Athens University of Economics and Business

Department of

Accounting & Finance

"BACKTESTING DIFFERENT MODELS OF VALUE-AT-RISK"

by

DAVIDOULA DIMOPOULOU

A Thesis submitted to the Department of Accounting and Finance of the Athens University of Economics and Business as partial fulfillment of the Requirements for the Master's Degree

Athens, 30/11/2017

We approve the Thesis of
Davidoula Dimopoulou

Drakos Konstantinos: ______________

Spyrou Spyros: ______________

Tsekrekos Andrianos: ______________

Date: 30/11/2017

CERTIFICATION OF THESIS PREPARATION

"I hereby declare that this particular thesis has been written by me, in order to obtain the Postgraduate Degree in Accounting and Finance, and has not been submitted to or approved by any other postgraduate or undergraduate program in Greece or abroad. This thesis presents my personal views on the subject. All the sources I have used for the preparation of this particular thesis are mentioned explicitly with references being made either to their authors, or to the URL's (if found on the internet)."

Davidoula Dimopoulou: ______________

Supervised by Dr Konstantinos Drakos

Acknowledgement

I take this opportunity to express gratitude to my supervisor Dr Konstantinos Drakos for his help and support. I am extremly thankful and indebted to him for sharing expertise, and sincere and valuable guidance and encouragement extended to me.

I would also like to thank my loved ones, who have supported me throughout entire process and encouraged me with their best wishes.

Davidoula Dimopoulou

Abstract

One of the most popular risk measurement techniques in finance is Value-at-Risk. This measure quantifies the worst expected loss over a given confidence level and target horizon, under normal market conditions. Even though VaR methods are commonly used, they are useful only when they predict accurately future risks. For this reason, backtesting models are of a great importance. Backtesting is a statistical procedure, where VaR estimates are continuously compared to the actual profits or losses of the investment.

In this thesis, the concept of VaR as an invaluable tool for financial risk management is explained, and a theoretical but detailed description of some of the methods of VaR computation are presented. Our primary objective is to identify by using different backtesting methods, which Value-at-Risk method is the most accurate. The performance of the VaR models is measured by applying several different tests of conditional and unconditional coverage to the exchange rate of the 10 major currencies towards euro. For our calculations, we used daily data from 04/01/1999 until 04/07/2017. The outcomes of the statistical backtests show that the model of VaR with the best performance is the EWMA.

Contents

1 Introduction **1**

1.1 History of VaR 1

1.2 Criticism about VaR 3

1.3 The Purpose of the Study 4

1.4 Questions answered by the Study 5

2 Literature Review **6**

2.1 Why VaR? 6

2.2 Basics of VaR 8

2.3 The choice of VaR Parameters 10

2.4 Measuring Returns 11

2.5 Different VaR Models 12

2.5.1 Variance/Covariance (Normal) Approach 13

2.5.2 Historical Simulation 15

2.5.3 GARCH Model 16

2.5.4 EWMA Model 17

3 Backtesting Methods **19**

3.1 Unconditional Coverage 22

3.1.1 Kupiec's Tests 22

3.1.2 Basel Committee's (1996) Traffic Light Approach 25

3.2 Conditional Coverage 28

3.2.1 Christoffersen's Interval Forecast Test 28

3.2.2 Mixed Kupiec-Test 31

4 Data and Methodology **33**
4.1 Portfolio Composition and Data 34
4.2 Calculation Process of VaR 35
4.3 Backtesting Process 37

5 Empirical Results **38**
5.1 VaR Data Consolidated Output 39
5.2 Frequency of Exceptions 41
5.2.1 Basel Committees (1996) "Traffic Light" Approach 41
5.2.2 Kupiec's Tests 44
5.3 Independence of Exceptions 46
5.3.1 Independence test of Christoffersen 46
5.3.2 Independence Test of the Mixed Kupiec-Test 48
5.4 Tests of Unconditional Coverage and Independence 50
5.4.1 Christoffersen's Interval Forecast Test 50
5.4.2 Mixed Kupiec's-Test 52
5.5 Final Results and Discussion 53

6 Conclusions **56**

Appendix A Critical Values of the χ^2 Distribution **62**

Appendix B Analytical Outcomes of Backtesting Process **63**

Chapter 1

Introduction

A main characteristic of the financial markets is change. Changes could either have positive or negative sense for the investors. As a result, risk, which is precisely interwoven with gain or loss, becomes something inevitable in financial markets. The acceptance of this concept does not imply, the effort to eliminate risk, which is impossible, nor does it imply the acceptance of consequent losses fatalistically. As Wriston (2009) claimed, "All of life is the management of risk, not its elimination." Thus, the main concept is to decide what risks to avoid and how could this happen and what risk to accept and the terms of their acceptance. Under these assumptions, the two main pillars of risk management are introduced: Firstly, establish a risk quantification method and secondly, develop and implement a valid backtesting method to evaluate the validity of the estimated risk numbers.

1.1 History of VaR

Over the last decades risk management is considered as a distinct sub-field in finance theory. The problem of measuring risk is an old one in statistics, economics and finance, however VaR did not emerge as a distinct project until the late 1980's. The stock market crash of 1987 was the triggering event. This was the first crisis in which a lot of economists began to worry about the firm-wide survival. Nevertheless the growth of risk management traces back to the 1970's, where the breakdown of Bretton Woods system of fixed exchange rates and the adoption of new theory, such as the Black-Scholes model, contributed to the "revolution" of risk management. Further factors that should be taken into consideration is the increase of trading activity all these years (Linsmeier & Pearson, 1996, Dowd, 1998) [1], [2] and the growth of the dollar value of outstanding derivatives positions. Indicatively, from 1.1 trillion dollars in 1986 to 72 trillion dollars in 1999 (Jorion, 2001) [3]. All these facts combined with the unpredictable financial disasters of Barings Bank, Orange County and Daiwa intensified the need of new risk measurement tools. Consequently, Value at Risk is presently the most used risk management technique by financial institutions, non-financial institutions, and regulators alike. VaR is mostly used by commercial and investment banks to quantify potential loss in value of financial derivatives (or traded portfolios) from adverse market conditions, over a specific period of time, however it can be used by any firm so as to measure its exposure to risk.

The mathematical form of risk management had already been developed by Harry Markowitz in the context of portfolio theory, in 1950's. In 1970's and 1980's some financial institutions started constructing their own risk management models. However it was not until the pioneering work from J.P Morgan and their publication of RiskMetrics system in 1994 that made VaR the industry-wide standard (Dowd, 1998, Jorion, 2001) [2], [3]. The system, which was available for free on the internet providing data feed for computing market risk, was the first that allowed public access to data and information that was once proprietary (Jorion, 2007) [4]. Furthermore, the Basel Committee on Banking Supervision (BCBS) regulatory guidelines have required banks with substantial trading activity to set aside a proportion of capital so as to insure against extreme portfolio losses. Because of RiskMetrics system, regulators became interested in VaR, allowing banks to utilize their own internal risk models in computing and reporting their VaR, for the estimation of regulatory capital requirements (Linsmeier & Pearson, 1996) [1]. The VaR methodology is now being utilized to quantify credit risk, operational risk and liquidity risk, leading to the 'Holy Grail' of risk management (Jorion, 2007) [4].

1.2 Criticism about VaR

"A 99% Value at Risk calculation does not evaluate what happens in the last one percent... This is like an airbag that works all the time, except when you have a car accident."- David Einthorn- [5]

Value at Risk is a very popular and widely used risk management tool. However several criticisms have arisen concerning VaR methods, which are still debated. The knowledge of the limitations of VaR is crucial for the avoidance of future financial disasters. In an interview in Derivatives Strategy magazine, Nassim Taleb (1997) [6] delivered a blistering attack on value at risk. His claims could be supported by many economists and they are examined in detail in numerous papers. Amongst some of the issues raised are the following:

1. Using VaR could provide a false sense of security that could lead to excessive risk taking and use of leverage. (Nassim Taleb, 1997) [6]. There is a misconception by many investors that Value at Risk expresses "the most they could lose". Especially in cases where a 99% confidence level is used, this phenomenon is quiet often. However, even when investors are fully aware of the true concept of VaR, subconsciously the 99% confidence may be misleading for their estimations. In real terms, unfortunately 99% is very far from 100% and this misunderstanding could be fatal.

2. Value at Risk does not measure the worst case loss. For example, a 99% VaR indicates us that in 1% of cases, the expected loss will not exceed the VaR amount. However, this metric does not provide us the magnitude of the loss incurred within this 1% of trading days nor the maximum possible loss. . According to David Einhorn (2008) [5], VaR focuses on risks near the center of the distribution that are more manageable and ignores the tails. In those cases the worst case loss could be fatal, such as the bankruptcy of Lehman Brothers or a terrorist attack.

3. Another criticism which dominates in literature as well is the model perfor-

mance. This is the result of the adoption of VaR according to the BCBS about capital regulatory purposes (Pritsker, 1997) [7]. Even though the greatest attention was given to model performance, there appears to be little coverage on the comparison of the estimation performance of VaR techniques. Different methods of computation could lead to different results for the same portfolio, making the representativeness of VAR ambiguous.

4. Method accuracy vs computational time, which is examined in detail by Pritsker (1997) [7], is another issue. It is of a great importance, methods of computing VaR to be both accurate and available on a time basis. In his study Pritsker verified that there is likely to be an inherent tradeoff between these objectives since more rapid methods tend to be less accurate.

1.3 The Purpose of the Study

This study has two main objectives. First of all, it provides a theoretical comparison of different methods of computing VaR. It is known, that there are many different methodologies for this purpose, however each one of them has its own pros and cons, derived from the specific underlying hypothesis of them. Our goal is to provide the reader the fundamental knowledge of the most common VaR methods. In that point we should mention that even thought the concept of VaR seems straightforward, the implementation is not that simple. More precisely, the study considers the following methods:

1. Variance-Covariance Approach.
2. Historical Simulation.
3. Calculation of VaR using GARCH model for the volatility.
4. Calculation of VaR using EWMA model for the volatility.

With the theoretical comparison of the above approaches, we place more emphasis on their shortcomings, which provides a motivation for the backtesting of VaR.

Therefore, a theoretical comparison of some of the traditional methods of backtesting is presented. The following backtesting techniques will be discussed namely; Kupiec's proportion of failures test, Kupiec's time until first failure tests, independence test, joint-test, and Basel Committee's 'traffic light' approach. In the second part of this study, an application of the theoretical background for both the VaR calculation methodologies and the backtesting techniques, takes place. This will be conducted to the exchange rate of the 10 major currencies towards euro, with daily data from 04/01/1999 until 04/07/2017.

The above analysis takes place, in order to identify, according to the statistical backtests, which VaR model shows the most accurate performance.

1.4 Questions answered by the Study

Risk management is a topic of a great importance for financial institutions and regulators as well. As mentioned above, the effort to assure that uncertainty does not deflect the endeavor from the business goals is not easy to implement. Some of the questions that arise by those who are concerned, and which are part of our study are listed below.

1. What are the underlying assumptions in some models of Value at Risk?
2. How can we validate the accuracy of VaR estimates?
3. What are some methods of backtesting VaR?
4. Which are the key statistical assumptions of backtesting models and how can we implement them?
5. How do some of the VaR backtesting techniques fare in validating the accuracy of VaR estimates in forex market?

Chapter 2

Literature Review

Unfortunately, literature on VaR model backtesting is recent and relatively limited. This chapter begins with a synopsis about the drivers of VaR and its definition. Following, we present the calculation of returns and the parameters we used for the implementation of VaR. Different VaR models, their characteristics and shortcomings are then reviewed. The chapter ends with a presentation of some of the backtesting techniques for validating the accuracy of VaR models.

2.1 Why VaR?

"VAR is like a wobbly speedometer. Even so, it gives a rough indication of speed. Derivatives disasters have occurred because drivers or passengers did not worry at all about their speed. Of course, there can be other sources of crashes. Like blown tires, for instance. Such accidents can be compared to operational risks, against which VAR provides no direct protection. Still, a wobbly speedometer is better than nothing." - Philippe Jorion- [8]

Value at Risk is the most popular tool for measuring and managing risk in the financial industry. Even though this risk management model is one of the statistical probability theories that have been known to perform poorly when applied to the financial markets, its popularity is due to some special characteristics. Besides, the model would not be used at all if it didn't confer benefits to those major financial banks and institutions that use it and have the expertise to do so. Jorion (2007)

[4] in an article for “Derivatives Strategy” magazine named “In Defence of VaR”, reported some of these.

One of the most important advantages of VaR, is that as a concept it is simple to understand, interpret and further use in analyses. Since VaR is expressed both in price units and as a percentage of portfolio value, gives us the ability to get a rough idea about the extent of risk in a portfolio, with just one number. The comparison of different assets and portfolios is another beneficial characteristic of VaR, since it is applicable to any assets with price such as stocks, bonds, currencies and the most important, derivatives. VaR is so widespread and commonly used, that is considered the gold standard for risk management. Thus, when competitors use it, clients require it, and regulators recommend it, there is no reason to reject it.

However the key word, which was mentioned above as well, is “derivatives”. Over the last decades, the importance of derivatives in finance has increased rapidly, since they are used in all “key” areas of the field. They are added to bond issues and capital investment opportunities, they are used to transfer risk, in executive or employee compensation plans and the list goes on. The proliferation of derivatives has been accompanied by an increase in the trade of cash instruments and the proliferation of different financing opportunities. This fact coincided with the growth of foreign trade and the rise of international financial relations between companies (Linsmeier & Pearson, 1996) [1]. Even though some economists use derivatives for speculation, they are mostly used to hedge financial risk. This is applied by transferring a wide range of risks in the economy from one entity to another.

It goes without saying that the financial industry has reached a stage where it is imperative for those working in the industry, to understand how derivatives work, the manner in which they are used, and more importantly, how to quantify their risk. As a result of the sheer numbers and complexity of some of these instruments, the magnitude of risks in companies’ portfolios are often not obvious. As a consequence, there has been demand for portfolio level quantitative measures of market risk, and VaR has become a significant component of such risk measures (Jorion, 2007) [4].

2.2 Basics of VaR

Risk plays an important role in business activities. We could define risk as "the possibility that something bad or unpleasant will happen". In financial terms, we could divide risk into the following three categories: market, credit and liquidity risk. According to Jorion (2001) [3], VaR was developed originally to gauge market risk, which is caused by movements in the volatility of asset prices. Subsequently, Dowd (1998) [2] claims that market risk can be subdivided into 4 categories: interest rate risks, equity price risks, exchange rate risks and commodity price risks. More mathematically, Jorion (2001) [3] gives the following definition of VaR:

"VaR describes the quantile of the projected distribution of gains and losses over the target horizon. If c is the selected confidence level, VaR corresponds to the 1-c lower-tail level."

To sum up, VaR is a number that summarizes the worst loss over a target horizon given a specific confidence level, under normal market conditions. In fact, it is a "probability boundary" of potential losses, a quantitative measure of portfolios downside risk. Under the worst case scenario VaR will not indicate us exactly the magnitude of the potential loss. VaR is a boarder value that will not be crossed more that a given number of times, depending on the confidence level.

For instance, with a 95% confidence level, VaR should be such that it exceeds 5% of the total number of observations in the distribution.

VaR has two important and appealing characteristics. First, it provides a simple and consistent measure of risk for different types of instruments and positions. Second, it takes into account the correlation between different risk factors. This characteristic is crucial whenever computing risk metrics for a portfolio consisting of more than one instrument (Dowd, 1998) [2]. In mathematical terms, VaR is calculated as follows:

$$VaR = \alpha \cdot \sigma \cdot W$$

Where:

α: the confidence level.

