

Quantitative Methods Induction Course

Lecture 3: Probability Theory & Statistical Inference

Department of Accounting and Finance

MSc in Financial Management
Athens University of Economics and Business

Induction Program

Uncertainty, Risk Modeling, Moments & Fat Tails, Parametric Distributions, CLT, Estimation & Hypothesis Testing

Lecture 3 Table of Contents

- 1 Probability Foundations & Set Theory
- 2 Conditional Probability & Bayes' Rule
- 3 Random Variables, Moments & Fat Tails
- 4 Core Parametric Distributions in Finance
- 5 Sampling Distributions & Central Limit Theorem
- 6 Statistical Estimation & Confidence Intervals
- 7 Hypothesis Testing Fundamentals
- 8 Review & Exam Practice Problems

Why Probability & Statistics in Finance?

- Finance is fundamentally the study of **allocating capital under uncertainty**:
 - **Asset Returns**: Security payoffs cannot be predicted deterministically; they are governed by probability distributions.
 - **Credit Risk**: Default events are stochastic arrivals modeled via Bayes' rule and Poisson processes.
 - **Risk Management (Value at Risk - VaR)**: Quantifying the maximum potential loss at a 95% or 99% confidence level requires modeling distributional tails.
 - **Empirical Finance & Econometrics**: Evaluating whether an investment manager generates genuine abnormal return ($\alpha > 0$) or pure luck requires rigorous **hypothesis testing**.

Core Paradigm

Probability theory provides the forward mathematical architecture of risk; statistical inference provides the backward framework to estimate truth from empirical market data.

Fundamental Definitions of Probability

Core Terminology

- **Random Experiment:** A process leading to an uncertain outcome (e.g., observing tomorrow's return on the S&P 500).
- **Basic Outcome (O_i):** A single, elementary result of a random experiment.
- **Sample Space (S or Ω):** The collection of **all possible basic outcomes**.
- **Event (A):** Any subset of basic outcomes from the sample space ($A \subseteq S$).

Set Operations on Events:

- **Intersection ($A \cap B$):** Outcomes belonging to **both** A and B .
- **Union ($A \cup B$):** Outcomes belonging to A , B , or both.
- **Complement (A^c or $\bar{A}$):** All outcomes in S not in A .

Mutually Exclusive (Disjoint) Events

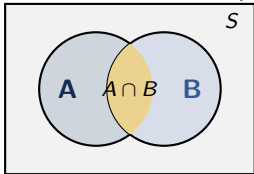
A and B have no outcomes in common:

$$A \cap B = \emptyset \implies P(A \cap B) = 0$$

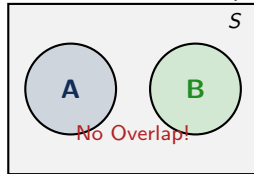
Example: A company cannot simultaneously file for bankruptcy and report record-high dividend payout tomorrow.

Venn Diagrams: Union, Intersection & Disjoint Sets

General Intersecting Events ($A \cap B \neq \emptyset$)



Mutually Exclusive Events ($A \cap B = \emptyset$)



The Addition Rule of Probability

For any two events A and B:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If A and B are mutually exclusive: $P(A \cup B) = P(A) + P(B)$.

Exam Problem: Rolling Two Dice

What is the probability of rolling a total of 7 on two fair six-sided dice?

- Total outcomes in sample space:

$$|S| = 6 \times 6 = 36.$$

- Favorable outcomes for sum = 7:

$$\{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$$

- Exactly 6 favorable combinations!

$$P(\text{Sum} = 7) = \frac{6}{36} = \frac{1}{6}$$

Exam Problem: Deck of Cards

In a standard deck of 52 cards, what is the probability of drawing:

- 1 An Ace:

$$P(\text{Ace}) = \frac{4}{52} = \frac{1}{13}$$

- 2 A Red Card OR a Face Card:

- Red cards: 26 (13 hearts + 13 diamonds).
- Face cards: 12 (J, Q, K in 4 suits).
- Red Face cards ($A \cap B$): 6.

