Understanding implied volatilities

- The implied volatility of an option is the volatility that, when substituted into Black-Scholes-Merton formula, gives the price of the option in the market.

$$c = S_0 e^{-qT} N(d_1) - K e^{-rT} N(d_2)$$

- $d_1 = \frac{\ln(S_0/K) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}$
- $d_2 = d_1 - \sigma\sqrt{T}$

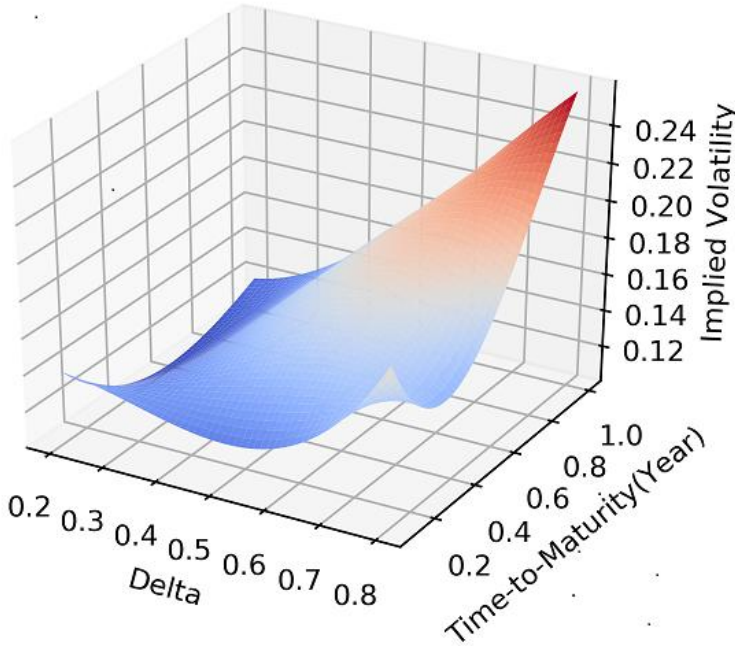
Meaning of the variables

Symbol	Meaning
S_0	Current underlying price
K	Strike price
r	Risk-free interest rate
q	Continuous dividend yield
σ	Volatility
T	Time to maturity
$N(\cdot)$	Standard normal CDF

- If Black-Scholes-Merton was used for pricing, all options on an asset would have the same implied volatility
- In fact, there is quite a variation in implied volatilities
- Nevertheless, implied volatilities are used to communicate prices and it is therefore important for traders to monitor implied volatilities
- The volatility surface shows implied volatilities as a function of
    - Moneyness or how likely the option is to be exercised (Traders measure this by the delta, the sensitivity of the option to the underlying asset's price)
    - Time to maturity, T

Volatility Surfaces for S&P 500

Jan 31, 2019



$\Delta_{call} = \frac{\partial C}{\partial S} = e^{-qT} N(d_1)$

X-axis: Delta

Delta measures the option's sensitivity to the underlying asset price.

Delta	Option type / moneyness
Low delta (~0.2–0.3)	Deep OTM option
Around 0.5	ATM option
High delta (~0.8–1.0)	Deep ITM option

Y-axis: Time-to-Maturity

This is the remaining life of the option in years.

- Small values → short-dated options
- Large values → long-dated options

Z-axis: Implied Volatility

- This is the market-implied volatility extracted from option prices.
- Higher values mean the market expects larger future uncertainty.

Main patterns of the graph

1. Implied volatility (IV) rises with maturity

As maturity increases, IV generally increases especially for high-delta options This is called the **term structure of volatility**. **Interpretation:** Markets expect more uncertainty over longer horizons.