σ: the standard deviation of the portfolio returns

W: the initial portfolio value

For example let's consider the following situation. We have a portfolio whith initial value of 100 million euros and the annual volatility of the portfolio returns is 30%. The 10-day VaR at 99% confidence level for this portfolio is:

$$VaR_{99\%} = -2.33 \cdot 30\% \cdot \sqrt{10/250} \cdot 100 = -13.98 \quad million$$

The time horizon and confidence level are of great importance in interpreting VaR outcomes. The holding period indicates how far into the future we are looking; the longer the holding period the larger the potential losses. Investors such as financial firms, who have actively traded portfolios, use a 1-day period, while institutional investors and non-financial corporations prefer longer time horizons. According to Dowd (1998) [2], the choice of the holding period should depend on the time it takes for the firm to liquidate the portfolio. However, we should also take into consideration the properties of the specific VaR method. For instance, for methods with use of normal approximations, a short term horizon should be preferable.

The confidence level on the other hand determines with how much certainty the measurement is made. Higher confidence levels mean higher potential losses. Regarding the choice of confidence level, we should mention that depends on the purpose. In the case of capital requirements, confidence levels depend on the risk aversion of the manager. Thus, risk averse managers choose higher confidence levels. In our case study, we used a 99% confidence level for the computation of VaR.

The events which occur when the returns of a portfolio exceed the estimated VaR measure are called VaR breaches. The number of breaches, typically, should be as close as possible to the indicative number defined by the confidence level. Therefore, one way of testing the accuracy of VaR models, is to conduct backtesting, considering the VaR breaches, which is the empirical focus of our study.

2.3 The choice of VaR Parameters

Value at Risk is applicable to many different portfolios. The holding period and confidence level are the most essential quantitative parameters, without which VaR estimates are meaningless. In each diverse situation different criteria are applied for the choice of holding period and confidence level. If VaR number is used to compare risks among different markets, the choice of parameters is arbitrary as long as consistency is maintained. However, if we use VaR to measure potential loss, the parameters should be determined by the nature of the portfolio.

Liquidity should be one of the criteria used for the choice of the holding period (Dowd, 1998) [2]. More specifically, financial institutions determine the holding period due to the amount of time required so as to liquidate the portfolio. For instance, banks and financial institutions with actively traded portfolios use the one day VaR measure. On the other hand, investors and non-financial firms choose longer VaR horizons. Another criterion that should be considered is the specific properties of VaR methods used. More precisely, the methods which assume that the returns of the portfolio are normally distributed are accurate only if short time horizons are used. In the same time, backtesting procedure can be conducted only with the application of a short holding period. Jorion (2007) [4] submits that the Basel Committee on Banking Supervision imposes that banks and financial institutions should perform backtesting over a one day period, despite the 10 day horizon used for regulatory capital.

Concerning the choice of the confidence level, purpose at hand is crucial. Dowd (1998) [2] states that the choice depends on whether the VaR number is for capital requirements, to provide input for internal risk management, or it is for making comparisons among different portfolios. As for the assessment of capital holdings, in case of the internal capital, the confidence level depends on internal risk aversion, while in case of regulatory capital, the choice depends on the prescribed levels of the regulation. Nevertheless, if the main purpose is backtesting VaR models, it is recommended to avoid high confidence levels. This is due to the fact that high confidence levels tend to reduce the number of observations in the tail of the distributions of returns. As a result the power of the test is reduced as well (Jorion, 2007) [4]. Another aspect is to consider confidence levels for accounting and comparison purposes. Different institutions report their VaR estimates using varying confidence levels (Dowd, 1998) [2].

2.4 Measuring Returns

By definition Value at Risk estimate is the downside of the portfolio return distribution. As a result we proceed with the definition of portfolio return. As portfolio return δP, we define the difference between the value of the portfolio at subsequent time intervals, i.e. $\delta P = P_t - P_{t-1}$ where P_t and P_{t-1} are the portfolio values at time t and $t-1$ respectively. There are two expressions for the rate of return: the arithmetic and geometric. The arithmetic rate of return, R_a, is given by the difference between the current portfolio price and the previous period's price, all divided by the previous period's portfolio price:

$$R_a = \frac{P_t - P_{t-1}}{P_{t-1}}$$

The geometric or log return of an asset is instead defined as:

$$R_g = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

Where $ln(\cdot)$ denotes the natural logarithm. Here we assume all income payments such as dividends are zero or reinvested in the portfolio such that they are reflected in the portfolio price.

The two returns are typically fairly similar, as can be seen from:

$$R_{t+1} = \ln(S_{t+1}) - \ln(S_t) = \ln\left(\frac{S_{t+1}}{S_t}\right) = ln(1 + r_t + 1) \approx r_t + 1$$

The approximation holds because $\ln(x) \approx x - 1$ when x is close to 1. More precisely:

$$R_g = ln\left(\frac{P_t}{P_{t-1}}\right) = ln(1 + R_a).$$

If R_a is small, then by Taylor expansion, $R_g = R_a - \frac{R_a^2}{2} + \frac{R_a^3}{3} - ...$, which implies $R_g \approx R_a$, if R_a is small. Thus, in practice, as long as returns are small, arithmetic returns and geometric returns converge. On the other hand, in times of high volatility or long holding periods, the choice of the type of returns which is used is crucial, since this may have a negative impact in VaR estimates.

In most cases it makes economic sense to use geometric returns, since if geometric returns are normally distributed, it is not possible to get a price that is negative. On the other hand, normally distributed arithmetic returns could generate negative asset prices. This is meaningless since stocks or portfolios have limited downside risk.

2.5 Different VaR Models

"One of the most difficult aspects of calculating VaR is selecting among the many types of VaR methodologies and their associated assumptions." (Minnich, 1998) [9]

Even though VaR is an easy and intuitive concept, its measurement is a challenging statistical problem. In practice, the main objective of calculating VaR is to provide a reasonably accurate estimate of downside risk at a reasonable cost. This requires choosing for the specific portfolio the most appropriate VaR method, among many different industry methods. VaR estimates obtained through alternative methodologies prove to have economically significant differences (Beder, 1995; Hendricks, 1996) [10], [11]. VaR can be estimated parametrically or non-parametrically with many different methodologies. However all these methodologies follow a common general structure, which can be summarized in three steps: a) Mark-to-Market the portfolio, b) Estimate the distribution of returns of the portfolio, c) Calculate the VaR. Parametric models assume the return distribution belongs to a parametric family, such as the normal distribution, while non-parametric models do not make any assumptions regarding the distributional shape of portfolio returns and therefore tend to require simulation. As a result, in parametric approach VaR can be computed directly from the standard deviation of the portfolio's return distribution together with a multiplicative factor that is based on the VaR confidence level (Jorion, 2007) [4].

This chapter presents the fundamentals of some of the most common VaR computation methods: Variance-Covariance, Historical simulation, EWMA model and GARCH model. The major aim of the discussion is not to provide a detailed description of the methods, but rather, to emphasize on the strengths and weaknesses of each of the approaches. For a detailed and more comprehensive discussion on the various VaR methodologies we refer the reader to, for instance; Linsmeier & Pearson (1996) [1], Dowd (1998) [2] or Jorion (2007) [4].

2.5.1 Variance/Covariance (Normal) Approach

The Variance/Covariance method is the easiest to implement. The basic assumption is that the portfolio consists of only securities with jointly normal distribution. Since the portfolio return is the linear combination of normal variables, it is also normally distributed (Jorion, 2001) [3]. The assumption of normality is the most basic and straightforward approach, as a result this method is ideal for portfolios that consist of only linear instruments (Dowd, 1998) [2]. Estimating VaR is therefore attained by simply multiplying the current portfolio price/value (P_0) by the portfolio standard deviation (σ) and a multiplicative factor (α) from the normal distribution for the chosen confidence level, i.e.

$$VaR = \alpha \cdot \sigma \cdot P_0$$

Using this approach, the portfolio standard deviation is assumed to be a linear combination of the volatilities and covariances of portfolio elements, and thus determined using the variance/covariance matrix. For this reason the method is called Variance-Covariance approach. At this point we should mention that the famous RiskMetrics model of J. P. Morgan (1996) [12] is an analytic variance-covariance method.

For a foex portfolio, which is our case, the plain standard deviation would be useful to calculate the required VaR. In case of a portfolio consisting of different securities, the first step is to 'map' individual investments into a set of simple and standardized market instruments. Then each instrument is stated as a set of positions in these standardized market instruments. The following step is the calculation of the variances and covariances of the instruments. The final step is the calculation of VaR by using the estimated variances and covariances and the weights on the positions. The pros and cons of Variance-Covariance method are both consequences of the main underlying assumptions, on which it is based. The main of these is the assumption about the linear relationship among market risk factors and the assumption that portfolio returns are joint normally distributed.

Simplicity is the main advantages of Variance/Covariance method. The assumption of Normality allows us to use all the mathematical properties of the Normal distribution and also take advantage of the translatability between different holding periods and confidence levels.

Even though the simplicity of the method is tempting, the method can be subject to a number of criticisms, since the huge body of contemporary empirical evidence has shown that probability distributions of real life data exhibit "fatter tails" that the standard normal distribution. This means that extreme outcomes are more likely to happen, that the normal distribution imposes. In that case, according to Jorion (2007) [4] the method tends to overestimate risk for small confidence levels and underestimates risk for higher confidence levels. Another "drawback" of Variance/Covariance method, is that we cannot use it for portfolios which include instruments whose returns are non linear functions of risk, such as options. This is a problematic trait given the increasing use of non-linear assets in financial market portfolios. In that case Delta-Normal approach is a solution. This method implies is to take first order Taylor approximations to the returns of these instruments and then compute VaR by using the linear approximation. Since this method is effective when we have limited non-linearity in a portfolio, Delta-Gamma models were proposed by Britten-Jones and Schaefer (1999) [13], which use second order approximations. It is obvious that there is improvement by using first and second order approximations. On the other hand, this tends to introduce more complexity, thus losing some of the basic simplicity of the variance-covariance method due to the additional assumptions required in light of the normality loss (Damodaran, 2007) [14]. We are not going through these methods in detail, as we are not using them in this thesis.

2.5.2 Historical Simulation

The historical simulation method is a non-parametric approach that is both easy to understand and implement. The method of historical simulation provides an implementation of full valuation and some advantage over the normal method, as it does not impose any distributional assumptions and does not require calculations of variances and covariances. All that we need is historical data of the time series. This method is based on the hypothetical assumption that the portfolio is held constant over the observation holding period. The estimation of VaR can then be attained by reading the desired quantile from the distribution of the portfolio returns. The method consists of going back in time and applying current weights to a time series of historical asset values (Jorion, 2001) [3].

$$R_{p,k} = \sum_{i=1}^{N} w_{i,t} R_{i,k} \qquad k = 1, ..., t$$

The returns of this formula "reconstruct" a new, hypothetical portfolio, by using the history and the current position. A great advantage of this method is that it allows for non-linear securities and the implementation is simply, by only using the historical data. Since the method does not rely on specific assumptions about valuation models, is not prone to model risk. As a result it can account for fat tails and excludes the need for any linear approximations (for instance first and/or second order Taylor approximations), which tend to lead to inaccurate VaR calculations. Thus, it can be utilized on any portfolio with all kinds of instruments, both linear and non-linear (Jorion, 2007) [4]. For these reasons historical simulations is perhaps the most widely used method by banks and financial institutions worldwide, to compute VaR (Jorion, 2001) [3]. The above advantages render historical simulation as superior over the variance-covariance approach, especially when dealing with non-linear portfolios, which have become a major feature of most financial market portfolios nowadays (Jorion, 2007) [4].

However, a serious assumption of historical simulations dictates that the past will repeat itself. Even if this assumption holds sometimes, in cases of high volatility the VaR estimate might be severely distorted. The rationale is that the period of data may omit important events, and of course, the sample may contain events that may not appear again in the future. Moreover a major complication emanates from the inclusion into the portfolio of 'new' market instruments that don't have a robust amount of historical market data. Even though the same critique could be expressed for any other VaR methodology as well, in this case it is more important, since the calculation of VaR is largely based on historical data (Damodaran, 2007) [14]. Another disadvantage of historical simulations is that the users should choose wisely the time period of data. It is a challenge to decide how far the historical data should go and it is of a great importance to have a large period of data, especially if high confidence levels are used. Using large period of data, on the other hand, leads to a case where more emphasis is given to old time data, rather than new information. As a result we have distorted VaR estimates when the historical data set exhibits some huge market jumps. An efficient solution to these problems was suggested by Dowd (1998) [2]. He presented a convenient solution to the problems above with the use of weighted historical simulation, which gives lower weights in observations that lie further in the past (Dowd, 1998) [2]. In that way we could accurately recognize the market jumps in past periods.

2.5.3 GARCH Model

A cursory look in financial data suggests that some time periods are riskier than others, as a result the expected value of the magnitude of the errors differs. Moreover there is a degree of autocorrelation in the riskiness of financial returns, since risky times are not scattered randomly across the data. The variation of the amplitude of the returns from time to time is called "volatility clustering" and GARCH models, which stand for autoregressive conditional heteroscedasticity and generalized autoregressive conditional heteroscedasticity, are designed to deal with these issues. GARCH model uses the class of models developed by Engle (1982) [15] and Bollerslev (1986) [16]. As with the EWMA and the RM models, the return series is assumed to be conditionally normally distributed and VaR measures are calculated by multiplying the conditional standard deviation by the appropriate percentile point on the normal distribution. This model has become a widespread tool for dealing with time series heteroscedastic models.

More precisely, we define ht the variance of the residuals of a regression $r_t = m_t + \sqrt{h_t}\epsilon_t$. Here, by definition the variance of ϵ is equal to one. The form of the GARCH model for the variance is:

$$h_{t+1} = \omega + \alpha(r_t - m_t)^2 + \beta h_t = \omega + \alpha h_t \epsilon_t^2 + \beta h_t.$$

At this point the constants ω, α, β must be estimated. Updating requires the previous forecast h and residual. The weights are $(1-\alpha-\beta, \beta, \alpha)$ and the long run variance is $\sqrt{\frac{\omega}{1-\alpha-\beta}}$. This is applicable only if $\alpha + \beta < 1$ and makes sense if the weights are positive $(\alpha > 0, \beta > 0, \omega > 0)$.

The model that we describe is a GARCH (1,1) model. The first number in the parenthesis indicates us the number of autoregressive lags which appear in the equation described above, while the second number refers to the number of moving average lags. The GARCH (1, 1) model is the simplest and more robust of the family of volatility models and can be extended and modified in many ways.

2.5.4 EWMA Model

Under this model, the standard deviation of returns for date t is estimated over a window from date $t-k$ till date $t-1$ is:

$$\begin{aligned} \sigma_t &= \sqrt{(1-\lambda) \sum_{s=t-k}^{t-1} \lambda^{t-s-1} r_s^2} \\ &= \sqrt{\lambda \sigma_{t-1}^2 + (1-\lambda) r_{t-1}^2} \end{aligned} \tag{2.1}$$

EWMA may be seen as a special case of the GARCH (1,1) in which $\omega = 0$, $\alpha = 1-\lambda$ and $\beta = \lambda$. Once we estimate σ_t the returns are assumed to be normally distributed, so the VaR estimates are obtained using percentile points on the normal distribution: the 99% VaR is -2.33σ and the 95% VaR is -1.66σ. In this case $\lambda \in (0, 1)$ is known as the decay factor. This factor reflects how the impact of past observations decays while forecasting one day ahead σ_t. As the observations move towards the past, the impact decays exponentially, thus the most recent observation has the largest impact. A high value of λ leads to a lower decay of the weights and indicate persistence and long memory of past observations. On the other hand, low values of λ the weights attached to the returns, decay rapidly as we move further to the past. The EWMA approach to variance estimation was popularized by RiskMetrics, advocating the use of $\lambda = 0,94$ with daily financial returns.

