ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ
ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS

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Statistics for Business

Assignment 2: Suggested Solutions

Discrete Random Variables

1. The correct Table with the probability is

Years, y	3	4	5	6	7	8	9	10	11	12	13
$p(y)$	.03	.05	.07	**.10**	.14	.20	.18	.12	.07	.03	.01

With the "correct" probability distribution you would get:

(a) $\mu = \mathrm{E}(Y) = 3(.03) + 4(.05) + 5(.07) + \ldots + 13(.01) = 7.9$

(b) $\sigma^2 = \mathrm{Var}(Y) = \mathrm{E}(Y^2) - [\mathrm{E}(Y)]^2$

$$= 3^2(.03) + 4^2(.05) + 5^2(.07) + \ldots + 13^2(.01) - 7.9^2 = 67.14 - 62.41 = 4.73,$$

or

$$\mathrm{Var}(Y) = \sum_{y=3}^{13}(y-\mu)^2 P(y) = (3-7.9)^2(.03)+(4-7.9)^2(.05)+\ldots+(13-7.9)^2(.01) = 4.73.$$

So $\sigma = 2.17$.

(c) $(\mu - 2\sigma, \mu + 2\sigma) = (3.56, 12.24)$.So,

$$\Pr(3.56 < Y < 12.24) = \Pr(4 \leq Y \leq 12) =$$
$$= .05 + .07 + .10 + .14 + .20 + .18 + .12 + .07 + .03 = 0.96$$

(a′) Using the probabilities provides (e.g. $p(6) = 0.01$) we find:

$$3(.03) + 4(.05) + 5(.07) + 6(.01) + \ldots + 13(.01) = 7.36$$

(b′) $\sigma^2 = \mathrm{Var}(Y)$

$$\mathrm{Var}(Y) = \sum_{y=3}^{13}(y-\mu)^2 P(y) = (3-7.36)^2(.03)+(4-7.36)^2(.05)+\ldots++(13-7.36)^2(.01) = 4.86$$

(c′) $(\mu - 2\sigma, \mu + 2\sigma) = (2.95, 11.774)$.So,

$$\Pr(2.95 < Y < 11.774) = \Pr(3 \leq Y \leq 11) =$$
$$= 0.03 + .05 + .07 + .01 + .14 + .20 + .18 + .12 + .07 = 0.87$$

2. (a) The mean of X will be larger than the mean of Y. We have

$$\mathrm{E}(X) = \mathrm{E}(Y + 1) = \mathrm{E}(Y) + 1 = \mu + 1.$$

   (b) The variances of X and Y will be the same (the addition of 1, a constant, doesn't affect variability). Note that

$$\mathrm{Var}(X) = \mathrm{Var}(Y + 1) = \mathrm{Var}(Y) = \sigma^2.$$

3. Let $X = \#$ that recover from stomach disease. Then, Y is binomial with $n = 20$ and $p = 0.8$. To find these probabilities, I just use Excel.

   (a) $\Pr(X \geq 10) = 1 - \Pr(X \leq 9) = 1 - 0.001 = 0.999.$
   (b) $\Pr(14 \leq X \leq 18) = \Pr(X \leq 18) - \Pr(X \leq 13) = 0.931 - 0.087 = 0.844.$
   (c) $\Pr(X \leq 16) = 0.589.$

4. Let $Y = \#$ of successful operations. Then Y is binomial with $n = 5$.

   (a) With $p = 0.8$, $\Pr(Y = 5) = \frac{5!}{5!(5-5)!}(.8)^5(1 - .8)^{5-5} = (.8)^5 = 0.328.$
   (b) With $p = 0.6$, $\Pr(Y = 4) = \frac{5!}{4!(5-4)!}(.8)^4(1 - .8)^{5-4} = 5(.6)^4(.4) = 0.259.$
   (c) With $p = 0.3$, $\Pr(Y < 2) = \Pr(Y = 1) + \Pr(Y = 0) = 0.528.$

5. Let $X = \#$ of customers that arrive during the hour. Then, X is Poisson with $\lambda = 7$.

   (a) $\Pr(X \leq 3) = .0818$
   (b) $\Pr(X \geq 2) = 1 - \Pr(X \leq 3) = 1 - .0818 = .9927$
   (c) $\Pr(X = 5) = .1277$

6. Note that over a one–minute period, $Y = \#$ of cars that arrive at the toll booth is Poisson with $\lambda = 80/60 = 4/3$. Then

$$\Pr(Y \geq 1) = 1 - \Pr(Y = 0) = 1 - e^{-4/3} = 0.7364$$

7. Define: $X = \#$ of cars through entrance I, $Y = \#$ of cars through entrance II. Thus, X is Poisson with $\lambda = 3$ and Y is Poisson with $\lambda = 4$. Then,

$$\begin{aligned} & \Pr(\text{three cars arrive}) \\ = \; & \Pr(X = 0, Y = 3) + \Pr(X = 1, Y = 2) + \Pr(X = 2, Y = 1) + \Pr(X = 3, Y = 0). \end{aligned}$$

By independence,

$$\begin{aligned} & \Pr(\text{three cars arrive}) \\ = \; & \Pr(X{=}0)\Pr(Y{=}3) + \Pr(X{=}1)\Pr(Y{=}2) + \Pr(X{=}2)\Pr(Y{=}1) + \Pr(X{=}3)\Pr(Y{=}0) \\ = \; & 0.0521 \end{aligned}$$

Continuous Random Variables

8. Let X have an exponential distribution with $1/\lambda = 2.4$.

   (a) $\Pr(X > 3) = 1 - \Pr(Y < 3) = 1 - (1 - e^{-\frac{1}{2.4}3}) = e^{-\frac{3}{2.4}} = 0.2865.$

   (b) $\Pr(2 \leq X \leq 3) = (1 - e^{-\frac{1}{2.4}3}) - (1 - e^{-\frac{1}{2.4}2}) = 0.1481.$

9. Let $Y =$ time between fatal airplane accidents. So, Y is exponential with $1/\lambda = 44$ days.

   (a) $\Pr(Y \leq 31) = 1 - e^{-\frac{1}{44}31} = .5057.$

   (b) $\text{Var}(Y) = 1/\lambda^2 = (1/\lambda)^2 = 44^2 = 1936.$

10. Just looking up the relevant table, we find

    (a) $z_0 = 0.5$

    (b) $z_0 = 1.10$

    (c) $z_0 = 1.645$

    (d) $z_0 = 2.576$

11. A GPA 3.0 is

$$\frac{3.0 - 2.4}{0.8} = 0.75$$

standard deviations above the mean. So

$$\Pr(Z > 0.75) = 1 - \Pr(Z \leq 0.75) = .2266.$$

12. Let X = width of a bolt of fabric, so X has a normal distribution with $\mu = 950$mm and $\sigma = 10$mm.

   (a) $\Pr(947 \leq X \leq 958) = \Pr(\frac{947-950}{10} \leq \frac{X-950}{10} \leq \frac{958-950}{10}) = \Pr(-0.3 \leq Z \leq 0.8) = .406$

   (b) It is necessary that $\Pr(X \leq c) = .8531$. Note that for the standard normal, we find that $\Pr(Z \leq z_0) = .8531$ when $z_0 = 1.05$. So, $c = \mu + z_0 \cdot \sigma = 950 + (1.05)(10) = 960.5$mm