# Financial Derivatives
# Chapters #8 #9 #10
# Options: Properties & Trading Strategies

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# 1 Preview

- Brief revision of option contracts
- Option types and underlying securities
- Parameters that determine option prices
- Introduction to the Greeks
- Rational option pricing bounds
- Empirical evidence
- Trading strategies involving options
  - Combining options and underlying assets
  - Same–type options combinations
  - Different–type options combinations

# 2 Option contracts

- A contract that gives its holder the right (without the corresponding obligation) to buy or sell an asset by a certain date for a certain price.
- Options are traded both on exchanges and OTC.
- Exchange–traded options are actively traded on:
  - stocks (CBOE, PHLX, PACIFEX)
  - stock indices (CBOE)
  - foreign currencies (PHLX)
  - futures contracts (at futures exchanges)

- OTC options can be traded on any asset an investor might wish
  - Bonds
  - Swaps
  - Commodities
  - Interest rate
- The right to buy the asset is a *call* option; the right to sell the asset is a *put* option.
- Contrary to forward and futures (where two parties commit to take some future action), options give a right with no obligation; an option holder does not have to exercise this right. This is why it costs nothing to enter a forward/futures contract, whereas the purchase of an option requires an up–front payment. This payment is known as *option price* or *option premium*.

- The price specified in the contract is known as the *exercise* or *strike price*; those are chosen by the exchange and are usually spaced \$2.50, \$5, or \$10 apart depending on the underlying asset's price.
- The date in the contract is known as the *expiration date* or *maturity*. Expiration dates are usually on a January, February, or March cycle. The last day on which options are traded is the third Friday of the expiration month. Delivery of exercise (if any) happens on the Saturday immediately following the third Friday of the expiration month.

# 3 Option types

*European option contract*

- A European–style option is an agreement made at time $t = 0$ that gives the option holder the right (without the corresponding obligation) to buy (or sell) an asset $S$ (the underlying) at a future prespecified date $T > 0$ (maturity or expiry) at a fixed price $K > 0$ (strike or exercise price)
- European call ($c$): option to buy European put ($p$): option to sell

*American option contract*

- An American–style option is an agreement made at time $t = 0$ that gives the option holder the right (without the corresponding obligation) to buy (or sell) the underlying asset $S$ at any time up to (and including) the expiry date $T > 0$ (maturity) at a fixed exercise price $K > 0$
- American call ($C$): option to buy American put ($P$): option to sell

*Bermudan option*

- A contract that gives the option holder the right to buy (or sell) the underlying asset $S$ at one of many specific times up to and including the expiry date $T$.

*Chooser option*

- A contract that gives the option holder the right to choose between buying or selling the underlying asset $S$ (i.e. a call or a put) in the future

*Lookback option*

- A contract whose payoff depends on the maximum or minimum of the underlying asset $S$ over the option life $[0, T]$

*Compound option*

- A contract that gives the option holder the right to buy or sell another option contract (i.e. an option written on an option)

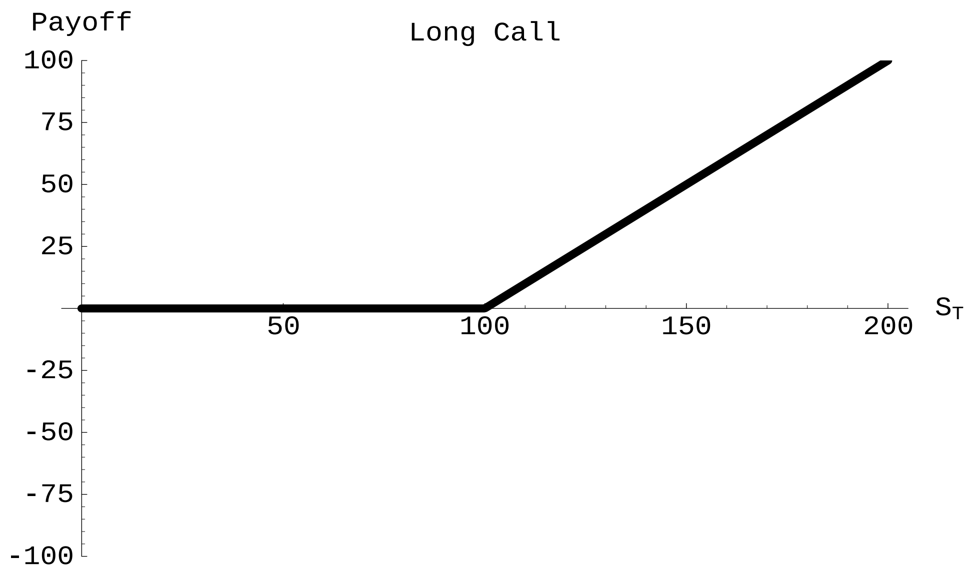
*Asian, Parisian, Russian, Passport*—the list goes on expanding

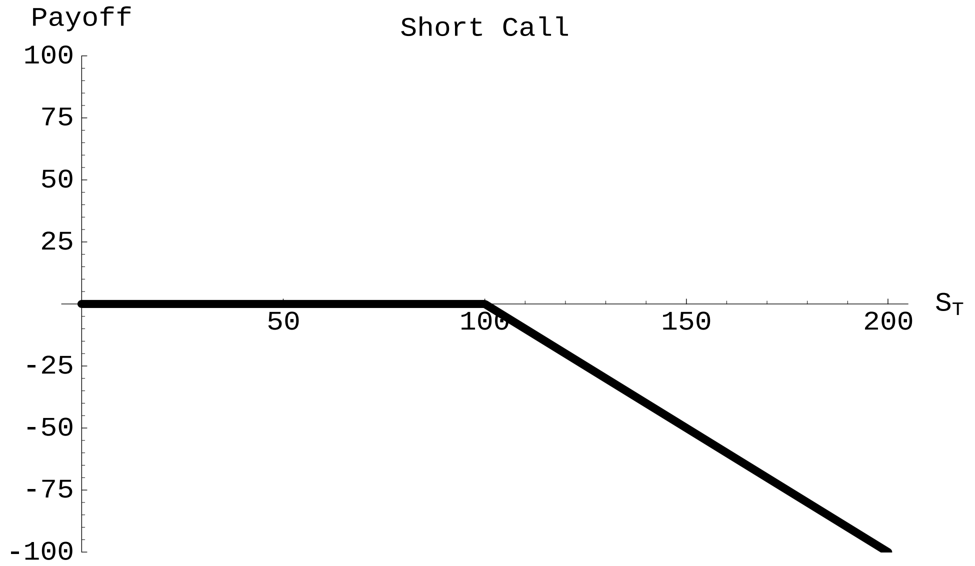
- Formalisation: You can be *long* (i.e. buy) a (call or put) option or *short* (i.e. sell) a (call or put) option. Being short an option is also referred to as *writing* an option. At maturity $T$, the option holder receives:*

| | Option | |
|---|---|---|
| | call | put |
| long | $\left(\widetilde{S}_T - K\right)^+$ | $\left(K - \widetilde{S}_T\right)^+$ |
| short | $-\left(\widetilde{S}_T - K\right)^+$ | $-\left(K - \widetilde{S}_T\right)^+$ |

where $\widetilde{S}_T$ is the price of the underlying asset at the maturity of the option and $K$ the exercise price.