In this method, the choice of window width k is critical. Short windows suffer from inferior statistical efficiency, however they do better in capturing short term volatility dynamics.

Chapter 3

Backtesting Methods

Given the existence of these alternative models for VaR estimation, and the importance of VaR to financial firms and financial regulators, evaluating the validity and accuracy of such measures has become an important question. There is a recent, and rapidly growing, literature on the evaluation of VaR models. From a regulatory perspective, it is highly desirable to treat the VaR model as a black box, and obtain inferences about its quality using the observed time series of the returns and the VaR estimates. Strategies for testing and model selection are said to be "model free" if the information set that they exploit is restricted to the time series of the returns and VaR estimates.

Up to this point we have mentioned the main methods of VaR computation. Since the aim of this thesis is to identify which method has the most accurate outcome, our next step is to conduct backtesting procedures for each particular model. Besides, a VaR model is useful only if it predicts future risks with accuracy. Indicatively, Hendricks (1996) [11] stated that the method of historical simulation provides exceptional performance compared to the Variance-Covariance approach and mainly when dealing with high confidence levels. Of course this observation is reasonable if we consider that the Variance-Covariance approach assumes normality, while in real terms most securities have "fat tailed" distributions. Other empirical studies have shown that the analytical approach (Variance-Covariance) provides accurate results for portfolios with a limited number of non- linear instruments (Campbell, 2005). Similarly, in the next section comparisons between the different methods of Value at Risk will follow, based on the empirical evidence of our results.

Backtesting is a statistical procedure, where VaR estimates are compared with actual profits and losses systematically. A straightforward and simple method of backtesting is to examine whether the frequency of VaR exceedances is in line with the number we expect, according to our confidence level. For instance, if a 99% confidence level is used to compute daily VaR, we expect 1 exception to occur every 100 days, under normal conditions. These types of tests are known as unconditional coverage tests, since they ignore conditioning, or time variation, in the data. When conducting unconditional coverage tests for a specific confidence level, higher number of exceptions than the number expected would indicate that the VaR measure systematically understates the portfolio's actual risk level. The case which we have fewer than expected VaR violations would be a sign that we have an overly conservative VaR measure that overstates risk. At this point we should mention that neither of the two extremes is desirable since they have capital implications.

Exceptions however could occur closely in time, which also should invalidate the model. In theory, exceedances should be spread evenly in time, which means they should be independent. Otherwise, this could be an indication that our model does not capture correctly the changes in market volatility and correlations. An ideal VaR model, theoretically, should be able to statistically measure the dispersion of the exceptions. These types of tests are called conditional coverage tests (i.e. conditional on current conditions) (Jorion, 2001) [3]. The clustering of exceptions renders an invalid VaR model, since it does not capture market volatility and correlations accurately. In times of recession, large losses occurring in succession are more likely to lead to disastrous events, like bankruptcy (Christoffersen, 1998) [17]. Consequently, tests of conditional coverage thus also test for conditioning, alternatively time variation, in the data (Jorion, 2007) [4].

Another form of conditional coverage tests is the so called joint tests, which combine conditional coverage tests, such as the independence property test with the unconditional tests. In backtesting verification systems should be able to satisfy both the unconditional coverage and independence properties (Christoffersen, 1998) [17].

Assuming that the number of exceedances is x and the size of our sample is T we can define the failure rate as x/T. Under the idealistic scenario, the failure rate would be equal to the confidence level. Therefore, the expected number of exceptions x in a total of T observations is $(1-c)T$. Certainly, the number of exceptions will not be exactly $(1-c)T$. Instead, it could swing within an acceptable range. In the backtesting method, the range for x will be calculated and thus the VaR model can be accepted or rejected (Campbell, 2006) [18]. Considering that each day will produce either a VaR exception or not with probability $p = (1-c)$, the exceptions express a classic Bernoulli Trial (Bernoulli process). As a result, their sum will follow the Binomial distribution:

$$f(x) = \binom{T}{x}(1-c)^x c^{T-x} \quad \forall x = 0, 1, 2, \dots .$$

At this point we should mention that a Binomial distribution x has expected value $E[X] = (1-c)T$ and variance $V[X] = (1-c)Tc$. Since we have a large enough sample size, T, by applying the central limit theorem we can approximate the Binomial distribution by a normal distribution;

$$z = \frac{x-\mu}{\sigma} = \frac{x(1-c)T}{\sqrt{(1-c)Tc}}.$$

Where $Z \sim N(0,1)$. As a result, given a confidence level c, there is a range for z, say $|z| \leq a$, where a is the number in the standard normal tables corresponding to $(1-c)$. Hence the range for x can be calculated as;

$$(1-c)T - \alpha\sqrt{(1-c)Tc} < x < (1-c)T + \alpha\sqrt{(1-c)Tc}.$$

The model is accepted if the number of exceptions x is within the range, and rejected otherwise (Dowd, 2006) [19].

A great number of different VaR backtesting methods have been proposed. In the following chapter, an insight in different backtesting methods is provided. Since the aim of this thesis is the accuracy of the performance of different models of VaR, the focus is on those backtests that we are using for the empirical evidence. More precisely, in this thesis we focus on the following techniques:

- Kupic's "Proportion of Failures" (POF) Test (1995)
- Kupic's "Time until first failure" (TUFF) Test
- Basel Committees (1996) Traffic Light approach
- Independence Test, Christoffersen (1998)
- Christoffersen's Conditional Coverage Test
- Mixed Kupiec-Test by Haas (2001)

3.1 Unconditional Coverage

3.1.1 Kupiec's Tests

POF - Test:

The most straightforward and widely known test, which was suggested by Kupiec (1995) [20]. This test examines whether the observed number of exceedances is in line with the expected number, due to the confidence level. Using this test we validate (backtest) the accuracy of the VaR model by recording the failure rate. That is, the proportion of times VaR is exceeded in a given sample. It is also known as POF-test (proportion of failures). To implement the test, we use Hypothesis testing, where the null hypothesis (H_0) imposes that our model is "correct".

$$H_0 : p = \hat{p} = \frac{x}{T}$$

Where:

p: the failure rate suggested by the confidence level

$\hat{p}$: the observed failure rate deviation of the portfolio returns

x: the number of exceedances

T: the size of our sample (the total number of observations)

The point is to identify whether the observed failure rate and the failure rate suggested by the confidence level statistically differ.

The same test can be conducted as a Likelihood Ratio test, which is a statistical test that calculates the ratio between the maximum probabilities of a result under two alternative hypotheses. The numerator is defined as the maximum probability of the observed result under null hypothesis and the denominator as the maximum probability of the observed result under the alternative hypothesis. This value is compared to the critical value of χ^2 distribution with one degree of freedom, and if it is larger, the null hypothesis is rejected and our model is inaccurate. Since the exceptions follow the Binomial distributions, the form of the LR test is the following:

$$LR_{POF} = -2\ln[(1-p)^{(T-x)}p^x] + 2\ln[(1-x/T)^{T-x}(x/T)^x] \tag{3.1}$$

An immediate observation is that the interval for exceptions is dependent on the test confidence level $p = (1-c)$. As we increase the confidence level p the value of α becomes smaller and thus we have a smaller interval for x. In that way it is easier to reject the VaR model. On the other hand, smaller value of p leads to larger interval for x and as a result, becomes easier to accept the current VaR model (Jorion, 2007) [4].

One more observation about this test is that usually the interval for x is large. This means that for $c = 99\%$ and $T = 251$ trading days, as long as $x < 7$ the model is accepted. However there is high probability that even though the number of exceptions for this confidence level is less that 7, the model is inaccurate. This type of error is intertwined with Type I and Type II errors. Type I error refers to the probability of rejecting a correct model, while Type II error expresses the probability of not rejecting an incorrect model. Our main purpose is to find a test statistic that would minimize the possibility of either or both errors occurring (Jorion, 2007) [4].
The table below shows a summary of the non-rejection regions for the Kupiec POF test statistic (1995) [20] for different observation periods and confidence levels.

Table 3.1: Non-rejection Region for Number of Failures x

VaR confidence Level	Probability Level p = (1-c)	T=251 days	T=510 days	T=1000 days
99.0%	0.01	$x<7$	$1<x<11$	$4<x<17$
97.5%	0.025	$2<x<12$	$6<x<21$	$15<x<36$
95.0%	0.05	$6<x<20$	$16<x<36$	$37<x<65$
92.5%	0.075	$11<x<28$	$27<x<51$	$59<x<92$
90.0%	0.1	$16<x<36$	$38<x<65$	$81<x<120$

From the table above it is clear that as we increase the sample size the power of the test increases. For instance at 90% confidence level, the interval x/T for accepting the model with 251 observations is in the range [16/251] = 0.06; 36/251 = 0.14] compared to [81/1000 = 0.08; 120/1000 = 0.12] for 1000 observations, which is much tighter.

However Kupiec's POF-test has some weaknesses. First of all, the test is statistically weak for sample sizes consistent with the regulatory framework, which is one year. This weakness has already been acknowledged by Kupiec himself (Jorion, 2007) [4]. Furthermore, this test takes into consideration only the number of exceptions and not the frequency of their occurrence. Thus, it may fail to reject a model that produces clustered exceptions (serially dependent violations), which is a common weakness of unconditional coverage models (Campbell, 2006) [18].

TUFF-Test:

"Time until first failure" is another type of test suggested by Kupiec. This time, by using the Likelihood Ratio statistic, we compute the time (v) it takes for the first exception to occur.

$$LR_{TUFF} = -2\ln\left(\frac{p(1-p)^{\nu-1}}{\left(\frac{1}{\nu}\right)\left(1-\frac{1}{\nu}\right)^{\nu-1}}\right) \tag{3.2}$$

By following the same steps, we reject our model if the value of LR-TUFF is larger than the critical value of χ^2 distribution.

The main drawback of this test is that it does not identify unsuitable VaR models. For instance, it might not reject a model that reports exceedance in day 7 with 99% daily VaR (Dowd, 2006) [19]. Due to the lack of power, Kupiec's TUFF-test is best used only as a preliminary to the POF-test when there is no larger set of data available. Finally, we could also use it for testing independence of exceptions in the mixed Kupiec-test by Haas (2001) [21], which we are going to discuss later on.

3.1.2 Basel Committee's (1996) Traffic Light Approach

Since 1998, Regulatory Framework imposes that banks should set aside a specific amount of capital, so as to cover potential portfolio losses. This amount of required capital is defined by the bank's VaR estimations. Due to this framework, a strict measure of backtesting is needed, so as to prevent banks from understating their risk estimates. The Basel Committee decided to allow banks to use their own VaR estimates for the calculation of the capital requirement (Jorion, 2001)[3].

The backtesting method used for the regulatory framework, is carried out by comparing the last 250 daily 99% VaR estimates with corresponding daily trading outcomes. The model is evaluated by counting the VaR exceptions for this period. Specifically, the regulatory risk based capital requirements are a function of the larger of either the bank's current assessment of the 99% confidence level VaR over a 10 day holding period or a multiple of the bank's average reported 99% confidence level VaR over the preceding 60 day holding period plus an additional amount that reflects the underlying credit risk (c) of the bank's portfolio (Basel Committee, 1996)[22].

The size of the Market risk capital requirement depends on the outcome of backtesting:

$$MRC_t = \max\left[VaR_t(0.01), S_t \cdot \frac{1}{60}\sum_{i=0}^{59} VaR_{t-i}(0.01)\right] + c$$

$$S_t = \begin{cases} 3 & \text{if } x \leq 4 \text{ green} \\ 3 + 0.2(x-4) & \text{if } 5 \leq x \leq 9 \text{ yellow} \\ 4 & \text{if } 10 \leq x \text{ red} \end{cases}$$

Where:

S_t: the scaling factor

x: the number of exceptions over the 250 days

In essence S_t is like a "penalty" factor, which increases as the number of exceptions increase. This happens, since the accepted value of Market risk capital is the most conservative. The risk based capital requirement rises when a VaR model indicates more risk. What may be less intuitive from equation above is that the risk based capital requirement also depends on the accuracy of the VaR model. Importantly, the multiplication factor, k, varies with backtesting results. According to the scaling factor, the Basel Committee classifies the outcomes of backtesting in three categories: green, yellow and red. In the first category, the VaR method is considered accurate and accepted. The yellow zone outcomes could be produced by both accurate and inaccurate models with relatively high probability, even though they are more likely for inaccurate models. In that case, if the bank is able to demonstrate that the VaR model is 'fundamentally sound' and suffers, for example, from "bad luck" (i.e. not due to normal market conditions), supervisors may consider revising their requirements. Finally, the red zone indicates a clear problem with the VaR model and as a result, immediate rejection of the model.

The table below displays the probabilities of obtaining a given number of exceptions for a correct model with 99% confidence level and T=250 days.

Table 3.2: Basel 'Traffic Light' Probabilities of Obtaining Exceptions

ZONE	Number of excpeptions(x)	Scaling Factor(k)	Probability P(X = x)	Cumulative Prob. P(X <x)
Green zone	0	0	8.11%	8.11%'
	1	0	20.47%	28.58%
	2	0	25.74%	54.32%
	3	0	21.49%	75.81%
	4	0.4	13.41%	89.22%
Yellow zone	5	0.5	6.66%	95.88
	6	0.65	2.75%	98.63
	7	0.75	0.97%	99.60%
	8	0.85	0.30%	99.90%
	9	1	0.08%	99.98%
Red zone	10+	1	0.02%	100.00%

Source: Calculation. Given we have c=99% (p=1%) and T=250 days,by substituting this in the Binomial distribution, we can determine the probability with which certain number of exceptions can be realized and the cumulative probability as well.

From the exception table (Table 3.2) it is evident that there is a probability of 2.75% that we will have exactly six exceptions in 250 days, and that there is a probability of 98.63% that the number of exceptions will be less or equal to six.

Despite the simplicity of this method, Basel Traffic light approach is not accurate so as to evaluate a VaR model. One of the reasons is that it does not take into account the independence of the exceptions. Furthermore, the framework cannot easily distinguish between accurate and inaccurate models. For these reasons the method is best used as a preliminary test for the accuracy of VaR.

3.2 Conditional Coverage

The unconditional coverage tests and the Traffic Light approach take into consideration only the number of the exceptions and not the independence of their occurrence. Efficient VaR models are able to react in changes in volatility in correlations, otherwise a sequence of consecutive exceptions is occurred. It is very important to detect exceptions' clustering, since the occurrence of large losses in succession is more likely to lead to disastrous events than individual exceptions occurring occasionally. (Christoffersen & Pelletier, 2004)[23]

The tests of conditional coverage provide us a solution to this problem. Following we present two conditional coverage tests: Christoffersen's (1998)[17] interval forecast test and the mixed Kupiec-test by Haas (2001)[21].

3.2.1 Christoffersen's Interval Forecast Test

This test is an independence test (or Markov test), suggested by Christoffersen (1998)[17]. It is the most known test of conditional coverage. Similar to Kupiec's test, it uses the log-likelihood testing, however the test is extended so as to cope with the independence of exceptions. Furthermore, the test examines whether the exceptions of every day is independent from the outcome of the previous day. For example, if the likelihood of a VaR exception increased on a day preceeding a previous VaR exception, then this would point towards a need to raise VaR level estimates, as successive losses would imply higher risk exposure. Christoffersen's test applies the same Likelihood-Ratio statistical testing framework as Kupiec for the independence of exceptions.

