ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ

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Statistics for Business: Formulas

- **Sample Variance, Covariance and Correlation**

$$
\begin{aligned}
s_x^2 &= \frac{\sum_{i=1}^n (x_i-\bar{x})^2}{n-1} = \frac{\sum_{i=1}^n x_i^2 - n\bar{x}^2}{n-1}; \; s_{xy} = \frac{\sum_{i=1}^n (x_i-\bar{x})(y_i-\bar{y})}{n-1} = \frac{\sum_{i=1}^n x_i y_i - n\bar{x}\bar{y}}{n-1}; \\
r &= \frac{s_{xy}}{\sqrt{s_x^2}\sqrt{s_y^2}} = \frac{s_{xy}}{s_x s_y}
\end{aligned}
$$

- **Combinatorics**:

$$
n! = n(n-1)(n-2)....(2)(1); \quad P_r^n = \frac{n!}{(n-r)!}; \quad C_r^n = \binom{n}{r} = \frac{n!}{r!(n-r)!}
$$

- **Probability**
  - $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B); \quad \Pr(A \cap B) = \Pr(B)\Pr(A|B) = \Pr(A)\Pr(B|A)$
  - Let $E_1, E_2, ..., E_k$ be a finite partition of the sample space S and $A \subset S$. Then

$$
\Pr(A) = \Pr(A \cap E_1) + \Pr(A \cap E_2) + ... + \Pr(A \cap E_k).
$$

- **Expected Value and Variance and**

$$
\begin{aligned}
\mathrm{E}(X) \equiv \mu &= \sum\nolimits_{i=1}^n \Pr(X = x_i) \times x_i \\
\mathrm{E}[g(x)] &= \sum\nolimits_{i=1}^n \Pr(X = x_i) \times g(x_i) \\
\mathrm{Var}(X) \equiv \sigma^2 &= \sum\nolimits_{i=1}^n \Pr(X = x_i) \times [x_i - \mathrm{E}(X)]^2
\end{aligned}
$$

- **Variance, Covariance and Correlation**

$$
\begin{aligned}
\mathrm{Var}(X) &= \mathrm{E}[(X-\mu)^2] = \mathrm{E}(X^2) - [\mathrm{E}(X)]^2 = \mathrm{E}(X^2) - \mu^2; \\
\mathrm{Cov}(X,Y) &= \mathrm{E}[(X-\mu_X)(Y-\mu_Y)] = \mathrm{E}(XY) - \mathrm{E}(X)\mathrm{E}(Y) = \mathrm{E}(XY) - \mu_X\mu_Y; \\
\rho &= \mathrm{Corr}(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)}\sqrt{\mathrm{Var}(Y)}} \\
\mathrm{Var}(aX+bY) &= a^2\mathrm{Var}(X) + b^2\mathrm{Var}(Y) + 2ab\mathrm{Cov}(X,Y); \\
\mathrm{Cov}(a+bX, c+dY) &= bd\mathrm{Cov}(X,Y)
\end{aligned}
$$

- **Discrete Random Variables**

$$
\begin{aligned}
\text{Bernoulli} &: \Pr(X=x) = p^x(1-p)^{1-x}; \; x = 0, 1 \\
\text{Binomial} &: \Pr(X=x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}; \; x = 0, 1, 2, ..., n
\end{aligned}
$$

- **Sampling Distributions – Confidence Intervals – Hypothesis tests**
  - If $X \sim N(\mu, \sigma^2)$, and $\{X_1, ..., X_n\}$ is a random sample, then

$$\begin{aligned} \text{if } \sigma^2 \text{ is known } \underline{\textbf{or}}\ n \text{ is large} &: \quad \frac{\bar{X}-\mu}{\sigma_X/\sqrt{n}} \sim N(0,1) \ \text{ or } \ \frac{\bar{X}-\mu}{S_X/\sqrt{n}} \to N(0,1) \\ \text{if } \sigma^2 \text{ is unknown } \underline{\textbf{and}}\ n \text{ is small} &: \quad \frac{\bar{X}-\mu}{S_x/\sqrt{n}} \sim t_{n-1} \end{aligned}$$