$$P(R \cup F) = \frac{26}{52} + \frac{12}{52} - \frac{6}{52} = \frac{32}{52} = \frac{8}{13} \quad (\text{Exam Q})$$

Conditional Probability & Independence

Definition of Conditional Probability

The conditional probability of event A given that event B has already occurred is:

$$P(A | B) = \frac{P(A \cap B)}{P(B)} \quad (\text{for } P(B) > 0)$$

The Multiplication Rule:

$$P(A \cap B) = P(A | B) \cdot P(B) = P(B | A) \cdot P(A)$$

Exam Problem

If $P(A|B) = 1/3$ and $P(B) = 1/2$, then:

$$P(A \cap B) = \left(\frac{1}{3}\right) \left(\frac{1}{2}\right) = \frac{1}{6}$$

Statistical Independence

Two events A and B are **independent** if knowing B provides zero information about A :

$$P(A | B) = P(A) \iff P(A \cap B) = P(A) \cdot P(B)$$

Past Exam Calculation: If $P(A) = 0.3$, $P(B) = 0.4$, and A, B are independent:

$$P(A \cap B) = (0.3)(0.4) = \mathbf{0.12}$$

Law of Total Probability & Bayes' Theorem

Law of Total Probability

Let $B_1, B_2, \dots, B_k$ be a partition of sample space S ($\bigcup B_i = S$ and $B_i \cap B_j = \emptyset$). For any event A :

$$P(A) = \sum_{i=1}^k P(A \cap B_i) = \sum_{i=1}^k P(A | B_i)P(B_i)$$

Bayes' Theorem (Bayesian Updating)

Reverses the conditioning from prior probability $P(B_j)$ to posterior probability $P(B_j | A)$:

$$P(B_j | A) = \frac{P(A \cap B_j)}{P(A)} = \frac{P(A | B_j)P(B_j)}{\sum_{i=1}^k P(A | B_i)P(B_i)}$$

Financial Meaning

Prior Belief $P(\text{Default})$ updated with new market evidence A (e.g., credit rating downgrade) $\implies$
Posterior Risk Assessment $P(\text{Default} | \text{Downgrade})$.

Bayes' Theorem: Financial Credit Risk Example

Credit Rating Agency Problem

- Historical probability of corporate bond default: $P(D) = 2\% = 0.02 \implies P(ND) = 0.98$.
- Probability of being downgraded before default: $P(\text{Down} \mid D) = 80\% = 0.80$.
- Probability of being downgraded if healthy: $P(\text{Down} \mid ND) = 5\% = 0.05$ (False Alarm).

Question: If a company is downgraded, what is the actual probability of default $P(D \mid \text{Down})$?

Step 1: Total Probability of a Downgrade $P(\text{Down})$:

$$P(\text{Down}) = P(\text{Down} \mid D)P(D) + P(\text{Down} \mid ND)P(ND) = (0.80)(0.02) + (0.05)(0.98) = 0.016 + 0.049 = 0.065$$

Step 2: Posterior Probability via Bayes' Rule:

$$P(D \mid \text{Down}) = \frac{P(\text{Down} \mid D)P(D)}{P(\text{Down})} = \frac{0.016}{0.065} = \frac{16}{65} \approx \mathbf{24.62\%}$$

Takeaway

Even after a downgrade, the probability of default is 24.6% (not 80%), because defaults are rare baseline events (base-rate effect)!

Random Variables: Discrete vs. Continuous

Mathematical Definition

A random variable X is a function that assigns a real number to each outcome in sample space S .

Discrete Random Variables

Takes on countable values $x_1, x_2, \dots$

■ Probability Mass Function (PMF):

$$p(x) = P(X = x)$$

$$\sum_i p(x_i) = 1, \quad p(x_i) \geq 0$$

- **Example:** Number of defaults in a credit portfolio, dice rolls.

Continuous Random Variables

Takes on values on an unbroken interval.

