

## Assignment 2

Answer the following questions and provide your answers in a typed document. The answers are to be handed by Tuesday, September 15, 2015.

### Discrete Random Variables

1. The maximum patent life for a new drug is 17 years. Subtracting the length of time required by the FDA for testing and approval of the drug provides the actual patent life for the drug – that is, the length of time that the company has to recover research and development costs and to make a profit. The distribution of the lengths of actual patent lives for new drugs is given below:

Years, $y$	3	4	5	6	7	8	9	10	11	12	13
$p(y)$	0.03	0.05	0.07	0.01	0.14	0.2	0.18	0.12	0.07	0.03	0.01

- a. Find the mean patent life for a new drug.
  - b. Find the standard deviation of  $Y$  = the length of life of a randomly selected new drug.
  - c. What is the probability that the value of  $Y$  falls in the interval  $\mu \pm 2\sigma$ ?
2. Suppose that  $Y$  is a discrete random variable with mean  $\mu$  and variance  $\sigma^2$  and let  $X=Y+1$ .
    - a. Do you expect the mean of  $X$  to be larger than, smaller than, or equal to  $\mu = E(Y)$ ? Why?
    - b. Recalling that the variance is a measure of spread or dispersion, do you expect the variance of  $X$  to be larger than, smaller than, or equal to  $\sigma^2 = \text{Var}(Y)$ ? Why?
  3. The probability that a patient recovers from a stomach disease is .8. Suppose 20 people are known to have contracted this disease. What is the probability that
    - a. exactly 14 recover?
    - b. at least 10 recover?
    - c. at least 14 but not more than 18 recover?
    - d. at most 16 recover?
  4. A new surgical procedure is successful with a probability of  $p$ . Assume that the operation is performed five times and the results are independent of one another. What is the probability that
    - a. all five operations are successful if  $p = .8$ ?
    - b. exactly four are successful if  $p = .6$ ?
    - c. less than two are successful if  $p = .3$ ?
  5. Customers arrive at a checkout counter in a department store according to a Poisson distribution at an average of seven per hour. During a given hour, what are the probabilities that

- a. no more than three customers arrive?
  - b. at least two customers arrive?
  - c. exactly five customers arrive?
6. Cars arrive at a toll both according to a Poisson process with mean 80 cars per hour. If the attendant makes a one-minute phone call, what is the probability that at least 1 car arrives during the call?
7. A parking lot has two entrances. Cars arrive at entrance I according to a Poisson distribution at an average of three per hour and at entrance II according to a Poisson distribution at an average of four per hour. What is the probability that a total of three cars will arrive at the parking lot in a given hour? (Assume that the numbers of cars arriving at the two entrances are independent.)

## Continuous Random Variables

8. The magnitude of earthquakes recorded in a region of North America can be modeled as having an exponential distribution with mean 2.4, as measured on the Richter scale. Find the probability that an earthquake striking this region will
- a. exceed 3.0 on the Richter scale.
  - b. fall between 2.0 and 3.0 on the Richter scale.
9. Historical evidence indicates that times between fatal accidents on scheduled American domestic passenger flights have an approximately exponential distribution. Assume that the mean time between accidents is 44 days.
- a. If one of the accidents occurred on July 1 of a randomly selected year in the study period, what is the probability that another accident occurred that same month?
  - b. What is the variance of the times between accidents?
10. If  $Z$  is a standard normal random variable, find the value  $z_0$  such that
- a.  $\Pr(Z > z_0) = 0.5$
  - b.  $\Pr(Z < z_0) = .8643$
  - c.  $\Pr(-z_0 < Z < z_0) = .90$
  - d.  $\Pr(-z_0 < Z < z_0) = .99$
11. The grade point averages (GPAs) of a large population of college students are approximately normally distributed with mean 2.4 and standard deviation .8. What fraction of the students will possess a GPA in excess of 3.0?
12. The width of bolts of fabric is normally distributed with mean 950 mm (millimeters) and standard deviation 10 mm.
- a. What is the probability that a randomly chosen bolt has a width of between 947 and 958mm?
  - b. What is the appropriate value for  $C$  such that a randomly chosen bolt has a width less than  $C$  with probability .8531?