*Denote $(x)^+ \equiv \max[x, 0], \forall x \in \mathbb{R}$


Payoff
Long Call
100
75
50
25
-25
-50
-75
-100
50
100
150
200
S_T


Payoff
Short Call
100
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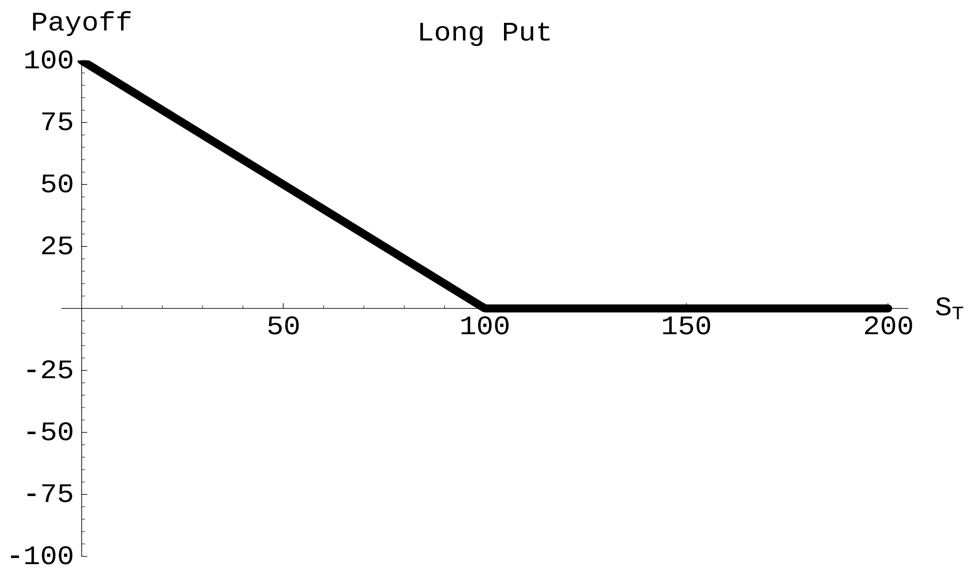
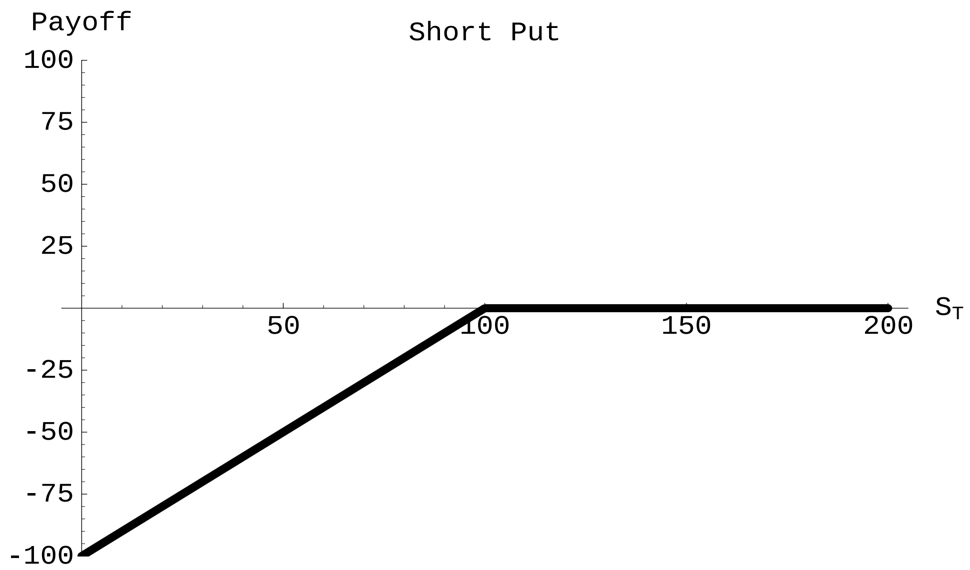
200
S_T



Payoff
Long Put
100
75
50
25
-25
-50
-75
-100
50
100
150
200
$S_T$

Payoff
Short Put
100
75
50
25
-25
-50
-75
-100
50
100
150
200
$S_T$

# 4 Terminology

- For any given asset at any given time, many different option contracts may be trading. All options of the same type are referred to as an *option class*
- An *option series* consists of all the options of a given class with the same expiration date and strike price
- Options are referred to as *in–the–money* (ITM), *at–the–money* (ATM) or *out–of–the–money* (OTM)
  - An ITM option would give the holder a positive cash flow if it were exercised immediately (call: $S > K$, put: $S < K$)
  - An ATM option would lead to a zero cash flow if it were exercised immediately (call: $S = K$, put: $S = K$)
  - An OTM option would lead to a negative cash flow if it were exercised immediately (call: $S < K$, put: $S > K$)

- The *intrinsic value* of an option is defined as the maximum of zero and the value the option would have if it were exercised immediately (call: $(S-K)^+$, put: $(K-S)^+$)
- An ITM American option must be worth at least as much as its intrinsic value
- Often it is optimal for the holder of an ITM American option to wait rather than exercise immediately. The option is then said to have *time value*
- Suppose that the price of a call with 2 months to maturity is \$3, when the stock price is \$30 and the strike price \$28. The intrinsic value of the option is $30-28=\$2$, and the time value is $3-2=\$1$
- Generally

$$\text{Option Value} = \text{Intrinsic Value} + \text{Time Value}$$

  and the time value of an option is zero when (a) the option has reached maturity or (b) it is optimal to exercise immediately

# 5 Dividends and stock splits

- Suppose you own $N$ options with a strike price of $K$:
  - No adjustments are made to the option terms for cash dividends
  - When there is an $n$-for-$m$ stock split,
    - the strike price is reduced to $mK/n$
    - The no. of options is increased to $nN/m$
  - Stock dividends are handled in a manner similar to stock splits
  - Example: Consider a call option to buy 100 shares for $20/share How should terms be adjusted (a) for a 2-for-1 stock split? (b) for a 5% stock dividend?

# 6 Market makers and margins

- Most exchanges use market makers to facilitate options trading
- A market maker quotes both bid and ask prices when requested
- The market maker does not know whether the individual requesting the quotes wants to buy or sell
- Margins are required when options are sold
- When a naked option is written the margin is the greater of:
  - A total of 100% of the proceeds of the sale plus 20% of the underlying share price less the amount (if any) by which the option is out of the money
  - A total of 100% of the proceeds of the sale plus 10% of the underlying share price
- For other trading strategies there are special rules

# 7 Warrants

- Warrants are options that are issued by a corporation or a financial institution
- The number of warrants outstanding is determined by the size of the original issue and changes only when they are exercised or when they expire
- The issuer settles up with the holder when a warrant is exercised
- When call warrants are issued by a corporation on its own stock, exercise will usually lead to new treasury stock being issued

# 8 Executive stock options

- Executive stock options are a form of remuneration issued by a company to its executives
- They are usually at the money when issued
- When options are exercised the company issues more stock and sells it to the option holder for the strike price
- They become vested after a period of time (usually 1 to 4 years)
- They cannot be sold
- They often last for as long as 10 or 15 years
- Accounting standards now require the expensing of executive stock options

# 9 Parameters that determine option prices

- There are six parameters that determine the prices of options
  - the current price of the underlying asset, $S$ or $S_0$
  - the exercise price of the option, $K$
  - the time left until the option matures, $T$
  - the volatility of the underlying asset price changes, $\sigma$
  - the risk–free rate of return, $r$
  - the payments (e.g. dividends) that the underlying asset makes over the life of the option, $\delta$

| Parameter | European | | American | |
|---|---|---|---|---|
| | Call, $c$ | Put, $p$ | Call, $C$ | Put, $P$ |
| Underlying, $S$ | $+$ | $-$ | $+$ | $-$ |
| Strike price, $K$ | $-$ | $+$ | $-$ | $+$ |
| Maturity, $T$ | $+^{\dagger}$ | $+^{\dagger}$ | $+$ | $+$ |
| Volatility, $\sigma$ | $+$ | $+$ | $+$ | $+$ |
| Riskless rate, $r$ | $+$ | $-$ | $+$ | $-$ |
| Dividends, $\delta$ | $-$ | $+$ | $-$ | $+$ |