■ Probability Density Function (PDF):

$$P(a \leq X \leq b) = \int_a^b f(x) dx$$

$$\int_{-\infty}^{\infty} f(x) dx = 1, \quad f(x) \geq 0$$

- Note: $P(X = x_0) = 0$ for any single point!
- **Example:** Asset returns, interest rates.

The Cumulative Distribution Function (CDF)

Definition of CDF

For any random variable X (discrete or continuous), the CDF $F(x)$ is:

$$F(x) = P(X \leq x)$$

Four Fundamental Properties of Any CDF:

1 Monotonically Non-Decreasing (Exam Q!):

$$\text{If } x_1 < x_2 \implies F(x_1) \leq F(x_2)$$

2 Lower Asymptote: $\lim_{x \rightarrow -\infty} F(x) = 0$

3 Upper Asymptote: $\lim_{x \rightarrow +\infty} F(x) = 1$

4 Right-Continuous: $\lim_{h \rightarrow 0^+} F(x + h) = F(x)$

Financial Risk Use: Value at Risk (VaR)

The α -percentile Value at Risk is the inverse CDF (F^{-1}):

$$\text{VaR}_\alpha = -F^{-1}(\alpha)$$

VaR identifies the maximum loss quantile at confidence level $1 - \alpha$.

Expectation, Variance & Moments

Expected Value (Mean μ)

$$\mathbb{E}[X] = \sum x_i p(x_i) \quad \text{or} \quad \int_{-\infty}^{\infty} x f(x) dx$$

Properties of Expectation:

- $\mathbb{E}[c] = c$
- $\mathbb{E}[aX + b] = a\mathbb{E}[X] + b$
- $\mathbb{E}[X + Y] = \mathbb{E}[X] + \mathbb{E}[Y]$ (Always!)

Exam Question: Expected value of rolling a fair 6-sided die:

$$\mathbb{E}[X] = \frac{1 + 2 + 3 + 4 + 5 + 6}{6} = \mathbf{3.5}$$

Variance (σ^2) & Volatility (σ)

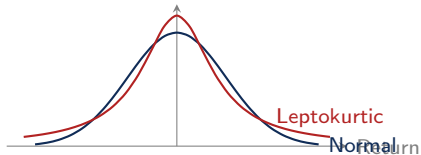
$$\text{Var}(X) = \mathbb{E}[(X - \mu)^2] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$$

Properties of Variance:

- $\text{Var}(X) \geq 0$ (Always non-negative!)
- $\text{Var}(c) = 0$
- $\text{Var}(aX + b) = a^2 \text{Var}(X)$ (Constants drop out; scalar is squared!)
- $\sigma_X = \sqrt{\text{Var}(X)}$ (Standard deviation)

Higher Moments: Skewness, Kurtosis & Fat Tails

Moment	Formula	Financial Risk Interpretation
1st: Mean (μ)	$\mathbb{E}[X]$	Expected asset return
2nd: Variance (σ^2)	$\mathbb{E}[(X - \mu)^2]$	Volatility / Total risk
3rd: Skewness (S)	$\frac{\mathbb{E}[(X - \mu)^3]}{\sigma^3}$	Asymmetry ($S < 0 \implies$ Market Crash Risk)
4th: Kurtosis (K)	$\frac{\mathbb{E}[(X - \mu)^4]}{\sigma^4}$	Tail thickness ($K = 3$ for Normal distribution)
Excess Kurtosis	$K - 3$	$> 0 \implies$ Leptokurtic (Fat Tails!)



Why Financial Asset Returns are NOT Normal

Empirical equity returns exhibit **excess kurtosis** ($K > 3$): extreme market crashes (Black Monday 1987, 2008 Lehman, 2020 Covid) happen thousands of times more often than a Gaussian model predicts!