The analytical form of the procedure is presented in Christoffersen (1998)[17].

At first we define a dummy variable which takes the value of 1 when an exception is occurred and 0 otherwise:

$$I_t = \begin{cases} 1 & \text{if violation occurs} \\ 0 & \text{if no violation occurs} \end{cases}$$

Let us also define n_{ij} as the number of days when condition j occurred assuming that condition i occurred on the previous day. As a result the following contingency table occurs:

Table 3.3: Contingency Table

	$I_{t-1} = 0$	$I_{t-1} = 1$	
$I_t = 0$	n_{00}	n_{10}	$n_{00} + n_{10}$
$I_t = 1$	n_{01}	n_{11}	$n_{01} + n_{11}$
	$n_{00} + n_{01}$	$n_{10} + n_{11}$	N

Furthermore π_i is the probability of an exception occurring conditional on state i on the previous day.

$$\pi_0 = \frac{n_{01}}{n_{00} + n_{01}}, \pi_1 = \frac{n_{11}}{n_{10} + n_{11}} \ \ and \ \ \pi = \frac{n_{01} + n_{11}}{n_{00} + n_{01} + n_{10} + n_{11}}$$

At this point, a hypothesis test is conducted. For an accurate model, the probability of an exception taking place today should be independent of whether an exception occurred or not the day before. This means that our null hypothesis imposes:

$$H_0 : \pi_0 = \pi_1$$

The equivalent likelihood ratio test is the following:

$$LR_{ind} = -2\ln\left(\frac{(1-\pi)^{n_{00}+n_{10}} \cdot \pi^{n_{01}+n_{11}}}{(1-\pi_0)^{n_{00}} \cdot \pi_0^{n_{01}} \cdot (1-\pi_1)^{n_{10}} \cdot \pi_1^{n_{11}}}\right)$$

That is, the chance of an exception occurring after a day of no exception is the same as occurring after a day of an exception (Campbell, 2005)[18]. If these proportions differ greatly from each other the validity of VaR is in doubt.

As with the POF-test the LR-statistic follows the the χ^2 (Chi-squared) distribution with 1-degree of freedom. As a result, if the value of the LR_{ind} -statistic is less than the critical value of χ^2 (Chi-squared) distribution, p with 1-degree of freedom, the model is considered accurate, otherwise it is rejected.

If we combine this statistic with Kupiec's POF Test, we obtain the conditional coverage test:

$$LR_{cc} = LR_{ind} + LR_{pof}$$

LRcc follows χ^2 distribution with 2 degrees of freedom. If the value of the LR_{cc} statistic is higher than the critical value the null hypothesis is rejected, and the model is inaccurate.

With Christoffersen's test, for an inaccurate VaR model, we can examine if the cause of inefficiency is inaccurate coverage, clustered exceptions, or even both. For this evaluation one could conduct independently the LR_{ind} and LR_{pof} tests. In that point we should mention that according to Campbell (2005)[18], in some cases a model passes the joint test, while fails in either the independence or the coverage test separately. Thus, it is preferable to conduct each one of the tests, even though the joint test is positive.

A crucial drawback of Christoffersen's method is that it only considers dependence between two successive days and as a result it has limited power against clustering. However it is possible that a VaR violation does not depend on the violation that occurred the day before, but many days ago. Thus this method is weak to produce robust results.

3.2.2 Mixed Kupiec-Test

Haas (2001)[21] introduced an improved method, which combines the ideas of Kupiec and Christoffersen. This test measures the time between the exceptions so it can capture more general forms of dependence. In essence this test is a mixed Kupiec-test which measures the time between exceptions, instead of observing only whether an exception today depends on the outcome of the previous day.

To conduct this test, Kupiec's TUFF-test is needed. According to Haas (2001)[21], Kupiec's TUFF-test, which was used to measure the time until the first exception, can be used to count the time between two exceptions. The form of the test statistic now becomes:

$$LR_i = -2\ln\left(\frac{p(1-p)^{\nu_i-1}}{\left(\frac{1}{\nu_i}\right)\left(1-\frac{1}{\nu_i}\right)^{\nu_i-1}}\right) \tag{3.3}$$

Where ν_i is the time between exceptions i and $i-1$. If we calculate the LR statistics for each exception we have an independence test, where the null hypothesis is that the exceptions are independent from each other. The form of the test statistic for n exceptions is the following:

$$LR_{ind} = \sum_{i=2}^{n}\left[-2\ln\left(\frac{p(1-p)^{\nu_i-1}}{\left(\frac{1}{\nu_i}\right)\left(1-\frac{1}{\nu_i}\right)^{\nu_i-1}}\right)\right] - 2\ln\left(\frac{p(1-p)^{\nu-1}}{\left(\frac{1}{\nu}\right)\left(1-\frac{1}{\nu}\right)^{\nu-1}}\right) \tag{3.4}$$

The statistic follows χ^2 distribution with n degrees of freedom and combined with the POF-test, we obtain the mixed Kupiec's test, which covers us both for independence and coverage:

$$LR_{mix} = LR_{pof} + LR_{ind}$$

The LR_{mix} -statistic is χ^2 distributed with $n+1$ degrees of freedom. Just like with other likelihood-ratio tests, the statistic is compared to the critical values of χ^2 distribution. If the test statistic is lower, the model is accepted, otherwise the model is rejected.

Chapter 4

Data and Methodology

The technical part of this thesis is carried out upon foreign exchange data. The objective of this study is to examine the accuracy of different VaR models that are currently used by great companies and financial institutions, based on different backtesting methods.

The performance of a VaR method depends a lot on the type of the portfolio being considered. For this reason, the decision about the VaR simulation method and the type of portfolio used in the study has not been randomly selected. The procedures of backtesting are conducted by comparing daily profits and losses with daily VaR estimates using a time period of one year (in our case 257 trading days). Similarly, VaR estimates are calculated using a one day moving window over a 257 day holding period. The accuracy of the results of VaR calculations is evaluated by applying the Basel framework and tests by Kupiec (1995)[20], Christoffersen (1998)[17] and Haas (2001)[21]. Due to some technical limitations that we are going to discuss later, it was not possible to have an arithmetic value for all the tests that we presented in previous chapters. However, the backtesting process here is thorough enough and provides a satisfactory view on the accuracy of the VaR models at this point.

At this chapter we describe briefly the portfolio composition and data, the calculation process of Value-at-Risk and finally the backtesting process.

4.1 Portfolio Composition and Data

All market and sample data is actual daily data for the period 4 January 1999 to 4 July 2017 (a sample of 4740 days). Our data is derived from the official site of the European Central Bank (www.ecb.europa.eu), which provided us with daily exchange rates against the euro for the following 10 major currencies of world: Australian dollar, Canadian dollar, Japanese yen, Norwegian krone, Russian ruble, Swedish krona, Swiss franc, Turkish lira, Pound sterling and US dollar. The sample under investigation includes highly volatile periods, since the years of the financial crisis of 2008 are included.

The reason we chose to conduct the technical part of this thesis upon foreign exchange data, is because the forex market is considered to be the largest financial market with over 5 trillion dollars in daily transactions. This number is greater than the futures and equity markets combined and makes forex market the biggest and most liquid market globally. International currency markets are made up of banks, commercial companies, central banks, investment management firms, hedge funds, retail forex brokers and investors as well. We could claim that the operation of forex market is fundamental for the international trade and global investing. As a result the valuation of the accuracy of risk management tools is of a great importance in this field.

At the same time, a second reason for focusing on the foreign exchange rate market is that we have a linear relationship in the assets and in that way we could make comparisons of various VaR models without restrictions and difficulty.

4.2 Calculation Process of VaR

According to Jorion (2007)[4] the choice of the confidence level for the calculation of VaR should not be arbitrary when backtesting is conducted. A confidence level of 95% is recommended so as to attain enough observations. With this confidence level, it is possible to observe enough VaR exceptions within the one year time period. However in this thesis we preferred a tighter bound for our estimations, and we used a 99% confidence level. The selection was due to the consideration of "real life" choices of financial institutions and investors, which use VaR as an indication of the amount of their capital cushion. The confidence level is crucial and should reflect the degree of risk aversion of the company and the cost of a loss of exceeding VaR. Higher risk aversion, or greater costs, implies that a greater amount of capital should cover possible losses, thus leading to a higher confidence level.

The VaR for each one of the 10 exchange rates was calculated for every day, after calculating the 1-day volatility, employing a rolling 1-year return sample. The calculations of different methods of VaR and backtesting as well were conducted by using mainly the Excel spreadsheet program, combined with the statistical software of Stata.

Since the main purpose of this thesis is to identify which VaR method performs the best, we computed VaR for 4 of the most commonly used methods of VaR: Parametric, Historical Simulation, EWMA and GARCH.

The Parametric (Variance/Covariance) was the easiest to implement. The basic assumption is that the portfolio consists of only securities with jointly normal distribution. Estimating VaR is therefore attained by simply multiplying the current portfolio price/value by the portfolio standard deviation and a multiplicative factor from the normal distribution for the chosen confidence level.

The historical simulation method is a non-parametric approach that is both easy to understand and implement. Since the method of historical simulation does not impose any distributional assumptions and does not require calculations of variances and covariances, all that we need is historical data of the time series. The estimation of VaR was attained by reading the desired quantile from the distribution of the portfolio returns. The method consists of going back in time and applying current weights to a time series of historical asset values.

With the EWMA method once we estimate σ_t the returns are assumed to be normally distributed, so the VaR estimates are obtained using percentile points on the normal distribution. This method is useful as it puts more weight to recent market developments. In addition, the user can define the weight that will be used in the estimation process. In this study we used the EWMA decay factor at 0.94, according to RiskMetrics (1996)[24] recommendation for daily data (Jorion, 2001)[3]. This method has relatively more emphasis on recent developments of market prices and at this point it is important to realize that the choice of this parameter has a significant effect on the outcome of the estimation. The following figure gives as a greater picture of this issue:

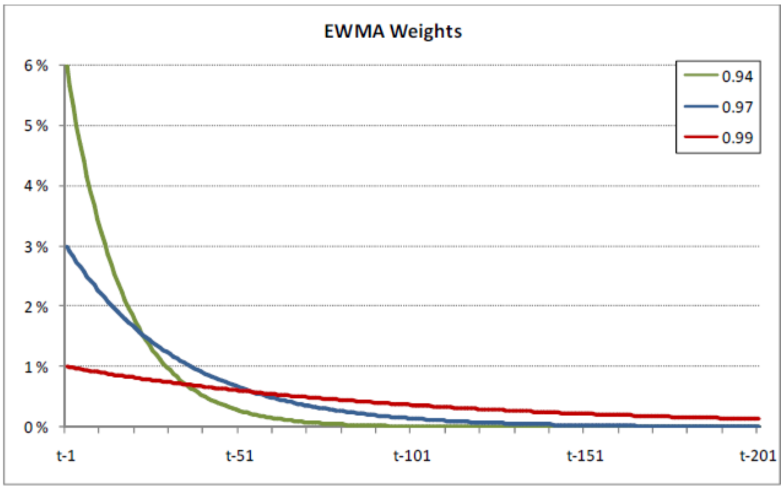

Figure 1: EWMA weights.

For example, using a decay factor 0.94 leads to a situation where the last observation $(t-1)$ is given a 6% weight and an observation one month ago $(t-21)$ only 1.74% weight. In practice, if the market experiences sudden jumps in volatility, VaR estimates react faster to these changes when using a lower decay factor.

Finally, GARCH model was implemented to deal with issues of volatility clustering. We know that there is a degree of autocorrelation in the riskiness of financial returns, since risky times are not scattered randomly across the data. In a GARCH model the return series is assumed to be conditionally normally distributed and VaR measures are calculated by multiplying the conditional standard deviation with the appropriate percentile point on the normal distribution. In this thesis we implemented a GARCH (1,1) model, after estimating the constants required by the model. For the calculation of VaR with this method we used Stata program which helped us estimate the variance of the residuals.

4.3 Backtesting Process

Using the Excel program we calculated the VaR estimates and the net profit or loss as well. The next step is to compare the actual profit or loss against the VaR estimates and count the number of exceptions against the VaR estimates. These exception figures would then be used in each backtest model to validate the VaR estimates.

If we denote $r_{t,t+1}$ the profit or loss of the portfolio over one day time interval the corresponding VaR estimate is then defined as VaR_t. VaR_t is calculated at the beginning of the period, i.e. using the closing prices of day t. For example, if the first VaR estimate is calculated with the closing prices of the day t then this estimate is compared to the outcome of the profit or loss that is realized at the end of the day $t+1$. In conducting the backtests, the number of exceptions or the test statistic values where compared to a mapping table with either the number of exceptions or the χ^2 (Chi-squared) distribution critical value, depending on the backtest method.

Chapter 5

Empirical Results

According to Haas (2001)[21] and Campbell (2005)[18] for the validation of Value at Risk models more than one backtesting technique should be used. Therefore the outcome of a test should be confirmed by another test. In this thesis, for every model of VaR (parametric, historical simulation, EWMA, GARCH) we utilize all backtesting methods that mentioned earlier for validating the accuracy of the VaR outcomes.

For the frequency of the exceptions we use the Basel's Committee "Traffic Light" Approach and Kupiec's POF-test. Christoffersen's interval forecast test and mixed Kupiec-test study the independence of exceptions. These tests are the traditional and most common tests, whose implementation only requires the total number of observations, the number of exceptions and the time when exceptions occurred.

Campbell (2005)[18], Niepolla (2009)[25] and Jorion (2007)[4], stated that high confidence levels should be avoided for backtesting purposes. For this reason we utilize a confidence level that is not so high. Therefore all backtests are conducted at 95% percentile of the χ^2 (Chi-squared) distribution as the critical value for the Likelihood-Ratio tests. The use of 95% confidence level implies that relatively strong evidence is required for the rejection of a VaR model.

5.1 VaR Data Consolidated Output

As mentioned to in the previous section, we compared the VaR estimate VaR_{t-1} with the realized profit and loss, r_t, at time t, so as to establish the number of exceptions as well as when they occur. Below the tables present the consolidated results of the number of exceptions from the analysis, which is the main output required for the backtesting procedure. Since we have 10 different exchange rates against the euro and a large data set, we present indicatively the tables for the 3 major currencies of the world: US dollar, Pound Sterling and Japanese Yen. The analytical tables for all the exchange rates are available in Appendix B.

Table 5.1: Number of VaR exceptions according to the type of model: US dollar

		US DOLLAR		
Year	Parametric	Historical Simulation	EWMA	GARCH
1	5	6	1	5
2	0	2	1	1
3	3	2	0	7
4	3	1	3	4
5	3	3	5	3
6	2	3	2	1
7	0	1	1	0
8	4	5	4	0
9	15	11	8	18
10	1	0	4	11
11	4	2	5	4
12	3	5	4	5
13	1	1	3	0
14	4	4	9	0
15	12	3	10	0
16	8	6	4	2
17	3	5	6	3

Table 5.2: Number of VaR exceptions according to the type of model:Pound Sterling

		POUND STERLING		
Year	Parametric	Historical Simulation	EWMA	GARCH
1	4	8	2	1
2	1	1	2	0
3	1	3	2	0
4	3	2	2	0
5	1	2	3	0
6	1	1	2	0
7	2	2	4	0
8	3	3	3	0
9	11	10	3	5
10	2	0	2	6
11	3	3	4	2
12	4	3	6	3
13	0	0	3	0
14	5	4	4	1
15	5	4	6	0
16	8	5	6	2
17	2	3	3	3

It is quite obvious from the tables that different VaR models yield different results. The number of exceptions varies, and in some cases significantly, between the models. This observation holds for all exchange rates, making the validation of our models indispensable.