$^{\dagger}$Not always

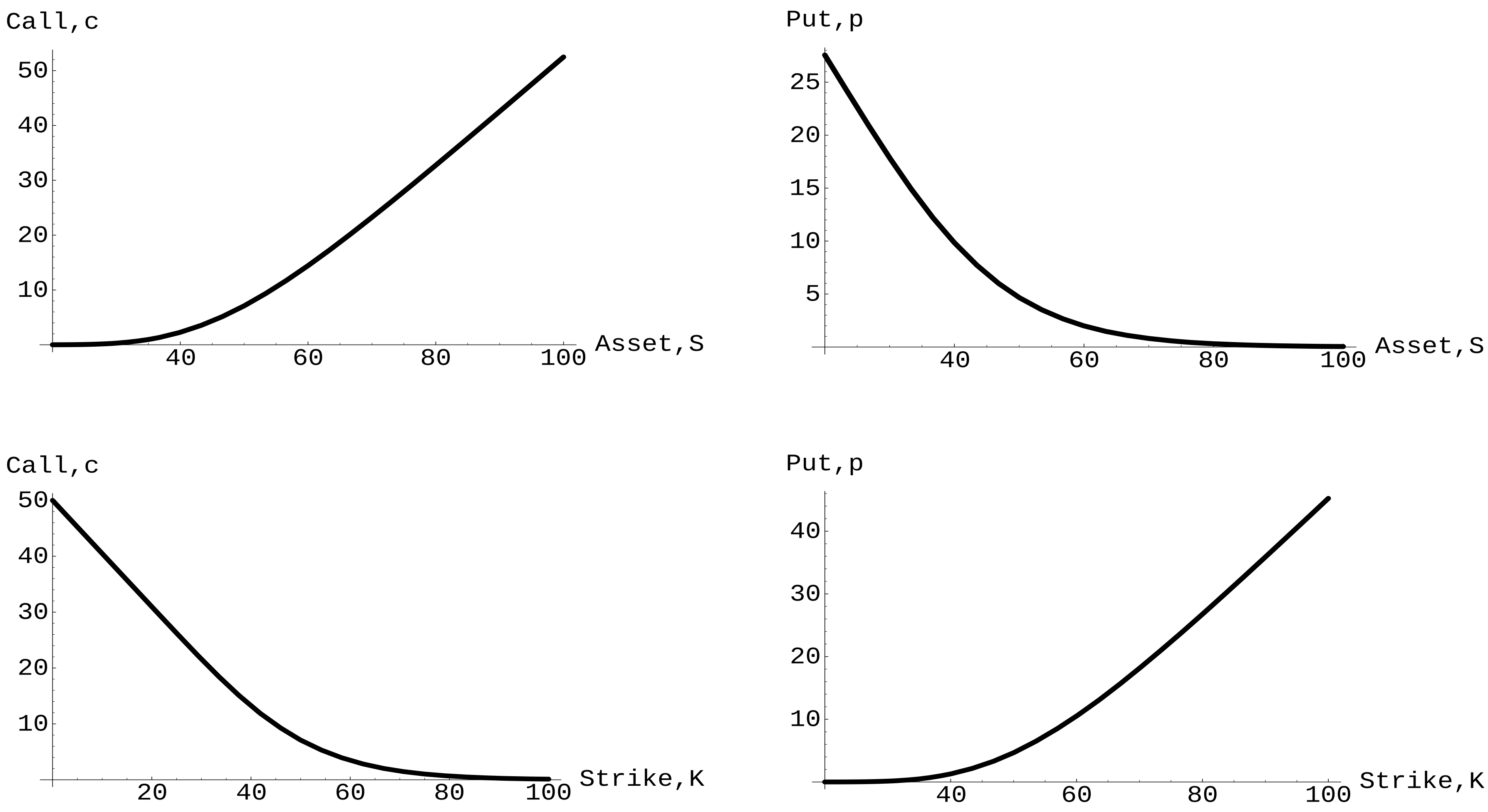
- In mathematical terms, if we write calls and puts as six–variable functions, $c(S, K, T, \sigma, r, \delta)$ and $p(S, K, T, \sigma, r, \delta)$, then the previous table implies

$$\frac{\partial c}{\partial S} > 0 \quad \frac{\partial c}{\partial K} < 0 \quad \frac{\partial c}{\partial T} > 0 \quad \frac{\partial c}{\partial \sigma} > 0 \quad \frac{\partial c}{\partial r} > 0 \quad \frac{\partial c}{\partial \delta} < 0$$

$$\Delta \qquad\qquad\qquad -\Theta \qquad \mathcal{V} \qquad \rho$$

$$\frac{\partial p}{\partial S} < 0 \quad \frac{\partial p}{\partial K} > 0 \quad \frac{\partial p}{\partial T} > 0 \quad \frac{\partial p}{\partial \sigma} > 0 \quad \frac{\partial p}{\partial r} < 0 \quad \frac{\partial p}{\partial \delta} > 0$$

$\Delta$ (delta), $\Theta$ (theta), $\mathcal{V}$ (vega) and $\rho$ (rho) are important option *hedging* statistics, usually referred to as "*the Greeks*" of an option.


Call,c
50
40
30
20
10
40
60
80
100
Asset,S
Put,p
25
20
15
10
5
40
60
80
100
Asset,S
Call,c
50
40
30
20
10
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60
80
100
Strike,K
Put,p
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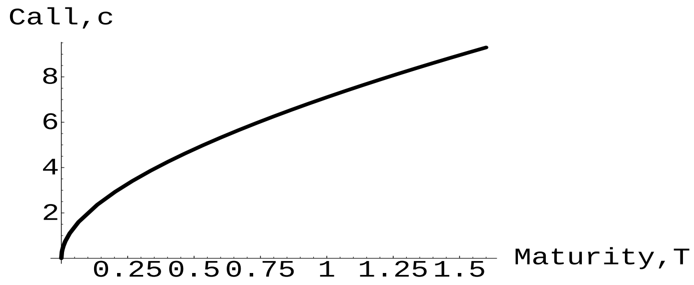
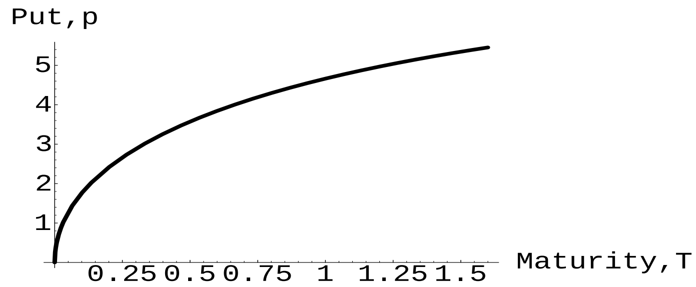
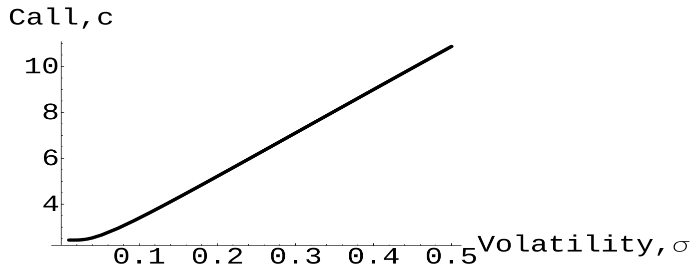
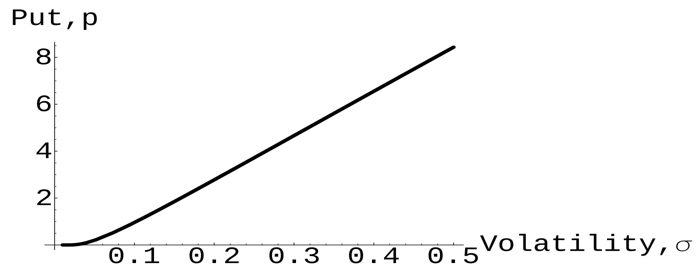
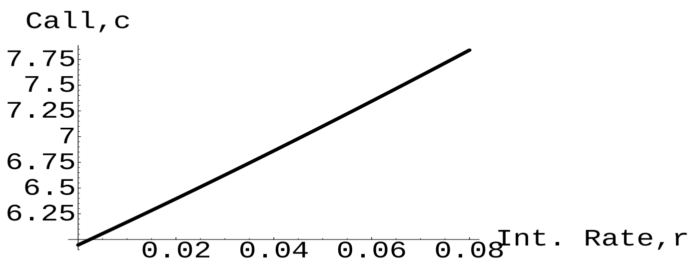
60
80
100
Strike,K