Joint Moments of Two Random Variables

- **Covariance:** Measures the directional co-movement of two asset returns:

$$\sigma_{XY} = \text{Cov}(X, Y) = \mathbb{E}[(X - \mu_X)(Y - \mu_Y)] = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$$

- **Pearson Correlation Coefficient** (ρ_{XY}): Standardized covariance bounded in $[-1, +1]$:

$$\rho_{XY} = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}, \quad -1 \leq \rho_{XY} \leq 1$$

Variance of a Linear Combination:

$$\text{Var}(aX + bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y) + 2ab \text{Cov}(X, Y)$$

If X and Y are independent: $\text{Cov}(X, Y) = 0$.

Crucial Exam Distinction

Independence $\implies$ Zero Covariance.

The converse is **NOT** generally true (zero covariance only implies no *linear* relationship)!

Discrete Distributions: Binomial & Poisson

Binomial Distribution $B(n, p)$

Counts successes in n independent Bernoulli trials with probability p :

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

$$\mathbb{E}[X] = np, \quad \text{Var}(X) = np(1 - p)$$

Application: Cox-Ross-Rubinstein (CRR) binomial tree for option pricing.

Poisson Distribution $\text{Pois}(\lambda)$

Counts rare event arrivals over a fixed time horizon with intensity λ :

$$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}, \quad k = 0, 1, 2, \dots$$

$$\mathbb{E}[\mathbf{X}] = \lambda, \quad \text{Var}(\mathbf{X}) = \lambda \quad (\text{Exam Q!})$$

Application: Modeling credit default arrivals, operational risk cyber events.

Continuous Distributions: Uniform & Lognormal

Uniform Distribution $U(a, b)$

Constant probability over interval $[a, b]$:

$$f(x) = \frac{1}{b-a}, \quad a \leq x \leq b$$

$$\mathbb{E}[X] = \frac{a+b}{2}, \quad \text{Var}(X) = \frac{(b-a)^2}{12}$$

Past Exam Question: If $U \sim U(0, 1)$, what is $P(0.2 \leq U \leq 0.8)$?

$$P(0.2 \leq U \leq 0.8) = \frac{0.8 - 0.2}{1 - 0} = \mathbf{0.6}$$

Lognormal Distribution

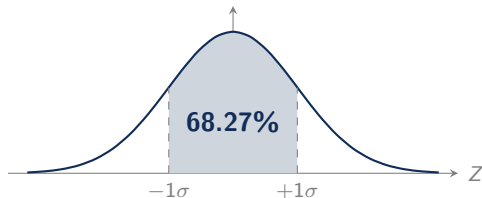
A variable S is lognormally distributed if $\ln(S) \sim N(\mu, \sigma^2)$.

- Always positive: $S > 0$.
- Right-skewed (cannot drop below zero).
- **Foundation of Black-Scholes:** Stock prices are assumed lognormal because stock returns are continuously compounded!

The Normal (Gaussian) Distribution $N(\mu, \sigma^2)$

Probability Density Function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \quad -\infty < x < \infty$$



The Empirical 68-95-99.7 Rule:

- $P(\mu - 1\sigma \leq X \leq \mu + 1\sigma) = 68.27\%$
- $P(\mu - 2\sigma \leq X \leq \mu + 2\sigma) = 95.45\%$
- $P(\mu - 3\sigma \leq X \leq \mu + 3\sigma) = 99.73\%$

Standardization (Z-score):

$$Z = \frac{X - \mu}{\sigma} \sim N(0, 1)$$

Population Parameters vs. Sample Statistics

Statistical Duality

We never observe the entire infinite population; we observe an empirical sample of size n .

Concept	Population Parameter (Unknown)	Sample Statistic (Observed)
Mean	$\mu = \mathbb{E}[X]$	$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$
Variance	$\sigma^2 = \mathbb{E}[(X - \mu)^2]$	$s^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$
Standard Deviation	σ	$s = \sqrt{s^2}$
Proportion	p	$\hat{p} = \frac{\text{Successes}}{n}$

Why Divide by $n - 1$ in Sample Variance?

