or Unsure to this question are provided below.

Response	Greece	Germany
Yes	187	197
No	334	411
Unsure	256	213

(a) Estimate the probability that an individual in Greece thinks the government is not protecting property rights adequately.

(b) Estimate the probability that an individual in Germany thinks the government is not protecting property rights adequately or is unsure the government is protecting property rights adequately.

(c) For a randomly selected individual from these two countries, estimate the probability that the person thinks the government is not protecting property rights adequately.

(d) Based on the survey results, does there appear to be much difference between the perceptions of individuals in Greece and investors in Germany regarding the issue of the government protecting property rights adequately?

5. Suppose that we have a sample space with five equally likely and mutually exclusive outcomes: S_1, S_2, S_3, S_4 and S_5. Let also

$$
\begin{aligned}
A &= \{S_1, S_2\} \\
B &= \{S_3, S_4\} \\
C &= \{S_2, S_3, S_5\}
\end{aligned}
$$

(a) Calculate $\Pr(A)$, $\Pr(B)$ and $\Pr(C)$.

(b) Calculate $\Pr(A \cup B)$. Are A and B mutually exclusive?

(c) Find $\bar{A}$, $\bar{C}$, and calculate $\Pr(\bar{A})$ and $\Pr(\bar{C})$ (with bars denoting complements).

(d) Find $A \cup \bar{B}$ and calculate $\Pr(A \cup \bar{B})$.

(e) Calculate $\Pr(A \cup B)$.

6. AUEB surveyed alumni to learn more about what they think of AUEB. One part of the survey asked respondents to indicate whether their overall experience at AUEB fell short

of expectations, met expectations, or surpassed expectations. The results showed that 4% of the respondents did not provide a response, 26% said that their experience fell short of expectations, and 65% of the respondents said that their experience met expectations.

(a) If we chose an alumnus at random, what is the probability that the alumnus would say their experience surpassed expectations?

(b) If we chose an alumnus at random. what is the probability that the alumnus would say their experience met or surpassed expectations?

7. Assume that there are two events A and B. that are mutually exclusive. Assume further that we know $\Pr(A) = 0.30$ and $\Pr(B) = 0.40$.

   (a) What is $\Pr(A \cap B)$?

   (b) What is $\Pr(A|B)$?

   (c) A fellow professor of politics argues that the concepts of mutually exclusive events and independent events are really the same and that if events are mutually exclusive they must be independent. Do you agree with this statement? Use the probability information in this problem to justify your answer.

   (d) What general conclusion would you make about mutually exclusive and independent events given the results of this problem?

8. The automobile industry sold 657,000 vehicles in the United States during January 2009. This volume was down 37% from January 2008 as economic conditions continued to decline. The Big Three U.S. automakers – General Motors, Ford, and Chrysler – sold 280,500 vehicles, down 48% from January 2008. A summary of sales by automobile manufacturer and type of vehicle sold is shown in the following table. Data are in thousands of vehicles. The non-U.S. manufacturers are led by Toyota, Honda, and Nissan. Thee category Light Truck includes pickup, minivan, SUV, and crossover models.

	Type	of Vehicle
	Car	Light Truck
U.S.	87.4	193.1
Non U.S.	228.5	148.0

   (a) Develop a joint probability table for these data and use the table to answer the following questions.

   (b) What are the marginal probabilities? What do they tell you about the probabilities associated with the manufacturer and the type of vehicle sold?

   (c) If a vehicle was manufactured by one of the U.S. automakers, what is the probability that the vehicle was a car? What is the probability that it was a light truck?

   (d) If a vehicle was not manufactured by one of the U.S. automakers, what is the probability that the vehicle was a car? What is the probability that it was a light truck?

(e) If the vehicle was a light truck, what is the probability that it was manufactured by one of the U.S. automakers?

(f) What does the probability information tell you about sales?

9. Giannis Antetokounmpo of the NBA Milwaukee Bucks is the best free-throw shooter on the team making 89.7% of his shots. Assume that late in a basketball game, Giannis Antetokounmpo is fouled and is awarded two shots.

   (a) What is the probability that he will make both shots?

   (b) What is the probability that he will make at least one shot?

   (c) What is the probability that he will miss both shots?

   (d) Late in a basketball game a team often intentionally fouls an opposing player in order to stop the game clock. The usual strategy is to intentionally foul the other team's worst free- throw shooter. Assume that the Milwaukee Bucks' center makes 58% of his free- throw shots. Calculate the probabilities for the center as shown in parts (a), (b), and (c), and show that intentionally fouling the Milwaukee Bucks' center is a better strategy than intentionally fouling Giannis Antetokounmpo.

10. A study of 31,000 hospital admissions in Athens, Attica found that 4% of the admissions led to treatment-caused injuries. One-seventh of these treatment-caused injuries resulted in death, and one-forth were caused by negligence. Malpractice claims were filed in 1 out of 7 .5 cases involving negligence, and payments were made in one out of every two claims.

   (a) What is the probability that a person admitted to the hospital will suffer a treatment-caused injury due to negligence?

   (b) What is the probability that a person admitted to the hospital will die from a treatment-caused injury?

   (c) In the case of a negligent treatment-caused injury, what is the probability that a malpractice claim will be paid?

11. A survey of MSc students, provided the following data for 2018 students.

   (a) For a randomly selected MSc student, prepare a joint probability table for the experiment consisting of observing the student's age and whether the student applied to one or more schools.

   (b) What is the probability that a randomly selected applicant is 23 or under?

   (c) What is the probability that a randomly selected applicant is older than 26?

   (d) What is the probability that a randomly selected applicant applied to more than one school?

   (e) Given that a person applied to more than one school, what is the probability that the person is 24–26 years old?

Age Group	Applied to More than One School	
	YES	NO
23 and under	207	201
24 – 26	299	379
27 – 30	185	268
31 – 35	66	193
36 and over	51	169

Table 1: Table: Survey Data

(f) Given that a person is in the 36-and-over age group, what is the probability that the person applied to more than one school?

(g) What is the probability that a person is 24–26 years old or applied to more than one school?

(h) Suppose a person is known to have applied to only one school. What is the probability that the person is 31 or more years old?

(i) Is the number of schools applied to independent of age? Explain.

12. The prior probabilities for events A_1,A_2 and A_3 are $\Pr(A_1) = 0.20$, $\Pr(A_2) = 0.50$, and $\Pr(A_3) = .30$. The conditional probabilities of event B given A_1,A_2 and A_3 are are $\Pr(B|A_1) = 0.50$, $\Pr(B|A_2) = 0.40$, and $\Pr(B|A_3) = 0.30$.

    (a) Calculate $\Pr(B \cap A_1)$, $\Pr(B \cap A_2)$ and $\Pr(B \cap A_3)$.

    (b) Apply Bayes' theorem and compute $\Pr(A_1|B)$, $\Pr(A_2|B)$, and $\Pr(A_3|B)$

13. Piraues Bank reviewed its business loans policy with the intention of selling some of its loans to a fund. In the past approximately 5% of its business lenders defaulted, leaving the bank unable to collect the outstanding balance. Hence, management established a prior probability of 0.05 that any particular business lender will default. The bank also found that the probability of missing a monthly payment is 0.20 for customers who do not default. Of course, the probability of missing a monthly payment for those who default is 1.

    (a) Given that a customer missed one or more monthly payments, what is the posterior probability that the customer will default?

    (b) The bank would like to sell its loan if the probability that a customer will default is greater than 0.20. Should the bank sell its loan if the customer misses a monthly payment? Why or why not?