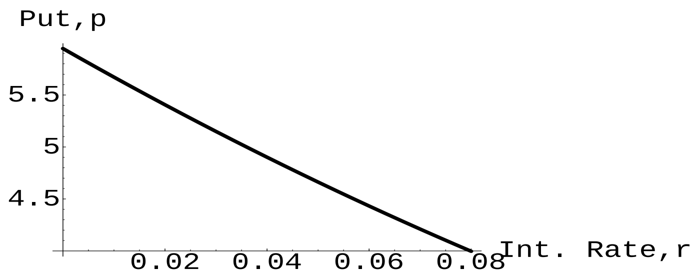

Call,c
8
6
4
2
0.25 0.5 0.75 1 1.25 1.5
Maturity,T


Put,p
5
4
3
2
1
0.25 0.5 0.75 1 1.25 1.5
Maturity,T


Call,c
10
8
6
4
0.1 0.2 0.3 0.4 0.5
Volatility,σ


Put,p
8
6
4
2
0.1 0.2 0.3 0.4 0.5
Volatility,σ


Call,c
7.75
7.5
7.25
7
6.75
6.5
6.25
0.02 0.04 0.06 0.08
Int. Rate,r


Put,p
5.5
5
4.5
0.02 0.04 0.06 0.08
Int. Rate,r

# 10 Bounds for option prices

- In possibly the most influential paper on option (warrant) pricing, Merton [9] derived bounds for option prices (premiums).
- If option prices were outside these bounds, then arbitrage opportunities would exist.
- The remarkable feature in Merton [9] is that all bounds derived *do not depend on any assumption* regarding the factors affecting option prices or the stochastic process that the underlying asset price follows.

1. *[Non–negativity]* Option prices (puts or calls, of any type and on any underlying) can never be negative.

$$c \geq 0 \qquad C \geq 0 \qquad p \geq 0 \qquad P \geq 0$$

2. *[Call upper bound]* A call option (European or American) can not be worth more than the underlying asset

$$c \leq S_0 \qquad C \leq S_0 \tag{1}$$

3. *[Put upper bound]* A put option (European or American) can never be worth more than the exercise price

$$p \leq K \qquad P \leq K$$

For European puts more specifically

$$p \leq K e^{-rT} \tag{2}$$

4. *[Call lower bound]* For a European call written on an asset that pays no flows over $[0, T]$, the following inequality holds

$$c \geq \left(S_0 - Ke^{-rT}\right)^+ \tag{3}$$

5. *[Put lower bound]* For a European put written on an asset that pays no flows over $[0, T]$, the following inequality holds

$$p \geq \left(Ke^{-rT} - S_0\right)^+ \tag{4}$$

6. *[Put–Call parity]* The price of a European call option can be deduced from the price of the corresponding put, and vice versa.

$$c - p = S_0 - Ke^{-rT} \tag{5}$$

Put–Call parity does not hold exactly for American options; the following is a similar result that does hold

$$S_0 - K \leq C - P \leq S_0 - Ke^{-rT} \tag{6}$$

7. *[American call & early exercise]* It is never optimal to early–exercise an American call written on a no–flow–paying underlying asset. In these instances

$$c = C$$

$$C > S_0 - K$$

8. *[American put & early exercise]* It can be optimal to early–exercise an American put written on a no–flow–paying underlying asset. Thus

$$P \geq K - S_0$$

Proof. [of equation (3)][‡]
Portfolio $A$ is always worth at least as much, and is sometimes worth more than, portfolio $B$ at time $T$, in any state of the world. For no arbitrage opportunities to exist, it must be worth at least as much today ($t = 0$), i.e.

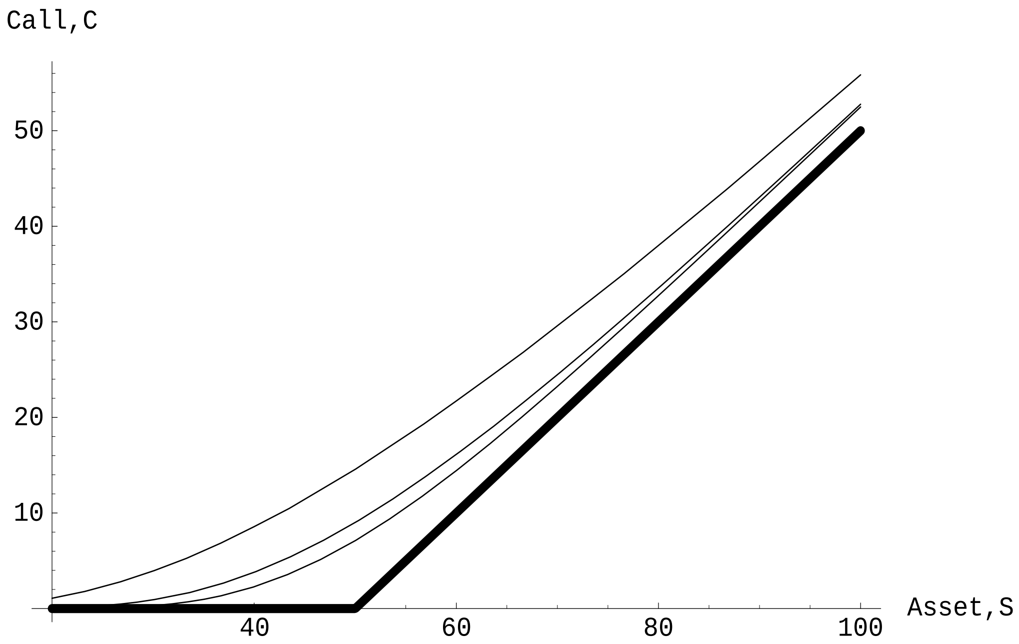
$$c + Ke^{-rT} \geq S_0$$

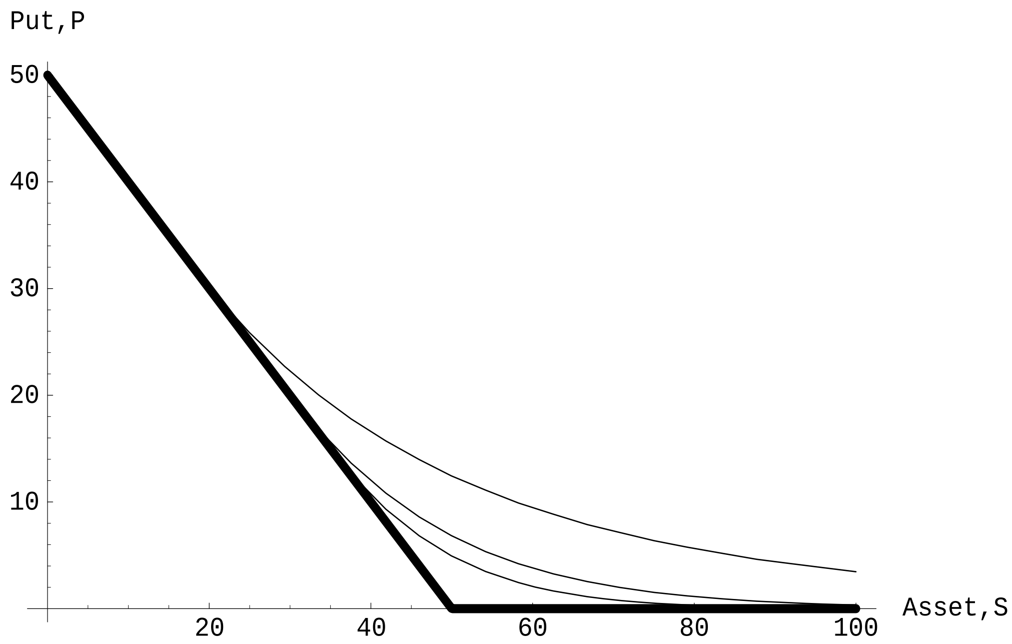
$$\Updownarrow$$

$$c \geq S_0 - Ke^{-rT}$$

■

[‡]Homework: Use portfolios $C$ and $D$ to prove equation (4) and $A$ and $C$ to prove (5)