Table 5.3: Number of VaR exceptions according to the type of model: Japanese Yen

		JAPANESE YEN		
Year	Parametric	Historical Simulation	EWMA	GARCH
1	3	1	3	17
2	1	3	3	12
3	1	0	3	8
4	8	7	4	0
5	6	6	4	3
6	4	4	8	1
7	4	3	8	0
8	12	8	9	1
9	16	7	10	19
10	0	0	4	24
11	5	3	5	24
12	3	3	4	23
13	0	0	0	0
14	5	5	3	10
15	5	2	8	5
16	5	2	5	1
17	6	5	5	3

5.2 Frequency of Exceptions

5.2.1 Basel Committees (1996) ”Traffic Light” Approach

The Basel Committees “Traffic light” approach is an unconditional coverage test which tests for the frequency of exceptions (failure rate). We should mention that the test is applicable only to banks. Nevertheless the Basel framework provides a useful exercise as a preliminary test before moving towards statistical hypothesis-based backtests. For regulatory purposes this backtesting method is mainly used at a 99% confidence level and exceptions boundaries are provided for this confidence level. However, for purposes of internal backtesting various confidence level exceptions can be computed. We can combine the VaR translation property and the tables of the Binomial distribution to find the exception ranges for the 90% and 95% confidence levels. The following Table 5.4 below displays the cut-off regions for the number of exceptions using the 'traffic light' approach for the three VaR confidence levels.

Table 5.4

	Zone		
Confidence Level	Green	Yellow	Red
99%	0-4	5-9	10+
95%	0-17	18-26	27+
90%	0-32	33-43	44+

At this point we recall that the green zone indicates an accurate VaR model, the yellow zone defines a model that is doubtful, while the red zone indicates an inaccurate model. We should also mention that outcomes close to zero for lower confidence levels are also a problem for the model, even though the green zone indicates an accurate VaR model. For instance, if we observe zero exceptions at a 95% confidence level over 250 days, our model is considered overly conservative and actually useless. On the other hand, regulators are only interested in identifying models that underestimate risk, thus even though these outcomes are false, from a regulator's point of view they are acceptable.

The findings show that in majority all VaR models are accurate. This outcome is expected since backtesting has taken place under a 95% confidence level and for this method the intervals of model acceptance are wider at lower confidence levels (i.e. 95% and 90%). In the table above all models are accurate, with exception the GARCH model for the US dollar in year 9, in which the model is under consideration.

Table 5.5: Traffic Light Approach results according to the type of model: US dollar

		US DOLLAR		
Year	Parametric	Historical Simulation	EWMA	GARCH
1	Green	Green	Green	Green
2	Green	Green	Green	Green
3	Green	Green	Green	Green
4	Green	Green	Green	Green
5	Green	Green	Green	Green
6	Green	Green	Green	Green
7	Green	Green	Green	Green
8	Green	Green	Green	Green
9	Green	Green	Green	Yellow
10	Green	Green	Green	Green
11	Green	Green	Green	Green
12	Green	Green	Green	Green
13	Green	Green	Green	Green
14	Green	Green	Green	Green
15	Green	Green	Green	Green
16	Green	Green	Green	Green
17	Green	Green	Green	Green

As you can see in the Appendix, in most of the cases the models are classified in the Green zone, rarely in Yellow zone and only in very few cases Traffic Light Approach determines a model as inaccurate by classifying it in the Red Zone. Usually these are the cases where we have a very large number of exceptions. For example in GARCH model for the Russian ruble at year 16 we had 28 VaR violations and the model was classified in the Red Zone. The analytical tables in Appendix B indicate that the cases where models are considered inaccurate are these where GARCH model for the computation of VaR is used.

As we will discuss later on, in general terms, the results from Traffic Light Approach are in line with the results of Kupiec's POF-Tests. The finding makes intuitive sense since both tests are based on the same testing framework looking at the failure rate.

5.2.2 Kupiec's Tests

We utilize Kupiec's POF-test to statistically test the model's accuracy in estimating the proportion of exceptions (unconditional coverage). It is used to examine whether the amount of exceptions is too large, as was suggested by the Basel traffic light approach, but this time in statistical terms. Even though the number of observations for one year is limited, the POF-test should yield some significant results, especially with lower confidence levels. Throughout the process of backtesting we use 95% percentile of the χ^2 distribution as the critical value for all the likelihood-ratio tests, which means that reasonably strong evidence is required in order to reject the model.

More or less the test should confirm the results obtained from the Traffic Light Approach. This is expected since the Basel Traffic Light framework is derived directly from the failure rate test. However, POF-Test proves that outcomes close to zero for lower confidence levels are a problem for the model, since our model is considered overly conservative and useless. This is the reason why Kupiec's POF-Test and Traffic Light Approach do not agree in cases where we have a small number of exceptions, like in the case of the US dollar. Thus Kupiec's test rejects the model in most of the years, while Traffic Light Approach accepts it. Even though POF-test has been criticized for having low statistical power in distinguishing accurate from inaccurate models, the results can be considered to be fairly reliable with one year of data and lower confidence levels of 95% and 90%, as in our case.

Table 5.6: Kupiec's Test outcome for the US dollar.

		US DOLLAR		
Year	Parametric	Historical Simulation	EWMA	GARCH
1	Reject	Reject	Reject	Reject
2	-	Reject	Reject	Reject
3	Reject	Reject	-	ACCEPT
4	Reject	Reject	Reject	Reject
5	Reject	Reject	Reject	Reject
6	Reject	Reject	Reject	Reject
7	-	Reject	Reject	-
8	Reject	Reject	Reject	-
9	ACCEPT	ACCEPT	ACCEPT	ACCEPT
10	Reject	-	Reject	ACCEPT
11	Reject	Reject	Reject	Reject
12	Reject	Reject	Reject	Reject
13	Reject	Reject	Reject	-
14	Reject	Reject	ACCEPT	-
15	ACCEPT	Reject	ACCEPT	-
16	ACCEPT	Reject	Reject	Reject
17	Reject	Reject	Reject	Reject

Another test of Kupiec that we conducted is the TUFF-test. The statistical significance of this test is limited, so no conclusions regarding the quality of a VaR model should be based on it. In fact the test provides quiet misleading results compared to POF-Test and Traffic Light Approach. Further evidence exists in the Appendix.

5.3 Independence of Exceptions

5.3.1 Independence test of Christoffersen

We use Christoffersen's independence test to examine if the exceptions are spread evenly over time or they are clustered. To conduct this test we calculate an LR_{ind} statistic, which we compare to the critical value of the χ^2 distribution with one degree of freedom, for a 95% confidence level. If the value of the LR statistic is below this value (3,84) the model is considered accurate. In order to calculate the LR statistic for independence first of all we need to identify the contingency table and of course the probabilities that come of table 5.7.

Table 5.7

Independence Test Data (for the US dollar-Parametric method-Year 9)						
n_{00}	n_{01}	n_{10}	n_{11}	π_0	π_1	π
226	14	14	1	0,058333	0,066667	0,058824

NB: n_{ij} is the number of days in which state j occurred in one day while it was at I the previous day, i.e. n_{11} is the number of consecutive exceptions. And π is the probability of an exception conditional on state i the previous day, i.e. π_0 is the conditional probability of an exception occurring given no exception the previous day.

If we substitute the number of conditional exceptions n_{ij} and the probability of conditional π exceptions from the results data (Table 4.5) into LR-test statistic (LR_{ind}) equation (2.6.9) gives:

$$LR_{ind} = -2\ln\left(\frac{(1-0.058824)^{(226+14)}\cdot(0.058824)^{(14+1)}}{(1-0.058333)^{226}\cdot 0.058333^{14}\cdot(1-0.066667)^{14}\cdot 0.066667^{1}}\right) = 0,017059686$$

Since the value of the LR_{ind} is below the critical value of 3,84 the accuracy of the model is accepted. The table below indicates the results of Independence Test for the US dollar.

Table 5.8: The results of Independence Test for the US dollar.

		US DOLLAR		
Year	Parametric	Historical Simulation	EWMA	GARCH
1	ACCEPT	ACCEPT	ACCEPT	ACCEPT
2	-	ACCEPT	ACCEPT	ACCEPT
3	ACCEPT	ACCEPT	-	ACCEPT
4	ACCEPT	ACCEPT	ACCEPT	ACCEPT
5	ACCEPT	ACCEPT	ACCEPT	ACCEPT
6	ACCEPT	ACCEPT	ACCEPT	ACCEPT
7	-	ACCEPT	ACCEPT	-
8	ACCEPT	ACCEPT	ACCEPT	-
9	ACCEPT	ACCEPT	ACCEPT	ACCEPT
10	ACCEPT	-	ACCEPT	ACCEPT
11	ACCEPT	ACCEPT	ACCEPT	ACCEPT
12	ACCEPT	ACCEPT	ACCEPT	ACCEPT
13	ACCEPT	ACCEPT	ACCEPT	-
14	ACCEPT	ACCEPT	ACCEPT	-
15	ACCEPT	ACCEPT	ACCEPT	-
16	ACCEPT	ACCEPT	ACCEPT	ACCEPT
17	ACCEPT	ACCEPT	ACCEPT	ACCEPT

In that case according to the test the majority of VaR models are accurate. In fact, for the most of the exchange rates of the whole study the test had a positive outcome. That means that it is extremely rare to have multiple exceptions occurring successive days.

Once again we applied this test for all the exchange rates of each one of the 4 different VaR methods. In many cases there are years that we had not exceptions occurring for two consecutive days. As a result π_1 (the probability that we have exceptions for two successive days) is zero and the value of the LR_{ind} test cannot be defined. Since we have no consecutive exceptions, it is reasonable to conclude that the model is accepted. Even though in many cases we had not an arithmetic value for the LR test it is clear that in the vast majority the outcome is positive to the acceptance of the model. Despite this outcome no instant conclusions should be derived from it, because the test does not capture more general forms of dependence between exceptions such as duration based dependence. Of course we could say that the model avoids the most severe time of dependence, which is multiple exceptions occurring consecutive days.

5.3.2 Independence Test of the Mixed Kupiec-Test

This test was suggested by Haas (2001)[21] and it is preferable than Christoffersen's test of independence, since it captures all forms of dependence between exceptions and not only these occurring in successive days. The implementation of the test includes an LR test which is distributed as an χ^2 distribution with n degrees of freedom, equal to the number of exceptions.

The implementation of the test for the US dollar, parametric method of VaR, in year 15 has the following outcome: $LR_{ind} = 9,806880918 + 0,801139262 = 10,60802018$. Since the value of LR_{ind} is below the critical value of 21,02606982, the model is accepted.

As we can see from the table below, the results for the acceptance of the model vary and depend on the method of VaR and the special characteristics of each the year. We know that the data include periods of high volatility and the investigation takes place when market conditions are not normal.

Table 5.9: The results of Independence Test of the Mixed Kupiec for the US dollar.

		US DOLLAR		
Year	Parametric	Historical Simulation	EWMA	GARCH
1	Reject	Reject	-	Reject
2	-	Reject	-	-
3	Reject	Reject	-	ACCEPT
4	ACCEPT	-	ACCEPT	ACCEPT
5	ACCEPT	ACCEPT	ACCEPT	ACCEPT
6	ACCEPT	ACCEPT	ACCEPT	-
7	-	-	-	-
8	Reject	ACCEPT	Reject	-
9	Reject	ACCEPT	ACCEPT	Reject
10	-	-	Reject	ACCEPT
11	ACCEPT	ACCEPT	Reject	Reject
12	ACCEPT	ACCEPT	ACCEPT	ACCEPT
13	-	-	Reject	-
14	ACCEPT	ACCEPT	Reject	-
15	ACCEPT	Reject	ACCEPT	-
16	-	-	Reject	-
17	Reject	Reject	Reject	Reject

By applying the test to our data, we see that we do not have an arithmetic value in cases where an exception occurred the first day of our year (in that case we have no value for the TUFF-Tests as well), or even when we had clustered exception for two successive days, where the summation cannot be calculated in our LR formula. In these two cases we could conclude of course that the model is inaccurate. Generally it is obvious that as a test is more conservative and strict than Christoffersen's since its results differ significantly from Christoffersen's test.

5.4 Tests of Unconditional Coverage and Independence

5.4.1 Christoffersen's Interval Forecast Test

With the combination of Kupiec's POF-Test and Christoffersen's independence test, we can perform a joint test of conditional coverage. The test statistic can be derived directly from the results of the previous backtests as follows:

$$LR_{cc} = LR_{ind} + LR_{pof}$$

The value of LR_{cc} is compared to the critical value of the χ^2 distribution with two degrees of freedom, for a 95% confidence level, which is 5,99. If this number is lower than 5,99 the model is rejected. For example, let's consider the LR_{cc} for the US dollar for Year 9 of the parametric model:

$$LR_{cc} = LR_{ind} + LR_{pof} = 0,01706 + 0,360181 = 0,37724$$

This value is less than the χ^2 (Chi-squared) critical value of 5.99 (95% percentile with 2-degrees of freedom), and therefore the model is accepted.

In cases where we already know that the POF-test produced results where critical values were exceeded significantly, the results from the joint test are not surprising, and that is the case. Generally, when the values of LR_{ind} or LR_{pof} are significantly larger than the critical values, it is highly possible that LRcc will also exceed the critical value and our model will be inaccurate. This conclusion is obvious in the case of US dollar.

The cases we accepted the VaR models, are these which the model was accepted by Kupiec's POF-Test and Christoffersen's independence test. Unfortunately, we are not able to conduct the LRcc test when we have no value for the LR_{ind} or the LR_{pof}, which is the reason why there are many gaps in our tables. A close look at the consolidated tables (Appendix B) will prove that for this specific test we have not a fixed outcome, since the conclusions for the validity of the model vary. Below the outcomes of the test for the US dollar are presented:

Table 5.10: Christoffersen's Interval Forecast Test results for the US dollar.

		US DOLLAR		
Year	Parametric	Historical Simulation	EWMA	GARCH
1	-	-	-	-
2	-	-	-	-
3	-	-	-	ACCEPT
4	-	-	-	-
5	-	-	-	-
6	-	-	-	-
7	-	-	-	-
8	-	-	-	-
9	ACCEPT	ACCEPT	ACCEPT	ACCEPT
10	-	-	-	ACCEPT
11	-	-	-	-
12	-	-	-	-
13	-	-	-	-
14	-	-	ACCEPT	-
15	ACCEPT	-	ACCEPT	-
16	ACCEPT	-	-	-
17	-	-	-	-

5.4.2 Mixed Kupiec's-Test

Mixed Kupiec's -test is capable of capturing more general forms of dependence between exceptions instead of just two successive days, thus it can be considered to be more informative and reliable than the joint test by Christoffersen. As with Christofferssen's Interval Forecast test above, the mixed Kupiec-test can also be conducted in a straightforward fashion since we already have the results of the POF-test and the independence test:

$$LR_{mix} = LR_{pof} + LR_{ind}$$

Let's calculate as an example, the LR_{mix} for the 3rd year of the parametric method of the US dollar:

$$LR_{mix} = 11,36376 + 11,18488 = 22,54864$$

This time we compare the value of LR_{mix} with the 95% percentile of χ^2 distribution with $n+1$ degrees of freedom, where n is the number of exceptions. The model is rejected if the value of LR_{mix} is lower than the critical value. This outcome is in a way expected, when we have a failure resulted also from the Christoffersen's test which is statistically weaker than the mixed Kupiec-test. For our example, LR_{mix} is higher than the critical value of 9,487729037 of the χ^2 distribution (with 4 degrees of freedom since that year we had 3 VaR exceptions), so the model is rejected.