  - If $\{X_1, ..., X_n\}$ and $\{Y_1, ..., Y_n\}$ are dependent (paired or matched) random samples and n **is small**, then with $d_i = x_i - y_i$;

$$t = \frac{\bar{d}-\mu_d}{s_d/\sqrt{n}} \sim t_{n-1}; \text{ where } \bar{d} = \frac{1}{n}\sum_{i=1}^n d_i; \text{ and } s_d = \sqrt{\frac{1}{n-1}\sum_{i=1}^n (d_i - \bar{d})^2}$$

  - If $X \sim N(\mu_x, \sigma_x^2), Y \sim N(\mu_y, \sigma_y^2)$ and $\{X_1, ..., X_n\}$ and $\{Y_1, ..., Y_n\}$ are independent random samples, then

$$\begin{aligned} \text{if } \sigma_x^2 \text{ and } \sigma_y^2 \textbf{ known} &: \quad \frac{(\bar{x}-\bar{y})-(\mu_x-\mu_y)}{\sqrt{(\sigma_x^2/n_x)+(\sigma_y^2/n_y)}} \sim N(0,1); \\ \text{if } \sigma_x^2 \text{ and } \sigma_y^2 \textbf{ unknown but } n = n_x + n_y \textbf{ is large} &: \quad \frac{(\bar{x}-\bar{y})-(\mu_x-\mu_y)}{\sqrt{(s_x^2/n_x)+(s_y^2/n_y)}} \to N(0,1); \\ \text{if } \sigma_x^2 = \sigma_y^2 \textbf{ unknown and } n = n_x + n_y \textbf{ is small} &: \quad \frac{(\bar{x}-\bar{y})-(\mu_x-\mu_y)}{\sqrt{(s_p^2/n_x)+(s_p^2/n_y)}} \sim t_{n_x+n_y-2}; \\ \text{where } s_p^2 &= \frac{(n_x-1)s_x^2+(n_y-1)s_y^2}{n_x+n_y-2} \end{aligned}$$

  - $\hat{\theta} \pm c \cdot \mathrm{SE}(\hat{\theta})$ [Point Estimate ± (Reliability Factor)·(Standard Error)]
  - Test Statistic: $TS = (\hat{\theta} - \theta_0)/\mathrm{SE}(\hat{\theta})$, where θ_0 the value of θ under the null.

- **Simple Regression Prediction Intervals**
  - Confidence interval estimate for the *expected value* of y given a particular x_0

$$\hat{y}_0 \pm t_{n-2,\alpha/2} \cdot s_e \sqrt{\frac{1}{n} + \frac{(x_0-\bar{x})^2}{\sum_{i=1}^n (x_i-\bar{x})^2}} = (b_0 + b_1 x_0) \pm t_{n-2,\alpha/2} \cdot s_e \sqrt{\frac{1}{n} + \frac{(x_0-\bar{x})^2}{\sum_{i=1}^n (x_i-\bar{x})^2}}$$

  - Confidence interval for an *actual observed value* of y given a particular x_0

$$\hat{y}_0 \pm t_{n-2,\alpha/2} \cdot s_e \sqrt{1 + \frac{1}{n} + \frac{(x_0-\bar{x})^2}{\sum_{i=1}^n (x_i-\bar{x})^2}} = (b_0 + b_1 x_0) \pm t_{n-2,\alpha/2} \cdot s_e \sqrt{1 + \frac{1}{n} + \frac{(x_0-\bar{x})^2}{\sum_{i=1}^n (x_i-\bar{x})^2}}$$

  where $s_e = \sum_{i=1}^n e_i^2/(n-2) = SSE/(n-2)$.