Call,C
50
40
30
20
10
40
60
80
100
Asset,S


Put,P
50
40
30
20
10
20
40
60
80
100
Asset,S

| Portfolio | Now $t = 0$ | Action | At maturity $T$ |
|---|---|---|---|
| A: One call, $Ke^{-rT}$ Cash | $c + Ke^{-rT}$ | Bank | $\max(S_T, K) = \begin{cases} S_T{}^{\S} & \text{if } S_T > K \\ K & \text{if } S_T \le K \end{cases}$ |
| B: One share | $S_0$ | Hold | $S_T$ |
| C: One put, One share | $p + S_0$ | Hold | $\max(S_T, K) = \begin{cases} S_T & \text{if } S_T \ge K \\ K^{\P} & \text{if } S_T < K \end{cases}$ |
| D: $Ke^{-rT}$ Cash | $Ke^{-rT}$ | Bank | $K$ |

$^{\S}(S_T - K)^+ + \left(Ke^{-rT}\right)e^{rT} = S_T - K + K = S_T$

$^{\P}(K - S_T)^+ + S_T = K - S_T + S_T = K$

# 11 Empirical research

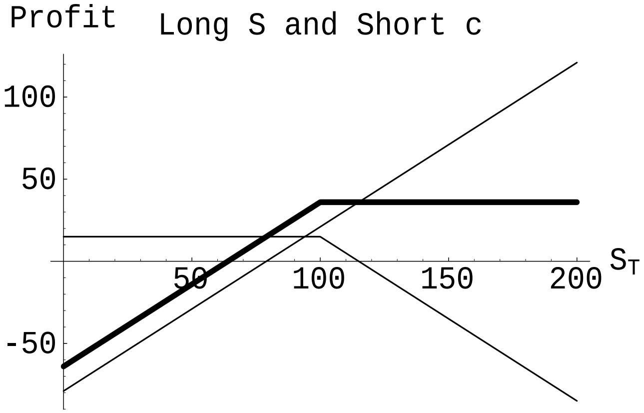
- Several researchers have empirically investigated the results we have just considered. The most influential studies have been those of Bhattacharya [1], Galai [2], Gould and Galai [3], Klemkosky and Resnick [7], [8] and Stoll [10]
- Some of the complications that arise in those studies are:
  - "Simultaneity" of option and asset prices
  - Can violations of the price bounds be economically exploited?
  - Do violations "survive" transaction cost inclusion?
  - How are dividends treated?

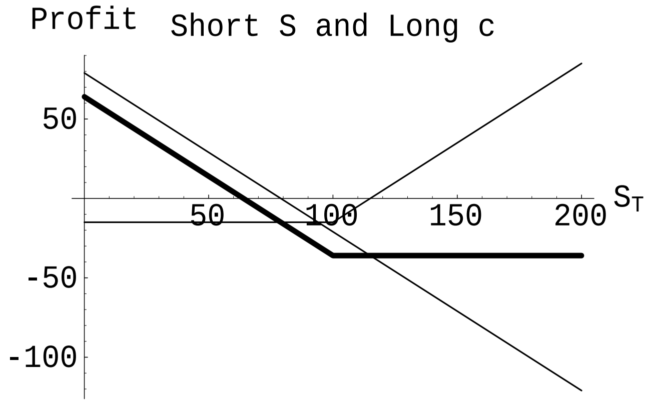
# 12 Advanced trading strategies

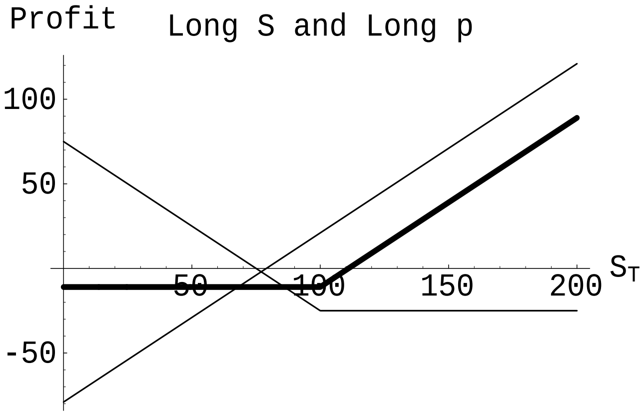
## 12.1 Portfolios of an option and its underlying

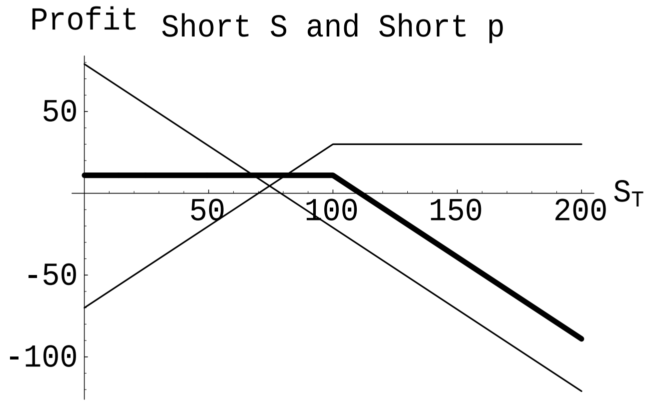
This strategies involve positions in a call or put option and the underlying asset. The profit patterns of these strategies can be inferred from put–call parity (equation (5)).

| Strategy Name | Position | Graph Panel | Approximate Payoff |
|---|---|---|---|
| Covered Call | Long | Upper Right | Long $p$ |
| | Short | Upper Left | Short $p$ |
| Protective Put | Long | Lower Left | Long $c$ |
| | Short | Lower Right | Short $c$ |


Profit
Long S and Short c
100
50
-50
50
100
150
200
S_T


Profit
Short S and Long c
50
-50
-100
50
100
150
200
S_T


Profit
Long S and Long p
100
50
-50
50
100
150
200
S_T


Profit
Short S and Short p
50
-50
-100
50
100
150
200
S_T

## 12.2 Portfolios of options of the same type: Spreads

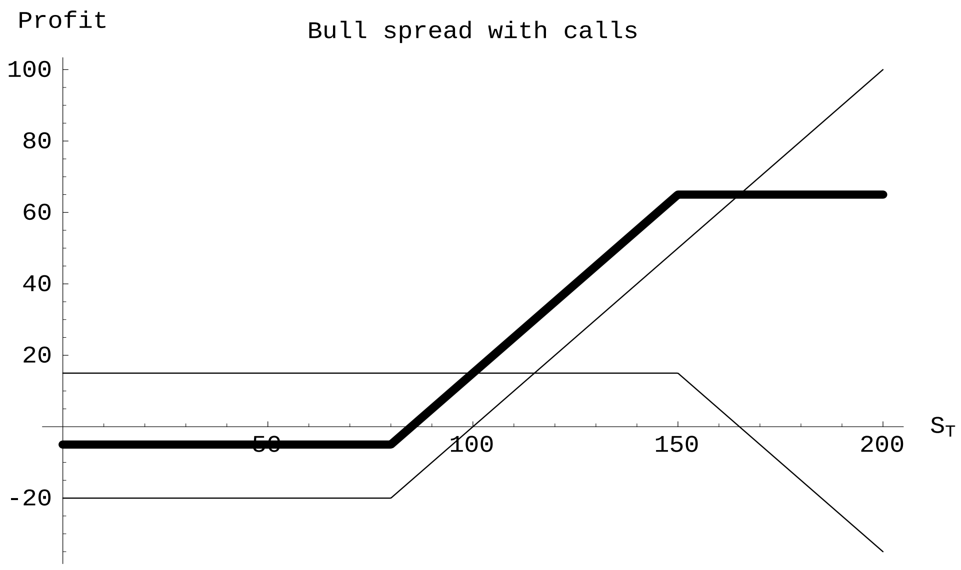
- *Bull spread*: This trading strategy is appropriate when you expect the underlying asset (e.g. stock, index) to increase. It involves positions in two options of the same type, but with different exercise prices. It can be created either by using calls or puts.
- *Bear spread*: This trading strategy is appropriate when you expect the underlying asset (e.g. stock, index) to decrease. It involves positions in two options of the same type, but with different exercise prices. It can be created either by using calls or puts.
- *Butterfly spread*: This trading strategy is appropriate when you have expectations about the underlying asset volatility. It involves positions in three options of the same type, but with different exercise prices. It can be created either by using calls or puts.