Indicatively, we present the results for the US dollar exchange rate, in which the majority of the outcomes indicate that we should reject VaR models. Among the other models, the EWMA method has two positive indications. We could say that the results are worrying.

Table 5.11: Mixed Kupiec's Test results for the US dollar.

		US DOLLAR		
Year	Parametric	Historical Simulation	EWMA	GARCH
1	Reject	Reject	-	Reject
2	-	Reject	-	-
3	Reject	Reject	-	Reject
4	Reject	-	Reject	Reject
5	Reject	Reject	Reject	Reject
6	Reject	Reject	Reject	-
7	-	-	-	-
8	Reject	Reject	Reject	-
9	-	ACCEPT	ACCEPT	-
10	-	-	Reject	ACCEPT
11	Reject	Reject	Reject	Reject
12	Reject	Reject	Reject	Reject
13	-	-	Reject	-
14	Reject	Reject	-	-
15	ACCEPT	Reject	ACCEPT	-
16	-	-	Reject	-
17	Reject	Reject	Reject	Reject

5.5 Final Results and Discussion

Even though we applied many different backtests in this study, the purpose is not to use the wide scale of tests in forthcoming testing. Out of all the backtests we conducted more attention should be paid on those that are considered to be the most reliable. Therefore we ought to be careful when interpreting backtesting results from Kupiec's TUFF-Test, and for the same reason one should recognize the shortcomings of Christoffersen's independence test as well. According to Haas (2001)[21] the most reliable and robust test is the mixed Kupiec's Test. Haas recommended that the procedure we should follow for model validation with back-testing is the following:

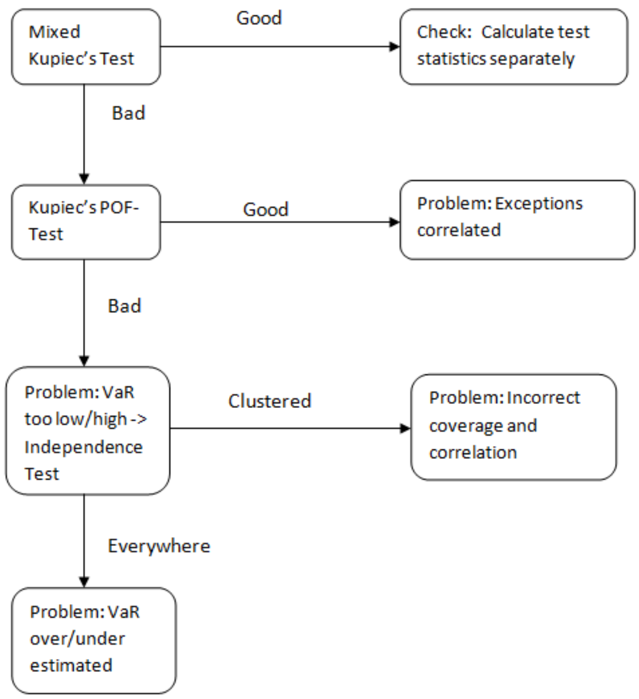

Figure 2: The procedure for model validation by Haas.

As we can see from the flowchart above, the testing procedure starts with the mixed Kupiec's test. A positive outcome should be confirmed by the separate tests of coverage and independence, since we know that joint tests may not always detect violations of these two properties. If the model is rejected due to the mixed Kupiec's test, the next step is to investigate whether the rejection is due to incorrect coverage between exceptions, dependence or even both.

All the tests we mentioned were applied in our dataset which is composed of 10 foreign exchange rates towards euro, for 17 years. VaR was calculated by 4 different methods and backtesting was conducted for each one of them, in order to identify the most efficient VaR method. Unfortunately the analytical results of both the VaR estimates and backtesting procedure cannot be analyzed in detail for every currency and every year, since the data set is very large. However we have created analytical tables that indicate the results from the Excel spreadsheet and are available in Appendix B.

Following the strategy of optimal backtesting suggested by Haas (2001)[21] and taking into consideration the outcomes of the backtests, we counted for each one of the VaR methods (Parametric, Historical simulation, EWMA, GARCH), for every currency and every year the number of accepted models. To our surprise, the proportion of accepted VaR models due to the number of years and assets was very small. The VaR model which was accepted most of the times was the EWMA. The second one was Historical simulation, and following GARCH and Parametric method.

The fact that the method of EWMA was proven to be the most efficient of these 4 was compatible to our intuition. This method puts more weight to recent market developments, thus even though it takes into consideration past events the method controls how much the future will be affected by the past. In essence this happens due to the decay factor λ. In this study we used the EWMA decay factor at 0.94, according to RiskMetrics (1996)[24] recommendation for daily data (Jorion, 2001)[3]. However, the user can define the weight that will be used in the estimation process, since this parameter has a significant effect on the outcome of the estimation. In practice, if the market experiences sudden jumps in volatility, VaR estimates react faster to these changes when using a lower decay factor.

It would be challenging to test whether different choice of parameters would yield better results. For instance, using a different decay factor is a potential idea for future testing. Especially the unusual circumstances in case of 2008 perhaps require a different approach by laying even more emphasis on recent developments, i.e. using a lower decay factor.

Chapter 6

Conclusions

"In short we ought to be able to identify most VaR bad models, but the worrying issue is whether we can find any good ones." (Dowd, 2006)[19]

VaR is one of the most prominent risk management techniques in finance, mostly used by financial institutions, non-financial institutions, and regulators alike. Nevertheless, it is important to mention that when implementing VaR systems, a number of simplifications and assumptions are involved. Even though VaR is widely used and commonly accepted as a risk management tool several criticisms have arisen concerning VaR methods, which are still debated. This makes the accuracy of VaR estimates dubious. There are a number of studies that have been conducted comparing the performance of the various VaR methods. In this study we focused on foreign exchange rates towards euro.

The theoretical part of this thesis provided a comparison of some of the VaR estimation techniques. More precisely, we compared the variance-covariance approach, historical simulation, EWMA and GARCH models. During the theoretical comparison of these methods we placed emphasis on their shortcomings, knowing their potential drawbacks provide motivation for the backtesting of VaR. Moreover, a theoretical comparison of some of the most common backtesting methods was presented.

We defined the fundamental properties of an accurate VaR model which are unconditional coverage, the independence property and conditional coverage and we discussed their relevance from the perspective of examining the accuracy of VaR model estimates. We reviewed the tests that examine the validity of the unconditional coverage property, the independence property, or joint properties. The backtesting techniques that were presented are; Kupiec's Proportion of Failures-Test, Basel Committees "Traffic light" Approach, Independence tests of Christoffersen and Kupiec and the joint-tests of Christoffersen and Kupiec.

The second part of this work was an empirical study that focused on applying the presented backtesting techniques in validating the accuracy of the VaR models mentioned above. The backtesting procedure was conducted at a 95% confidence level for a data set of 17 years. The results from backtests provided some indication of potential problems with the models. The tests of unconditional coverage suggested underestimation of risk in many cases for our data, while Christoffersen's test of independence indicated more positive results. This means that an exception which occurred today did not seem to have an effect on whether an exception will occur tomorrow. On the other hand, the mixed Kupiec's-Test produced a different outcome suggesting that the exceptions are not totally independent of each other. This test is more reliable since it captures more general forms of dependence.

The backtesting results raise concerns about the model's ability to estimate in satisfactory precision when market conditions are not normal. For example we can imagine the turbulent market of 2008 and especially the rising volatility during the autumn. Inevitably the crisis caused problems in estimating parameters that should describe future market movements. Besides all VaR models rely on historical market data, thus this issue concerns VaR systems in general. VaR is not able to capture abnormal market behavior, so even if we have strong evidence against our VaR models, we should be very cautious in accepting or rejecting the model.

During the empirical analysis, we provided some evidence that some of the backtesting models may produce misleading results and are unable to distinguish good

VaR models from bad ones. More precisely, the Christoffersen's framework is incapable of capturing exception dependence, and in addition, the TUFF-test by Kupiec produced very misleading results compared to the POF-test.

Finally, after taking into consideration all the parameters of backtesting procedure we came to the conclusion that the most accurate VaR model for our data is the EWMA. Of course this estimation does not hold for all kinds of assets or portfolios. In our investigation we used exchange rates for a long time horizon, since we have a linear relationship in the assets and in that way we could make comparisons of various VaR models without restrictions in Excel program. A study of different types of instruments might yield a different outcome. It would be very interesting to conduct the same study for equities, government bonds, commodities or interest rate derivatives. Moreover we should point that for the backtesting we used a 95% confidence level. Confidence levels of 99% or 90% may not agree with our outcomes. These are very interesting avenues for further research.

Systematic backtesting should be a regular part of VaR reporting in order to constantly monitor the performance of models. To this end, it is imperative that VaR estimates are never considered to be the Holy Grail of portfolio risk management, irrespective of how sophisticated the VaR model may be. The knowledge of the shortcomings associated with VaR models could lead to better interpretation of VaR numbers, and consequently improved financial risk management.

References

[1] Thomas J Linsmeier, Neil D Pearson, et al. Risk measurement: An introduction to value at risk. 1996.

[2] Kewin Dowd. Beyond value at risk: the new science of risk management. 1998.

[3] P. Jorion. *Value at Risk, The New Benchmark for Managing Financial Risk, 2nd Edition*. McGraw-Hill, United States., 2001.

[4] P. Jorion. *Value at Risk, The New Benchmark for Managing Financial Risk, 3rd Edition*. McGraw-Hill, United States., 2007.

[5] David Einhorn and Aaron Brown. Private profits and socialized risk. *Global Association of Risk Professionals*, 42:10–26, 2008.

[6] Nassim Taleb and P Jorion. Against var. *Derivatives Strategy*, 2:21–26, 1997.

[7] Matthew Pritsker. Evaluating value at risk methodologies: accuracy versus computational time. *Journal of Financial Services Research*, 12(2):201–242, 1997.

[8] Philippe Jorion. In defense of var. *Derivatives Strategy*, 2(4):20–23, 1997.

[9] Tanya Styblo Beder, Michael Minnich, Hubert Shen, and Jodi Stanton. Vignettes on var. *Journal of Financial Engineering*, 7:289–309, 1998.

[10] Tanya Styblo Beder. Var: Seductive but dangerous. *Financial Analysts Journal*, 51(5):12–24, 1995.

[11] Darryll Hendricks. Evaluation of value-at-risk models using historical data. 1996.

[12] JP Morgan. Reuters (december 18, 1996), riskmetrics-technical document, fourth edn, jp morgan and reuters, new york.

[13] Mark Britten-Jones and Stephen M Schaefer. Non-linear value-at-risk. *Review of Finance*, 2(2):161–187, 1999.

[14] Aswath Damodaran. *Strategic risk taking: a framework for risk management*. Pearson Prentice Hall, 2007.

[15] Robert F Engle. Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. *Econometrica: Journal of the Econometric Society*, pages 987–1007, 1982.

[16] Tim Bollerslev. Generalized autoregressive conditional heteroskedasticity. *Journal of econometrics*, 31(3):307–327, 1986.

[17] Peter F Christoffersen. Evaluating interval forecasts. *International economic review*, pages 841–862, 1998.

[18] Sean D Campbell. A review of backtesting and backtesting procedures. *The Journal of Risk*, 9(2):1, 2006.

[19] Kevin Dowd. Retrospective assessment of value at risk. *Risk Management: A Modern Perspective, Amsterdam et al*, pages 183–203, 2006.

[20] Paul H Kupiec. Techniques for verifying the accuracy of risk measurement models. *The journal of Derivatives*, 3(2):73–84, 1995.

[21] Marcus Haas. New methods in backtesting. *Financial Engineering Research Center, Bonn*, 2001.

[22] BASLE COMMITEE ON BANKING SUPERVISION. Amendment to the capital accord to incorporate market risks. *Basle, Switzerland, jan. 1996a.[Links]*, 1996.

[23] Peter Christoffersen and Denis Pelletier. Backtesting value-at-risk: A duration-based approach. *Journal of Financial Econometrics*, 2(1):84–108, 2004.

[24] JP Morgan. Riskmetricstm–technical document, morgan guaranty trust companies. *Inc. New York*, 1996.

[25] Olli Nieppola et al. Backtesting value-at-risk models. 2009.

Appendix A

Critical Values of the χ^2 Distribution

Alpha Risk

df	0.995	0.990	0.975	0.95	0.9	0.75	0.5	0.25	0.1	0.05	0.25	0.01	0.005	0.001
1	0.000039	0.000157	0.000982	0.00393	0.0158	0.102	0.455	1.323	2.706	3.841	1.323	6.635	7.879	10.828
2	0.010	0.020	0.051	0.103	0.211	0.575	1.386	2.773	4.605	5.991	2.773	9.210	10.597	13.816
3	0.072	0.115	0.216	0.352	0.584	1.213	2.366	4.108	6.251	7.815	4.108	11.345	12.838	16.266
4	0.207	0.297	0.484	0.711	1.064	1.923	3.357	5.385	7.779	9.488	5.385	13.277	14.860	18.467
5	0.412	0.554	0.831	1.145	1.610	2.675	4.351	6.626	9.236	11.070	6.626	15.086	16.750	20.515
6	0.676	0.872	1.237	1.635	2.204	3.455	5.348	7.841	10.645	12.592	7.841	16.812	18.548	22.458
7	0.989	1.239	1.690	2.167	2.833	4.255	6.346	9.037	12.017	14.067	9.037	18.475	20.278	24.322
8	1.344	1.646	2.180	2.733	3.490	5.071	7.344	10.219	13.362	15.507	10.219	20.090	21.955	26.124
9	1.735	2.088	2.700	3.325	4.168	5.899	8.343	11.389	14.684	16.919	11.389	21.666	23.589	27.877
10	2.156	2.558	3.247	3.940	4.865	6.737	9.342	12.549	15.987	18.307	12.549	23.209	25.188	29.588
11	2.603	3.053	3.816	4.575	5.578	7.584	10.341	13.701	17.275	19.675	13.701	24.725	26.757	31.264
12	3.074	3.571	4.404	5.226	6.304	8.438	11.340	14.845	18.549	21.026	14.845	26.217	28.300	32.909
13	3.565	4.107	5.009	5.892	7.042	9.299	12.340	15.984	19.812	22.362	15.984	27.688	29.819	34.528
14	4.075	4.660	5.629	6.571	7.790	10.165	13.339	17.117	21.064	23.685	17.117	29.141	31.319	36.123
15	4.601	5.229	6.262	7.261	8.547	11.037	14.339	18.245	22.307	24.996	18.245	30.578	32.801	37.697
16	5.142	5.812	6.908	7.962	9.312	11.912	15.338	19.369	23.542	26.296	19.369	32.000	34.267	39.252
17	5.697	6.408	7.564	8.672	10.085	12.792	16.338	20.489	24.769	27.587	20.489	33.409	35.718	40.790
18	6.265	7.015	8.231	9.390	10.865	13.675	17.338	21.605	25.989	28.869	21.605	34.805	37.156	42.312
19	6.844	7.633	8.907	10.117	11.651	14.562	18.338	22.718	27.204	30.144	22.718	36.191	38.582	43.820
20	7.434	8.260	9.591	10.851	12.443	15.452	19.337	23.828	28.412	31.410	23.828	37.566	39.997	45.315
21	8.034	8.897	10.283	11.591	13.240	16.344	20.337	24.935	29.615	32.671	24.935	38.932	41.401	46.797
22	8.643	9.542	10.982	12.338	14.041	17.240	21.337	26.039	30.813	33.924	26.039	40.289	42.796	48.268
23	9.260	10.196	11.689	13.091	14.848	18.137	22.337	27.141	32.007	35.172	27.141	41.638	44.181	49.728
24	9.886	10.856	12.401	13.848	15.659	19.037	23.337	28.241	33.196	36.415	28.241	42.980	45.559	51.179
25	10.520	11.524	13.120	14.611	16.473	19.939	24.337	29.339	34.382	37.652	29.339	44.314	46.928	52.620
26	11.160	12.198	13.844	15.379	17.292	20.843	25.336	30.435	35.563	38.885	30.435	45.642	48.290	54.052
27	11.808	12.879	14.573	16.151	18.114	21.749	26.336	31.528	36.741	40.113	31.528	46.963	49.645	55.476
28	12.461	13.565	15.308	16.928	18.939	22.657	27.336	32.620	37.916	41.337	32.620	48.278	50.993	56.892
29	13.121	14.256	16.047	17.708	19.768	23.567	28.336	33.711	39.087	42.557	33.711	49.588	52.336	58.301
30	13.787	14.953	16.791	18.493	20.599	24.478	29.336	34.800	40.256	43.773	34.800	50.892	53.672	59.703
40	20.707	22.164	24.433	26.509	29.051	33.660	39.335	45.616	51.805	55.758	45.616	63.691	66.766	73.402
50	27.991	29.707	32.357	34.764	37.689	42.942	49.335	56.334	63.167	67.505	56.334	76.154	79.490	86.661
60	35.534	37.485	40.482	43.188	46.459	52.294	59.335	66.981	74.397	79.082	66.981	88.379	91.952	99.607
70	43.275	45.442	48.758	51.739	55.329	61.698	69.334	77.577	85.527	90.531	77.577	100.425	104.215	112.317
80	51.172	53.540	57.153	60.391	64.278	71.145	79.334	88.130	96.578	101.879	88.130	112.329	116.321	124.839
90	59.196	61.754	65.647	69.126	73.291	80.625	89.334	98.650	107.565	113.145	98.650	124.116	128.299	137.208
100	67.328	70.065	74.222	77.929	82.358	90.133	99.334	109.141	118.498	124.342	109.141	135.807	140.169	149.449