Dividing by $n - 1$ (Bessel's correction) ensures that sample variance is an **unbiased estimator** of population variance: $\mathbb{E}[s^2] = \sigma^2$. Dividing by n would systematically underestimate true risk!

The Central Limit Theorem (CLT)

Theorem (Central Limit Theorem)

Let $X_1, X_2, \dots, X_n$ be independent and identically distributed (i.i.d.) random variables with mean μ and finite variance σ^2 . As $n \rightarrow \infty$:

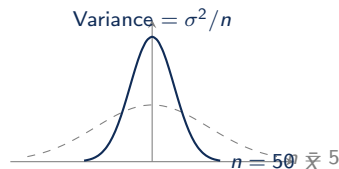
$$\bar{X} \xrightarrow{d} N\left(\mu, \frac{\sigma^2}{n}\right) \iff Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \xrightarrow{d} N(0, 1)$$

Key Insights for Finance:

- Holds **regardless of the underlying shape** of X_i (uniform, skewed, bimodal).
- The **Standard Error of the Mean** decays at rate $\sqrt{n}$:

$$SE(\bar{X}) = \frac{\sigma}{\sqrt{n}}$$

To cut estimation uncertainty in half, you need 4× larger sample!



Estimator Properties: Unbiasedness, Efficiency & Consistency

1. Unbiasedness

$$\mathbb{E}[\hat{\theta}] = \theta$$

The expected value of the estimator exactly equals the true parameter (no systematic bias).

2. Efficiency

$$\text{Var}(\hat{\theta}_1) < \text{Var}(\hat{\theta}_2)$$

Among all unbiased estimators, the efficient estimator has the **minimum variance** (tightest distribution).

3. Consistency

$$\text{plim}_{n \rightarrow \infty} \hat{\theta} = \theta$$

As sample size $n \rightarrow \infty$, the estimator converges in probability to the true parameter.

Point Estimate vs. Interval Estimate

- **Point Estimate:** A single number (e.g., sample mean $\bar{x} = 8.4\%$). Provides no indication of reliability.
- **Interval Estimate (Confidence Interval):** A range $[L, U]$ constructed from sample data that contains the true parameter with confidence $(1 - \alpha)$.

Confidence Intervals for the Mean (μ)

Case 1: Population σ Known (Z-Stat)

$$\bar{X} \pm Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Common Critical Values ($Z_{\alpha/2}$):

- 90% Confidence ($\alpha = 0.10$): $Z_{0.05} = \mathbf{1.645}$
- 95% Confidence ($\alpha = 0.05$): $Z_{0.025} = \mathbf{1.960}$
- 99% Confidence ($\alpha = 0.01$): $Z_{0.005} = \mathbf{2.576}$

Case 2: Population σ Unknown (t -Stat)

Replace σ with sample standard deviation s :

$$\bar{X} \pm t_{n-1, \alpha/2} \cdot \frac{s}{\sqrt{n}}$$

- Uses **Student's t -distribution** with $\nu = n - 1$ degrees of freedom.
- Has fatter tails than Z to account for extra estimation uncertainty in s .
- As $n \rightarrow \infty$, $t_{n-1} \rightarrow N(0, 1)$.

Worked Confidence Interval Example

Financial Application

An investment analyst evaluates the daily return of a hedge fund over $n = 36$ trading days. Sample mean return is $\bar{x} = 1.20\%$, and sample standard deviation is $s = 0.60\%$. **Task:** Construct a 95% confidence interval for the true mean daily return μ .

- Sample size: $n = 36 \implies \sqrt{n} = 6$.

- Standard Error of the mean:

$$SE = \frac{s}{\sqrt{n}} = \frac{0.60\%}{6} = 0.10\%$$

- Degrees of freedom: $df = n - 1 = 35$. For large n , $t_{35,0.025} \approx 2.03$ (or $Z = 1.96$).