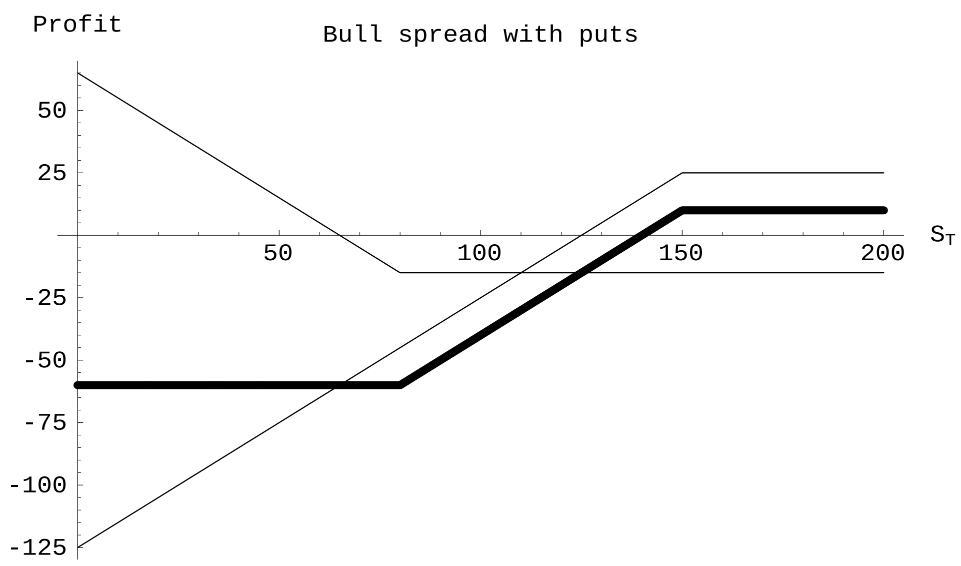
| Bull Spread (using calls) $K_l < K_h$ | Underlying price at maturity $T$ if $S_T \geq K_h$ | if $K_l < S_T < K_h$ | if $S_T \leq K_l$ |
|---|---|---|---|
| Buy call at $K_l$ (pay $c_l$) | $S_T - K_l$ | $S_T - K_l$ | 0 |
| Sell call at $K_h$ (receive $c_h$) | $K_h - S_T$ | 0 | 0 |
| Initial Investment $-c_l + c_h < 0$[‖] | | | |
| Total Payoff | $K_h - K_l$ | $S_T - K_l$ | 0 |

[‖]Since $K_h > K_l \Rightarrow c_l > c_h$. The negative sign implies a cash outflow.

| Bull Spread (using puts) $K_l < K_h$ | Underlying price at maturity $T$ if $S_T \geq K_h$ | if $K_l < S_T < K_h$ | if $S_T \leq K_l$ |
|---|---|---|---|
| Buy put at $K_l$ (pay $p_l$) | 0 | 0 | $K_l - S_T$ |
| Sell put at $K_h$ (receive $p_h$) | 0 | $S_T - K_h$ | $S_T - K_h$ |
| Initial Investment $-p_l + p_h > 0^{**}$ | | | |
| Total Payoff | 0 | $S_T - K_h$ | $K_l - K_h$ |

**Since $K_h > K_l \Rightarrow p_l < p_h$. The positive sign implies a cash inflow.

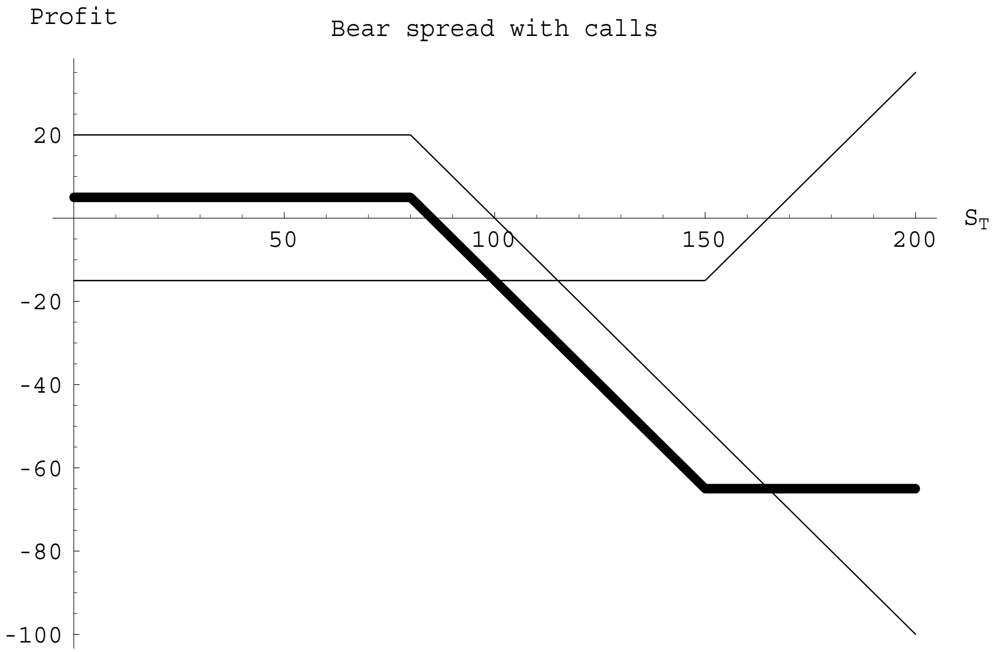

Profit
Bull spread with calls
100
80
60
40
20
-20
50
100
150
200
S_T


Profit
Bull spread with puts
50
25
-25
-50
-75
-100
-125
50
100
150
200
S_T

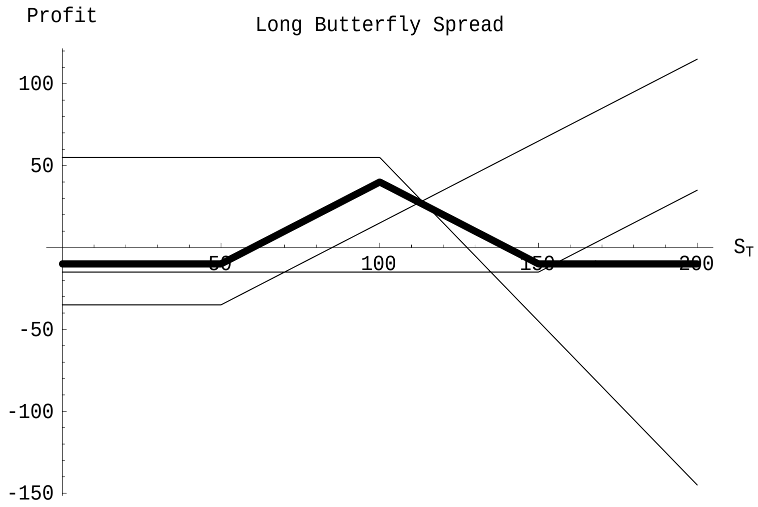
| Bear Spread (using calls)[††] $K_l < K_h$ | Underlying price at maturity $T$ if $S_T \geq K_h$ | if $K_l < S_T < K_h$ | if $S_T \leq K_l$ |
|---|---|---|---|
| Sell call at $K_l$ (receive $c_l$) | $S_T - K_l$ | $S_T - K_l$ | 0 |
| Buy call at $K_h$ (pay $c_h$) | $K_h - S_T$ | 0 | 0 |
| Initial Investment $c_l - c_h > 0$[‡‡] | | | |
| Total Payoff | $K_h - K_l$ | $S_T - K_l$ | 0 |

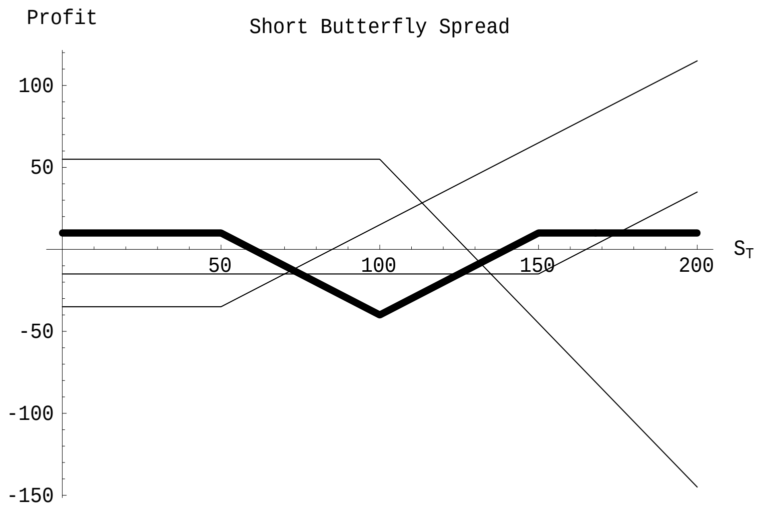
[††]Homework: create a bear spread using put options.