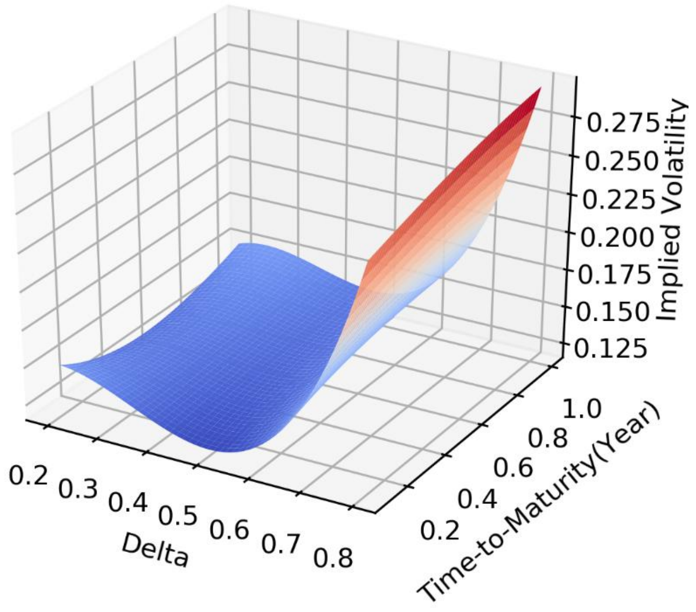
2. Volatility depends on delta

The surface is curved along the delta dimension. This reflects the **volatility skew**. Low-delta options have lower IV; high-delta options have higher IV . So the surface exhibits an upward skew.

3. Nonlinearity / curvature

- The surface is not flat or planar. There is visible **curvature**: around medium maturities across delta values. The curvature means the relationship between IV and delta is nonlinear.

Volatility Surface for S&P 500



June 25, 2019

Understanding Volatility Surface Movements

- To understand volatility surface movements we used data on S&P 500 call options to construct a neural network
- Input layer:
  - Daily asset price return
  - Moneyness (measured by delta)
  - Time to maturity
- Output layer:
  - Change in implied volatility

Understanding how the volatility surface moves is important for a number of reasons:

- It can help a trader hedge exposures more precisely
- It can help a quant determine a more sophisticated model reflecting how options are priced by the market.
- It can help a trader adjust implied volatilities in a market where asset prices are changing

Details

- 3 hidden layers
- 20 neurons per layer
- Observations from 2014-2019
- Randomly sampled 100 options per day
- 125,700 options in total
- 60% for training set
- 20% for validation set
- 20% for test set
- Z-score scaling

Sample of raw data

Date	*Return (%) on S&P 500*	*Maturity (years)*	*Delta*	*Change in implied volatility (bps)*
Jan 28, 2015	0.95	0.608	0.198	−31.1
Sept 8, 2017	1.08	0.080	0.752	−54.9
Jan 24, 2018	0.06	0.956	0.580	−1.6
Jun 24, 2019	−0.95	2.896	0.828	40.1

After scaling:

Date	*Return (%) on S&P 500*	*Maturity (years)*	*Delta*	*Change in implied volatility (bps)*
Jan 28, 2015	1.091	−0.102	−1.539	−31.1
Sept 8, 2017	1.247	−0.695	0.445	−54.9
Jan 24, 2018	0.027	0.289	−0.171	−1.6
Jun 24, 2019	−1.176	2.468	0.716	40.1

In constructing a machine learning model, it is always useful to have a simpler model as a benchmark. For this application, we can use the following model:

Expected change in implied volatility equals $R\dfrac{a + b\delta + c\delta^2}{\sqrt{T}}$

where R is the return on the asset (= change in price divided by initial price), T is the option's time to maturity, δ is the delta measure of the option's moneyness, and a, b, and c are constants (Hull and White, 2017)

- Linear Regression

Input

Variable	Formula	Captures
x_1	$R/\sqrt{T}$	Overall market shock scaled by maturity
x_2	$(R/\sqrt{T})\Delta$	Dependence on moneyness/delta
x_3	$(R/\sqrt{T})\Delta^2$	Nonlinear smile/skew curvature effects

Output

Change in implied volatility

Results

- Test set gave a modest 14% improvement over a simple analytic model proposed by Hull and White in 2017
- The behavior of the volatility surface is different in high and low volatility environments

Task

Repeat the analysis adding the value of the VIX index (observed on the first of the two days that the implied volatility and S&P 500 are observed) as a feature.