Appendix B

Analytical Outcomes of Backtesting Process

AUSTRALIAN DOLLAR	**PARAMETRIC VaR (Backtesting at 95% Confidence Level)**								
	YEAR		Frequency Tests			Independence Tests		Joint Tests	
		Number of Exceptions	Traffic Light (backtesting at 99% CL)	TUFF-Test	POF-Test	Christoffersen	Mixed Kupiec	Christoffersen	Mixed Kupiec
	1	3	GREEN	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	2	6	GREEN	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	3	2	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	4	2	green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	5	1	green	Reject	Reject	ACCEPT	Reject	-	Reject
	6	2	green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	7	2	green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	8	4	green	Reject	Reject	ACCEPT	Reject	-	Reject
	9	5	GREEN	Reject	Reject	ACCEPT	Reject	-	Reject
	10	0	Green	-	-	-	-	-	-
	11	2	Green	Rejec	Rej	ACCEPT	ACEE	-	Rej

				t	ect		PT		ect
	12	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	13	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	14	5	GREEN	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	15	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	16	5	GREEN	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	17	1	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	18	2	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
CANADIAN DOLLAR	1	3	green	ACCEPT	Reject		ACCEPT	-	Reject
	2	0	green	-	-	-	-	-	-
	3	3	green	Reject	Reject	ACCEPT	Reject	-	Reject
	4	3	green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	5	2	green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	6	4	green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	7	4	green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	8	4	gree	Reje	Reject	ACCE	Reje	-	Reject

			n	ct		PT	ct		
	9	10	red	ACCEPT	ACCEPT	Reject	Reject	Reject	-
	10	0	green	-	-	-	-	-	-
	11	8	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	-	ACCEPT
	12	1	green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	13	3	green	Reject	Reject	ACCEPT	Reject	-	Reject
	14	4	green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	15	6	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	ACCEPT
	16	5	Green	-	Reject	ACCEPT	-	-	-
	17	2	green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	18	2	green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
JAPANESE YEN	1	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	2	1	Green	-	Reject	ACCEPT	-	-	-
	3	1	Green	Reject	Reject	ACCEPT	Reject	-	-
	4	8	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	-	ACCEPT
	5	6	Gree	ACCE	Reject	ACCE	ACCE	-	ACCEPT

			n	PT		PT	PT		
	6	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	7	4	Green	-	Reject	ACCEPT	-	-	-
	8	12	Green	ACCEPT	ACCEPT	Reject	Reject	Reject	-
	9	16	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	-
	10	0	Green	-	-	-	-	-	-
	11	5	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	12	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	13	0	Green	-	Reject	-	-	-	-
	14	5	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	ACCEPT
	15	5	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	16	5	Green	-	Reject	ACCEPT	-	-	-
	17	6	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	ACCEPT
	18	0	green	-	-	-	-	-	-
NORWEGIAN KRONE	1	4	green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	2	5	Gree	ACCE	Reject	ACCE	ACCE	-	Reject

			n	PT		PT	PT		
	3	5	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	4	2	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	5	2	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	6	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	7	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	8	2	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	9	8	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	-
	10	2	Green	-	Reject	ACCEPT	-	-	-
	11	2	Green	Reject	Reject	ACCEPT	ACCEPT	-	Reject
	12	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	13	0	Green	-	-	-	-	-	-
	14	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	15	5	Green	Reject	Reject	ACCEPT	Reject	Reject	Reject
	16	3	Green	-	Reject	ACCEPT	-	-	-
	17	1	Green	Reject	Reject	ACCEPT	Reject	-	Reject

	18	1	Green	ACCEPT	Reject	ACCEPT	-	-	-
RUSSIAN RUBLE	1	2	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	2	0	Green	-	-	-	-	-	-
	3	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	4	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	5	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	ACCEPT
	6	1	Green	Reject	Reject	ACCEPT	-	-	-
	7	1	Green	Reject	Reject	ACCEPT	-	-	-
	8	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	9	14	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	-
	10	3	Green	ACCEPT	Reject	Reject	Reject	Reject	Reject
	11	4	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	12	0	Green	-	-	-	-	-	-
	13	2	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	14	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject

	15	11	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	-
	16	1	Green	Reject	Reject	ACCEPT	-	-	-
	17	0	Green	-	-	-	-	-	-
	18	0	Green	-	-	-	-	-	-
SWEDISH KRONA	1	6	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	2	4	Green	Reject	Reject	ACCEPT	ACCEPT	-	Reject
	3	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	4	4	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	5	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	6	4	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	7	0	Green	-	-	-	-	-	-
	8	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	9	14	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	10	3	Green	-	Reject	ACCEPT	-	-	-
	11	1	Green	Reject	Reject	ACCEPT	-	-	-

	12	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	13	4	Green	Reject	Reject	ACCEPT	ACCEPT	-	Reject
	14	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	15	1	Green	Reject	Reject	ACCEPT	-	-	-
	16	5	Green	-	Reject	ACCEPT	-	-	-
	17	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	18	2	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
SWISS FRANK	1	12	Green	ACCEPT	ACCEPT	Reject	Reject	Reject	-
	2	6	Green	Reject	Reject	Reject	Reject	Reject	-
	3	1	Green	Reject	Reject	ACCEPT	-	-	-
	4	8	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	-
	5	1	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	6	3	Green	Reject	Reject	ACCEPT	ACCEPT	-	Reject
	7	6	Green	ACCEPT	Reject	ACCEPT	Reject	Reject	-
	8	9	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	-

	9	13	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	10	0	Green	-	-	-	-	-	-
	11	24	Yellow	ACCEPT	Reject	ACCEPT	Reject	Reject	-
	12	7	Green	Reject	ACCEPT	ACCEPT	Reject	ACCEPT	-
	13	1	Green	Reject	Reject	ACCEPT	-	-	-
	14	12	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	15	1	Green	Reject	Reject	ACCEPT	-	-	-
	16	0	Green	-	-	-	-	-	-
	17	1	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	18	0	Green	-	-	-	-	-	-
TURKISH LIRA	1	2	Green	ACCEPT	Reject	ACCEPT	Reject	Reject	-
	2	1	Green	ACCEPT	Reject	ACCEPT	-	-	-
	3	0	Green	-	-	-	-	-	-
	4	2	Green	ACCEPT	Reject	Reject	Reject	Reject	-
	5	0	Green	-	-	-	-	-	-

	6	3	Green	ACCEPT	Reject	ACCEPT	Reject	Reject	-
	7	4	Green	Reject	Reject	ACCEPT	ACCEPT	-	Reject
	8	4	Green	Reject	Reject	ACCEPT	ACCEPT	-	Reject
	9	9	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	10	0	Green	-	-	-	-	-	-
	11	1	Green	ACCEPT	Reject	ACCEPT	-	-	-
	12	6	Green	ACCEPT	Reject	ACCEPT	Reject	Reject	
	13	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	14	6	Green	Reject	Reject	ACCEPT	ACCEPT	-	Reject
	15	3	Green	ACCEPT	Reject	Reject	Reject	Reject	-
	16	7	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	17	0	Green	-	-	-	-	-	-
	18	3	green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
POUND STERLING	1	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	2	1	Green	Reject	Reject	ACCEPT	-	-	-

	3	1	Gree n	Reje ct	Reject	ACCE PT	-	-	-
	4	3	Gree n	ACCE PT	Reject	Reje ct	Reje ct	Reject	-
	5	1	Gree n	Reje ct	Reject	ACCE PT	Reje ct	-	Reject
	6	1	Gree n	Reje ct	Reject	ACCE PT	-	-	-
	7	2	Gree n	-	Reject	ACCE PT	-	-	-
	8	3	Gree n	-	Reject	ACCE PT	-	-	-
	9	11	Gree n	ACCE PT	ACCEPT	ACCE PT	Reje ct	ACCEPT	-
	10	2	Gree n	ACCE PT	Reject	ACCE PT	ACCE PT	-	Reject
	11	3	Gree n	ACCE PT	Reject	ACCE PT	Reje ct	-	Reject
	12	4	Gree n	ACCE PT	Reject	ACCE PT	ACCE PT	-	Reject
	13	0	Gree n	-	-	-	-	-	-
	14	5	Gree n	ACCE PT	Reject	ACCE PT	ACCE PT	-	Reject
	15	5	Gree n	ACCE PT	Reject	ACCE PT	Reje ct	-	Reject
	16	8	Gree n	-	ACCEPT	ACCE PT	-	ACCEPT	-
	17	2	Gree n	Reje ct	Reject	ACCE PT	Reje ct	-	Reject
	18	0	Gree	-	-	-	-	-	-

			n						
US DOLLAR	1	5	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	2	0	Green	-	-	-	-	-	-
	3	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	4	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	5	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	6	2	Green	Reject	Reject	ACCEPT	ACCEPT	-	Reject
	7	0	Green	-	-	-	-	-	-
	8	4	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	9	15	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	-
	10	1	Green	ACCEPT	Reject	ACCEPT	-	-	-
	11	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	12	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	13	1	Green	Reject	Reject	ACCEPT	-	-	-
	14	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	15	12	Gree	ACCE	ACCEPT	ACCE	ACCE	ACCEPT	ACCEPT

			n	PT		PT	PT		
	16	8	Green	-	ACCEPT	ACCEPT	-	ACCEPT	-
	17	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	18	0	Green	-	-	-	-	-	-

AUSTRALIAN DOLLAR	Historical Simulation- VaR (Backtesting at 95% Confidence Level)								
	YEAR		Frequency Tests			Independence Tests		Joint Tests	
		Number of Exceptions	Traffic Light (backtesting at 99% CL)	TUFF-Test	POF-Test	Christoffersen	Mixed Kupiec	Christoffersen	Mixed Kupiec
	1	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	ACCEPT
	2	4	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	3	0	Green	Reject	Reject	-	-	-	-
	4	0	Green	-	-	-	-	-	-
	5	1	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	6	2	Green	ACCE	Rej	ACCEPT	Rejec	-	Rej

				PT	ect		t		ect
	7	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	8	7	Green	Reject	ACCEPT	ACCEPT	ACCEPT	-	Reject
	9	8	Green	Reject	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	10	0	Green	-	-	-	-	-	-
	11	2	Green	Reject	Reject	ACCEPT	ACCEPT	-	Reject
	12	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	13	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	14	6	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	ACCEPT
	15	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	16	4	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	17	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	18	2	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
CANADIAN DOLLAR	1	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	2	1	Green	ACCEPT	Reject	ACCEPT	-	-	-
	3	4	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject

	4	2	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	5	2	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	6	5	Green	-	Reject	ACCEPT	-	-	-
	7	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	8	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	9	9	Green	ACCEPT	ACCEPT	Reject	Reject	Reject	-
	10	0	Green	-	-	-	-	-	-
	11	7	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	12	1	Green	ACCEPT	Reject	ACCEPT	-	-	-
	13	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	14	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	15	5	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	16	3	Green	-	Reject	ACCEPT	-	-	-
	17	2	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	18	0	Green	-	-	-	-	-	-

JAPANESE YEN	1	1	Green	ACCEPT	Reject	ACCEPT	-	-	-
	2	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	3	0	Green	-	-	-	-	-	-
	4	7	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	5	6	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	ACCEPT
	6	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	7	3	Green	-	Reject	ACCEPT	-	-	-
	8	8	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	-
	9	7	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	Reject
	10	0	Green	-	-	-	-	-	-
	11	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	12	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	13	0	Green	-	-	-	-	-	-
	14	5	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	ACCEPT
	15	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	16	2	Gree	-	Reject	ACCE	-	-	-

			n			PT			
	17	5	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	ACCEPT
	18	0	Green	-	-	-	-	-	-
NORWEGIAN KRONE	1	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	2	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	3	1	Green	Reject	Reject	ACCEPT	-	-	-
	4	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	5	2	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	6	2	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	7	5	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	8	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	9	11	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	-
	10	0	Green	-	-	-	-	-	-
	11	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	12	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	13	0	Gree	-	-	-	-	-	-

			n						
	14	9	Green	Reject	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	15	6	Green	ACCEPT	Reject	ACCEPT	Reject	Reject	Reject
	16	2	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	17	1	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	18	1	Green	ACCEPT	Reject	ACCEPT	-	-	-
RUSSIAN RUBLE	1	2	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	2	1	Green	Reject	Reject	ACCEPT	-	-	-
	3	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	4	2	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	5	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	6	1	Green	Reject	Reject	ACCEPT	-	-	-
	7	1	Green	Reject	Reject	ACCEPT	-	-	-
	8	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	9	10	Green	ACCEPT	ACCEPT	Reject	Reject	ACCEPT	-
	10	2	Gree	ACCE	Reject	Reje	Reje	Reject	-