[‡‡]Since $K_h > K_l \Rightarrow c_l > c_h$. The positive sign implies a cash inflow.


Profit
Bear spread with calls
20
-20
-40
-60
-80
-100
50
100
150
200
S_T

| Butterfly Spread (using calls) | Underlying price at maturity $T$ | | | |
|---|---|---|---|---|
| $K_l < K_m < K_h$ | if $S_T \geq K_h$ | if $S_T \in (K_m, K_h)$ | if $S_T \in (K_l, K_m)$ | if $S_T \leq K_l$ |
| Buy call at $K_l$ (pay $c_l$) | $S_T - K_l$ | $S_T - K_l$ | $S_T - K_l$ | 0 |
| Sell 2 calls at $K_m$ (get $-2c_m$) | $2(K_m - S_T)$ | $2(K_m - S_T)$ | 0 | 0 |
| Buy call at $K_h$ (pay $c_h$) | $S_T - K_h$ | 0 | 0 | 0 |
| Initial investment $-c_l - c_h + 2c_m < 0$ | | | | |
| Total Payoff | $2K_m - K_l - K_h$ | $2K_m - K_l - S_T$ | $S_T - K_l$ | 0 |

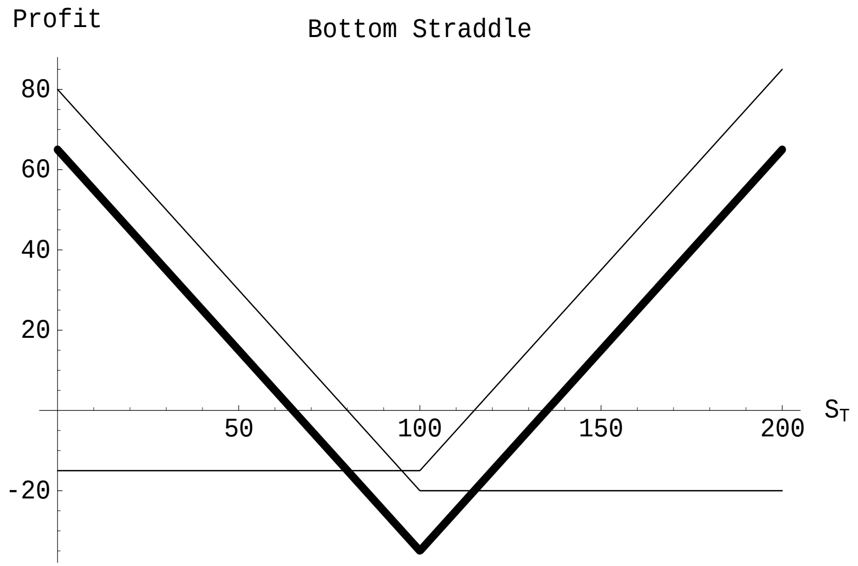

Profit
Long Butterfly Spread
100
50
-50
-100
-150
50
100
150
200
S_T

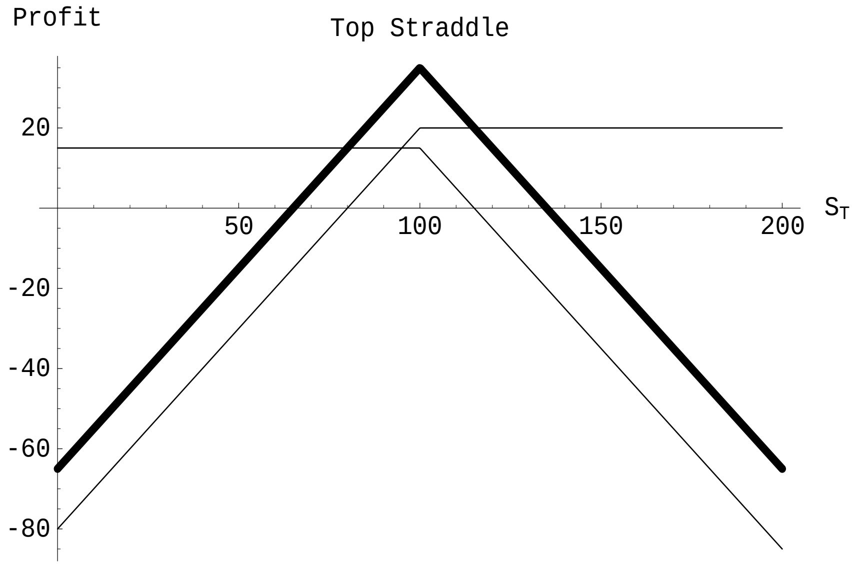

Profit
Short Butterfly Spread
100
50
-50
-100
-150
50
100
150
200
S_T

## 12.3 Combinations of option types

*Straddle*: Buy (bottom straddle) or sell (top straddle) a call and a put with the *same strike price and exercise date*. Appropriate strategy when high (bottom straddle) or low (top straddle) volatility is expected.

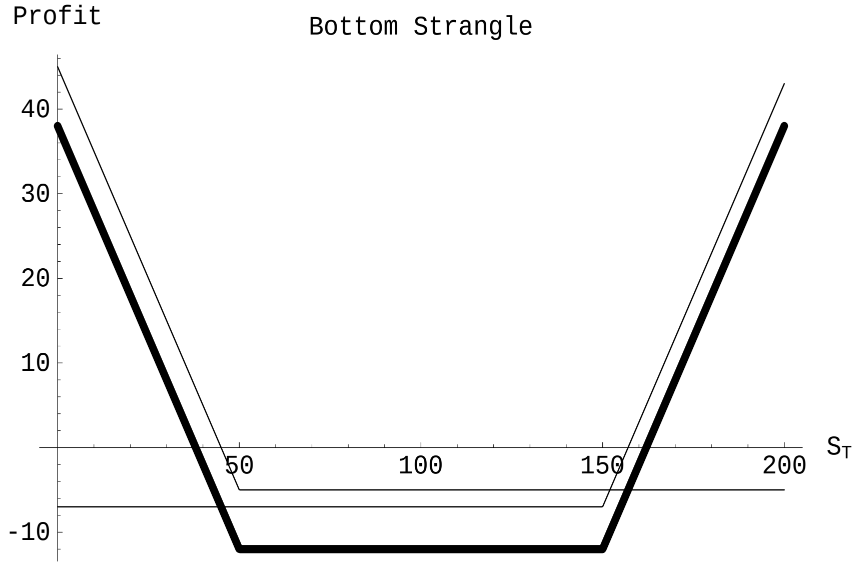
| Bottom Straddle | Underlying price at maturity $T$<br>if $S_T > K$ | <br>if $S_T \leq K$ |
|---|---|---|
| Buy call at $K$ (pay $c$) | $S_T - K$ | 0 |
| Buy put at $K$ (pay $p$) | 0 | $K - S_T$ |
| Total Investment: $-c - p$ | | |
| Total Payoff | $S_T - K$ | $K - S_T$ |