- Margin of Error (ME):

$$ME = 2.03 \times 0.10\% = 0.203\%$$

- 95% Confidence Interval:

$$[1.20\% - 0.203\%, 1.20\% + 0.203\%] = [\mathbf{0.997\%}, \mathbf{1.403\%}]$$

Interpretation: We are 95% confident that the fund manager's true expected daily return lies between 0.997% and 1.403%.

The Architecture of Hypothesis Testing

Null (H_0) vs. Alternative (H_1) Hypothesis

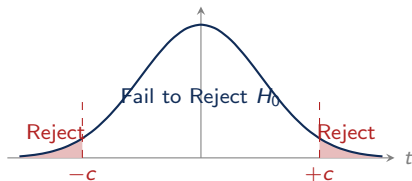
- **Null Hypothesis (H_0):** Benchmark state of no effect, zero alpha, or equality (presumed true until proven otherwise).
- **Alternative Hypothesis (H_1):** The research claim we seek empirical evidence to support.

Statistical Decision	H_0 True in Reality	H_0 False in Reality
Do Not Reject H_0	Correct Decision ($1 - \alpha$)	Type II Error (β)
Reject H_0	Type I Error (α) (Significance)	Correct Decision ($1 - \beta$) (Power)

The Two Classical Testing Errors

- **Type I Error (α):** False positive — concluding a manager generates alpha when it was just luck!
- **Type II Error (β):** False negative — failing to identify a truly skilled fund manager.

The Decision Rule: Critical Values vs. p -Values



Two Equivalent Decision Approaches:

1 Critical Value Approach:

Reject H_0 if $|t_{\text{stat}}| > t_{\text{crit}}$

2 p -Value Approach: p -value is the probability of observing a test statistic as extreme as the sample value, assuming H_0 is true.

Reject $H_0 \iff p\text{-value} < \alpha$

Financial Application: Testing for Alpha ($\alpha > 0$)

Portfolio Performance Evaluation

A mutual fund claims to generate superior abnormal monthly excess return (alpha). Over $n = 64$ months: sample mean excess return $\bar{x} = 0.50\%$, sample standard deviation $s = 1.60\%$. We test at significance level $\alpha = 0.05$:

$$H_0 : \mu = 0 \quad \text{vs.} \quad H_1 : \mu > 0 \quad (\text{One-Tailed Test})$$

Step 1: Compute Standard Error:

$$SE = \frac{s}{\sqrt{n}} = \frac{1.60\%}{\sqrt{64}} = \frac{1.60\%}{8} = \mathbf{0.20\%}$$

Step 2: Compute Test Statistic (t_{stat}):

$$t_{\text{stat}} = \frac{\bar{x} - \mu_0}{SE} = \frac{0.50\% - 0\%}{0.20\%} = \mathbf{+2.50}$$

Step 3: Decision Rule: For a one-tailed test at $\alpha = 0.05$, $Z_{0.05} = 1.645$.

Since $t_{\text{stat}} = 2.50 > 1.645 \implies \mathbf{\text{Reject } H_0!}$

Exam Practice 1: Event Definitions & Independence

Question 1 (Past Exam)

The general definition of an event is:

- A) An outcome of the experiment. B) A subset of the sample space.
- C) A set of outcomes with non-zero probability. D) The set of all outcomes.

Exam Practice 1: Event Definitions & Independence

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The general definition of an event is:

- A) An outcome of the experiment. B) A subset of the sample space.
- C) A set of outcomes with non-zero probability. D) The set of all outcomes.

Answer: B

An event is formally defined as any subset of the sample space ($A \subseteq S$).

Question 2 (Past Exam)

If $P(A) = 0.3$, $P(B) = 0.4$, and events A and B are independent, what is $P(A \cap B)$?

- A) 0.12 B) 0.70 C) 0.10 D) 0.20

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The general definition of an event is:

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If $P(A) = 0.3$, $P(B) = 0.4$, and events A and B are independent, what is $P(A \cap B)$?

- A) 0.12 B) 0.70 C) 0.10 D) 0.20

Answer: A

For independent events: $P(A \cap B) = P(A) \cdot P(B) = (0.3)(0.4) = 0.12$.