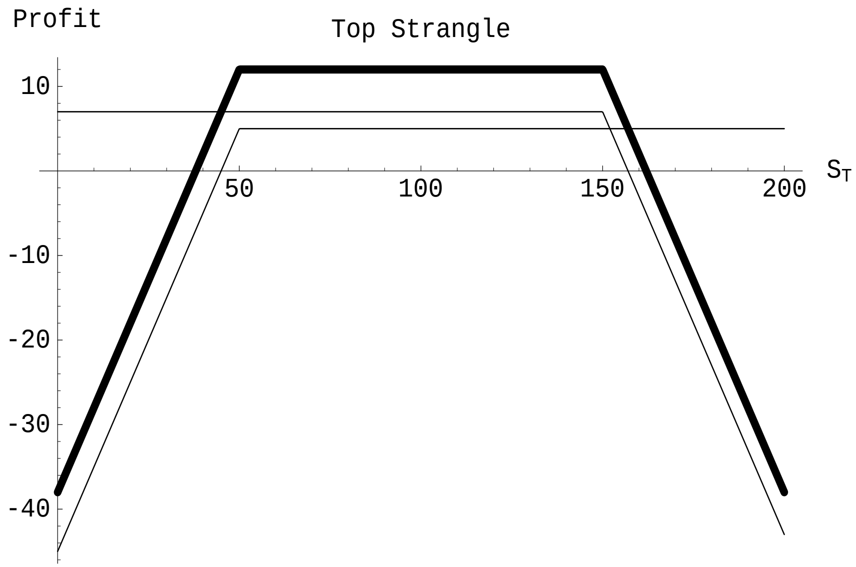

Profit
Bottom Straddle
80
60
40
20
-20
50
100
150
200
$S_T$


Profit
Top Straddle
20
-20
-40
-60
-80
50
100
150
200
$S_T$

*Strangle*: It involves buying (bottom vertical combination) or selling (top vertical combination) a call and a put with the *same expiry date but with different exercise prices*. This is also a volatility–based strategy. Compared to a straddle, it requires higher volatility to be profitable, but has less downside risk.

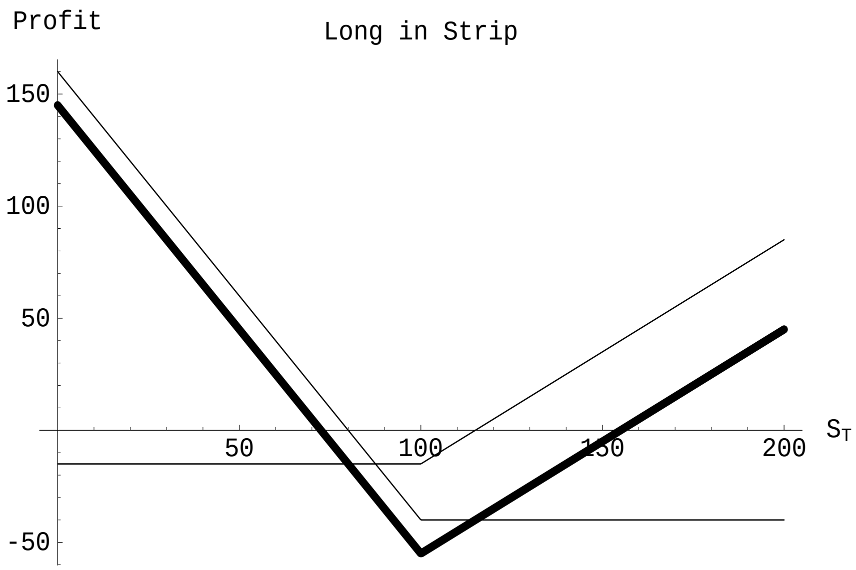
| Bottom Strangle | Underlying price at maturity $T$ | | |
|---|---|---|---|
| $K_p < K_c$ | if $S_T \geq K_c$ | if $K_p < S_T < K_c$ | if $S_T \leq K_p$ |
| Buy call at $K_c$ (pay $c$) | $S_T - K_c$ | 0 | 0 |
| Buy put at $K_p$ (pay $p$) | 0 | 0 | $K_p - S_T$ |
| Total Investment: $-c - p$ | | | |
| Total Payoff | $S_T - K_c$ | 0 | $K_p - S_T$ |

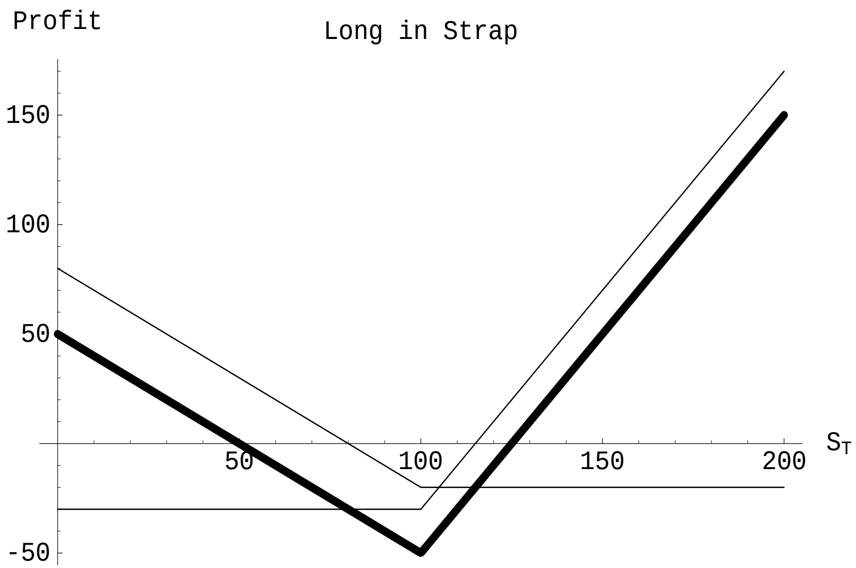

Profit
Bottom Strangle
40
30
20
10
-10
50
100
150
200
S_T


Profit
Top Strangle
10
-10
-20
-30
-40
50
100
150
200
S_T

*Strips and Straps*: Both of these strategies involve *three* options with the *same strike price and expiry date*; two puts and one call (strip) or two calls and one put (strap). Both are volatility–based strategies, but with a directional "bet": when a decrease (increase) in the underlying is more likely, a strip (strap) is appropriate.

| Strip | Underlying price at maturity $T$ | |
|---|---|---|
| | if $S_T > K$ | if $S_T \leq K$ |
| Buy call at $K$ (pay $c$) | $S_T - K$ | 0 |
| Buy 2 puts at $K$ (pay $2p$) | 0 | $2\,(K - S_T)$ |
| Total Investment: $-c - 2p$ | | |
| Total Payoff | $S_T - K$ | $2\,(K - S_T)$ |


Profit
Long in Strip
150
100
50
-50
50
100
150
200
$S_T$


Profit
Long in Strap
150
100
50
-50
50
100
150
200
$S_T$

# 13 Reading

- Background reading: Hull [4], Chapter 7
- Hull [5], Chapters 8 and 9
- Jarrow and Turnbull [6], Chapter 3
- Merton [9]. Do not get disappointed if you cannot follow all the math!

# References

[1] M. Bhattacharya. Transaction data tests of efficiency of the Chicago Board Options Exchange. *Journal of Financial Economics*, 12:161–185, 1983.

[2] D. Galai. Empirical tests of boundary conditions for CBOE options. *Journal of Financial Economics*, 6:187–211, 1978.

[3] J. P. Gould and D. Galai. Transaction costs and the relationship between put and call prices. *Journal of Financial Economics*, 1:105–129, 1974.

[4] J. C. Hull. *Options, Futures and other Derivatives*. Prentice Hall Inc., Upper Saddle River, New Jersey, 4th edition, 2000.

[5] J. C. Hull. *Options, Futures and other Derivatives*. Prentice Hall Inc., Upper Saddle River, New Jersey, 5th edition, 2003.

[6] R. A. Jarrow and S. Turnbull. *Derivative Securities*. South–Western College Publishing, Thomson Learning, 2nd edition, 2000.

[7] R. C. Klemkosky and B. G. Resnick. Put–Call parity and market efficiency. *Journal of Finance*, 34(5):1141–1155, 1979.

[8] R. C. Klemkosky and B. G. Resnick. An ex–ante analysis of put–call parity. *Journal of Financial Economics*, 8:363–378, 1980.

[9] R. C. Merton. The theory of rational option pricing. *Bell Journal of Economics*, 4(1):141–183, 1973.

[10] H. R. Stoll. The relationship between put and call option prices. *Journal of Finance*, 31:319–332, 1969.