Exam Practice 2: Unions & Uniform Variables

Question 3 (Past Exam)

If $P(A) = 0.5$, $P(B) = 0.6$, and $P(A \cap B) = 0.2$, what is $P(A \cup B)$?

- A) 0.70 B) 0.90 C) 0.80 D) 1.10

Exam Practice 2: Unions & Uniform Variables

Question 3 (Past Exam)

If $P(A) = 0.5$, $P(B) = 0.6$, and $P(A \cap B) = 0.2$, what is $P(A \cup B)$?

- A) 0.70 B) 0.90 C) 0.80 D) 1.10

Answer: B

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.6 - 0.2 = 0.90.$$

Question 4 (Past Exam)

If U is a uniform random variable on the interval $(0, 1)$, what is the probability that U is between 0.2 and 0.8?

- A) 0.60 B) 0.20 C) 0.80 D) 0.40

Exam Practice 2: Unions & Uniform Variables

Question 3 (Past Exam)

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- A) 0.70 B) 0.90 C) 0.80 D) 1.10

Answer: B

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.6 - 0.2 = 0.90.$$

Question 4 (Past Exam)

If U is a uniform random variable on the interval $(0, 1)$, what is the probability that U is between 0.2 and 0.8?

- A) 0.60 B) 0.20 C) 0.80 D) 0.40

Answer: A

$$P(0.2 \leq U \leq 0.8) = \frac{0.8-0.2}{1-0} = 0.60.$$

Exam Practice 3: Expectations & CDF Properties

Question 5 (Past Exam)

What is the expected value of rolling a fair six-sided die?

- A) 3.5 B) 3.0 C) 4.0 D) 2.5

Exam Practice 3: Expectations & CDF Properties

Question 5 (Past Exam)

What is the expected value of rolling a fair six-sided die?

- A) 3.5 B) 3.0 C) 4.0 D) 2.5

Answer: A

$$\mathbb{E}[X] = \frac{1+2+3+4+5+6}{6} = \frac{21}{6} = 3.5.$$

Question 6 (Past Exam)

A cumulative distribution function (CDF) is always:

- A) Monotonically increasing (non-decreasing) B) Everywhere greater than 0
- C) Everywhere less than 1 D) A continuous function of X

Exam Practice 3: Expectations & CDF Properties

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Answer: A

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Question 6 (Past Exam)

A cumulative distribution function (CDF) is always:

- A) Monotonically increasing (non-decreasing) B) Everywhere greater than 0
- C) Everywhere less than 1 D) A continuous function of X

Answer: A

CDFs accumulate probability from left to right and are strictly monotonically non-decreasing.

Lecture 3 Summary & Key Formulas

Core Probability Formulas:

- Addition: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Conditional: $P(A | B) = \frac{P(A \cap B)}{P(B)}$
- Independence: $P(A \cap B) = P(A)P(B)$
- Bayes: $P(B_j | A) = \frac{P(A|B_j)P(B_j)}{\sum P(A|B_i)P(B_i)}$
- Variance: $\text{Var}(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$
- Linear combo: $\text{Var}(aX + b) = a^2\text{Var}(X)$
- Poisson: $\mathbb{E}[X] = \text{Var}(X) = \lambda$

Inference & Testing Formulas:

- Central Limit Thm: $\bar{X} \sim N(\mu, \sigma^2/n)$
- Standard Error: $SE = \frac{\sigma}{\sqrt{n}}$ or $\frac{s}{\sqrt{n}}$
- 95% CI (Z): $\bar{X} \pm 1.96 \frac{\sigma}{\sqrt{n}}$
- 95% CI (t): $\bar{X} \pm t_{n-1, 0.025} \frac{s}{\sqrt{n}}$
- t-statistic: $t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$
- Decision rule: Reject $H_0 \iff p\text{-value} < \alpha$

Course Induction Complete! Best of luck in the MSc in Financial Management!