			n	PT		ct	ct		
	11	2	Gree n	Reje ct	Reject	ACCE PT	Reje ct	-	Reject
	12	1	Gree n	Reje ct	Reject	ACCE PT	-	-	-
	13	3	Gree n	ACCE PT	Reject	Reje ct	Reje ct	Reject	Reject
	14	2	Gree n	Reje ct	Reject	ACCE PT	Reje ct	-	Reject
	15	12	Gree n	ACCE PT	ACCEPT	ACCE PT	Reje ct	ACCE PT	-
	16	1	Gree n	Reje ct	Reject	ACCE PT	-	-	-
	17	0	Gree n	-	-	-	-	-	-
	18	0	Gree n	-	-	-	-	-	-
SWEDISH KRONA	1	5	Gree n	ACCE PT	Reject	ACCE PT	ACCE PT	-	Reject
	2	7	Gree n	Reje ct	ACCEPT	ACCE PT	ACCE PT	ACCE PT	ACCEPT
	3	3	Gree n	Reje ct	Reject	ACCE PT	Reje ct	-	Reject
	4	4	Gree n	Reje ct	Reject	ACCE PT	Reje ct	-	Reject
	5	1	Gree n	Reje ct	Reject	ACCE PT	-	-	-
	6	4	Gree n	ACCE PT	Reject	Reje ct	Reje ct	Reject	-
	7	1	Gree	Reje	Reject	ACCE	-	-	-

			n	ct		PT			
	8	5	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	ACCEPT
	9	12	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	10	2	Green	-	Reject	ACCEPT	-	-	-
	11	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	12	3	Green	ACCEPT	Reject	Reject	Reject	Reject	Reject
	13	4	Green	Reject	Reject	ACCEPT	ACCEPT	-	Reject
	14	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	15	4	Green	Reject	Reject	Reject	Reject	Reject	Reject
	16	3	Green	-	Reject	ACCEPT	-	-	-
	17	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	18	1	Green	Reject	Reject	ACCEPT	-	-	-
SWISS FRANK	1	6	Green	ACCEPT	Reject	Reject	Reject	Reject	Reject
	2	5	Green	Reject	Reject	ACCEPT	Reject	Reject	Reject
	3	0	Green	-	-	-	-	-	-
	4	6	Gree	ACCE	Reject	ACCE	Reje	Reject	Reject

			n	PT		PT	ct		
	5	1	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	6	2	Green	Reject	Reject	ACCEPT	ACCEPT	-	Reject
	7	3	Green	ACCEPT	Reject	Reject	Reject	Reject	Reject
	8	6	Green	ACCEPT	Reject	ACCEPT	Reject	Reject	Reject
	9	10	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	10	0	Green	-	-	-	-	-	-
	11	10	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	-
	12	7	Green	Reject	ACCEPT	ACCEPT	Reject	ACCEPT	-
	13	0	Green	-	-	-	-	-	-
	14	10	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	15	3	Green	Reject	Reject	Reject	Reject	Reject	Reject
	16	1	Green	-	-	ACCEPT	-	-	-
	17	2	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	18	0	Green	-	-	-	-	-	-
TURKISH	1	8	Gree	ACCE	ACCEPT	ACCE	Reje	ACCE	Reject

LIRA			n	PT		PT	ct	PT	
	2	11	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	-
	3	1	Green	Reject	Reject	ACCEPT	-	-	-
	4	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	5	2	Green	Reject	Reject	ACCEPT	ACCEPT	-	Reject
	6	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	7	8	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	8	4	Green	Reject	Reject	ACCEPT	ACCEPT	-	Reject
	9	5	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	10	0	Green	-	-	-	-	-	-
	11	2	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	12	5	Green	ACCEPT	Reject	ACCEPT	Reject	Reject	Reject
	13	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	14	5	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	15	3	Green	ACCEPT	Reject	Reject	Reject	Reject	Reject
	16	4	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject

	17	0	Green	-	-	-	-	-	-
	18	2	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
POUND STERLING	1	8	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	Reject
	2	1	Green	Reject	Reject	ACCEPT	-	-	-
	3	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	4	2	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	5	2	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	6	1	Green	Reject	Reject	ACCEPT	-	-	-
	7	2	Green	-	Reject	ACCEPT	-	-	-
	8	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	9	10	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	Reject
	10	0	Green	-	-	-	-	-	-
	11	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	12	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	13	0	Green	-	-	-	-	-	-

	14	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	15	4	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	16	5	Green	-	Reject	ACCEPT	-	-	-
	17	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	18	0	Green	-	-	-	-	-	-
US DOLLAR	1	6	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	2	2	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	3	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	4	1	Green	ACCEPT	Reject	ACCEPT	-	-	-
	5	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	6	3	Green	Reject	Reject	ACCEPT	ACCEPT	-	Reject
	7	1	Green	Reject	Reject	ACCEPT	-	-	-
	8	5	Green	Reject	Reject	ACCEPT	ACCEPT	-	Reject
	9	11	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	10	0	Green	-	-	-	-	-	-

	11	2	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	12	5	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	13	1	Green	Reject	Reject	ACCEPT	-	-	-
	14	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	15	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	16	6	Green	-	Reject	ACCEPT	-	-	-
	17	5	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	18	0	Green	-	-	-	-	-	-

AUSTRALIAN DOLLAR	**EWMA- VaR (Backtesting at 95% Confidence Level)**								
	YEAR		Frequency Tests			Independence Tests		Joint Tests	
		Number of Exceptions	Traffic Light (backtesting at 99% CL)	TUFF-Test	POF-Test	Christoffersen	Mixed Kupiec	Christoffersen	Mixed Kupiec
	1	4	Green	ACCE	Rej	ACCEPT	ACCE	-	Rej

				PT	ect		PT		ect
	2	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	3	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	4	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	5	1	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	6	4	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	7	1	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	8	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	9	4	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	10	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	11	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	12	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	13	5	Green	Reject	Reject	ACCEPT	ACCEPT	-	Reject
	14	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	15	5	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	16	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject

	17	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	18	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
CANADIAN DOLLAR	1	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	2	1	Green	ACCEPT	Reject	ACCEPT	-	-	-
	3	1	Green	Reject	Reject	ACCEPT	-	-	-
	4	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	5	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	6	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	7	5	Green	ACCEPT	Reject	ACCEPT	Reject	Reject	Reject
	8	6	Green	-	Reject	ACCEPT	-	-	-
	9	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	10	1	Green	Reject	Reject	ACCEPT	-	-	-
	11	7	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	12	4	Green	ACCEPT	Reject	ACCEPT	Reject	Reject	Reject
	13	5	Green	ACCEPT	Reject	ACCEPT	Reject	Reject	Reject

			n	PT		PT	ct		
	14	5	Green	Reject	Reject	ACCEPT	ACCEPT	-	Reject
	15	7	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	16	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	17	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	18	3	Green	-	Reject	ACCEPT	-	-	-
JAPANESE YEN	1	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	2	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	3	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	4	4	Green	ACCEPT	Reject	Reject	Reject	Reject	Reject
	5	4	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	6	8	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	-
	7	8	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	-
	8	9	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	-
	9	10	Green	-	ACCEPT	ACCEPT	-	ACCEPT	-
	10	4	Gree	ACCE	Reject	ACCE	Reje	-	Reject

			n	PT		PT	ct		
	11	5	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	12	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	13	0	Green	-	-	ACCEPT	-	-	-
	14	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	15	8	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	16	5	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	17	5	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	ACCEPT
	18	1	Green	ACCEPT	Reject	ACCEPT	-	-	-
NORWEGIAN KRONE	1	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	2	6	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	ACCEPT
	3	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	4	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	5	4	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	6	5	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	7	2	Gree	ACCE	Reject	ACCE	Reje	-	Reject

			n	PT		PT	ct		
	8	2	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	9	6	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	ACCEPT
	10	1	Green	Reject	Reject	ACCEPT	-	-	-
	11	0	Green	-	-	-	-	-	-
	12	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	13	4	Green	Reject	Reject	ACCEPT	ACCEPT	-	Reject
	14	1	Green	Reject	Reject	ACCEPT	-	-	-
	15	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	16	2	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	17	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	18	1	Green	ACCEPT	Reject	ACCEPT	-	-	-
RUSSIAN RUBLE	1	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	2	1	Green	Reject	Reject	ACCEPT	-	-	-
	3	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	4	4	Gree	ACCE	Reject	ACCE	ACCE	-	Reject

			n	PT		PT	PT		
	5	5	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	ACCEPT
	6	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	7	4	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	8	5	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	9	7	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	10	2	Green	Reject	Reject	ACCEPT	ACCEPT	-	Reject
	11	1	Green	ACCEPT	Reject	ACCEPT	-	-	-
	12	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	13	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	14	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	15	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	16	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	17	3	Green	Reject	Reject	Reject	Reject	Reject	-
	18	0	Green	-	-	-	-	-	-
SWEDISH	1	5	Gree	ACCE	Reject	ACCE	ACCE	-	Reject

KRONA			n	PT		PT	PT		
	2	3	Green	Reject	Reject	Reject	Reject	Reject	Reject
	3	4	Green	Reject	Reject	Reject	Reject	Reject	Reject
	4	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	5	7	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	Reject
	6	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	7	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	8	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	9	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	10	0	Green	Reject	Reject	ACCEPT	-	-	-
	11	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	12	2	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	13	6	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	ACCEPT
	14	5	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	15	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	16	5	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject

	17	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	18	4	Green	ACCEPT	Reject	Reject	Reject	Reject	Reject
SWISS FRANK	1	9	Green	ACCEPT	ACCEPT	Reject	Reject	Reject	-
	2	6	Green	Reject	Reject	ACCEPT	ACCEPT	-	Reject
	3	5	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	4	5	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	5	6	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	ACCEPT
	6	5	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	7	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	8	6	Green	ACCEPT	Reject	ACCEPT	Reject	Reject	Reject
	9	9	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	10	3	Green	Reject	Reject	Reject	Reject	Reject	Reject
	11	8	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	12	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	13	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject

	14	1	Gree n	ACCE PT	Reject	ACCE PT	-	-	-
	15	5	Gree n	ACCE PT	Reject	ACCE PT	Reje ct	-	Reject
	16	1	Gree n	Reje ct	Reject	ACCE PT	-	-	-
	17	4	Gree n	ACCE PT	Reject	ACCE PT	ACCE PT	-	Reject
	18	0	Gree n	-	-	-	-	-	-
TURKISH LIRA	1	2	Gree n	ACCE PT	ACCEPT	ACCE PT	ACCE PT	ACCEP T	ACCEPT
	2	2	Gree n	Reje ct	Reject	ACCE PT	Reje ct	-	Reject
	3	2	Gree n	Reje ct	Reject	ACCE PT	Reje ct	-	Reject
	4	3	Gree n	ACCE PT	Reject	ACCE PT	Reje ct	-	Reject
	5	1	Gree n	ACCE PT	Reject	ACCE PT	-	-	-
	6	1	Gree n	Reje ct	Reject	ACCE PT	-	-	-
	7	1	Gree n	ACCE PT	Reject	ACCE PT	-	-	-
	8	2	Gree n	Reje ct	Reject	ACCE PT	Reje ct	-	Reject
	9	4	Gree n	Reje ct	Reject	ACCE PT	ACCE PT	-	Reject
	10	4	Gree n	ACCE PT	Reject	ACCE PT	ACCE PT	-	Reject

	11	4	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	12	2	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	13	6	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	ACCEPT
	14	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	15	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	ACCEPT
	16	4	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	17	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	18	0	Green	-	-	-	-	-	-
POUND STERLING	1	2	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	2	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	3	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	4	2	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	5	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	6	2	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	7	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject

	8	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	9	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	10	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	11	4	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	12	6	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	ACCEPT
	13	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	14	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	15	6	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	-	ACCEPT
	16	6	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	17	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	18	1	Green	ACCEPT	Reject	ACCEPT	-	-	-
US DOLLAR	1	1	Green	ACCEPT	Reject	ACCEPT	-	-	-
	2	1	Green	Reject	Reject	ACCEPT	-	-	-
	3	0	Green	-	-	-	-	-	-
	4	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject

	5	5	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	6	2	Green	Reject	Reject	ACCEPT	ACCEPT	-	Reject
	7	1	Green	Reject	Reject	ACCEPT	-	-	-
	8	4	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	9	8	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	10	4	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	11	5	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	12	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	13	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	14	9	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	-
	15	10	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	16	4	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	17	6	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	18	0	Green	-	-	-	-	-	-

AUSTRALIAN DOLLAR	GARCH- VaR (Backtesting at 95% Confidence Level)								
	YEAR		Frequency Tests			Independence Tests		Joint Tests	
		Number of Exceptions	Traffic Light (backtesting at 99% CL)	TUFF-Test	POF-Test	Christoffersen	Mixed Kupiec	Christoffersen	Mixed Kupiec
	1	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	2	6	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	3	5	Green	-	Reject	ACCEPT	-	-	-
	4	0	Green	-	-	-	-	-	-
	5	1	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	6	0	Green	-	-	-	-	-	-
	7	0	Green	-	-	-	-	-	-
	8	0	Green	-	-	-	-	-	-
	9	9	Green	Reject	ACCEPT	ACCEPT	Reject	-	Reject
	10	12	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	11	8	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	-
	12	6	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	13	1	Green	Reject	Reject	ACCEPT	-	-	-

	14	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	15	1	Green	Reject	Reject	ACCEPT	-	-	-
	16	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	17	2	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	18	0	Green	-	-	-	-	-	-
CANADIAN DOLLAR	1	6	Green	Reject	Reject	Reject	Reject	Reject	-
	2	5	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	3	7	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	Reject
	4	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	5	1	Green	ACCEPT	Reject	ACCEPT	-	-	-
	6	0	Green	-	-	-	-	-	-
	7	0	Green	-	-	-	-	-	-
	8	1	Green	Reject	Reject	ACCEPT	-	-	-
	9	10	Green	Reject	ACCEPT	Reject	Reject	Reject	-
	10	7	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	-
	11	10	Gree	ACCE	ACCEPT	ACCE	ACCE	ACCE	ACCEPT

			n	PT		PT	PT	PT	
	12	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	13	1	Green	Reject	Reject	ACCEPT	-	-	-
	14	0	Green	-	-	-	-	-	-
	15	2	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	16	3	Green	-	Reject	ACCEPT	-	-	-
	17	1	Green	Reject	Reject	ACCEPT	-	-	-
	18	0	Green	-	-	-	-	-	-
JAPANESE YEN	1	17	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	-
	2	12	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	3	8	Green	-	ACCEPT	ACCEPT	-	ACCEPT	-
	4	0	Green	-	-	-	-	-	-
	5	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	6	1	Green	Reject	Reject	ACCEPT	-	-	-
	7	0	Green	-	-	-	-	-	-
	8	1	Gree	ACCE	Reject	ACCE	-	-	-

			n	PT		PT			
	9	19	Yellow	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	-
	10	24	Yellow	ACCEPT	Reject	ACCEPT	Reject	Reject	Reject
	11	24	Yellow	ACCEPT	Reject	ACCEPT	Reject	Reject	Reject
	12	23	Yellow	ACCEPT	Reject	ACCEPT	Reject	Reject	Reject
	13	0	Green	-	-	-	-	-	-
	14	10	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	15	5	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	16	1	Green	-	Reject	ACCEPT	-	-	-
	17	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	18	0	Green	-	-	-	-	-	-
NORWEGIAN KRONE	1	0	Green	-	-	-	-	-	-
	2	1	Green	Reject	Reject	ACCEPT	-	-	-
	3	0	Green	-	-	-	-	-	-
	4	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	5	6	Gree	ACCE	Reject	ACCE	ACCE	-	Reject

			n	PT		PT	PT		
	6	1	Green	ACCEPT	Reject	ACCEPT	-	-	-
	7	1	Green	Reject	Reject	ACCEPT	-	-	-
	8	2	Green	Reject	Reject	Reject	Reject	Reject	Reject
	9	10	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	-
	10	24	Yellow	-	Reject	ACCEPT	-	Reject	-
	11	8	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	12	8	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	-
	13	1	Green	Reject	Reject	ACCEPT	-	-	-
	14	6	Green	Reject	Reject	ACCEPT	ACCEPT	-	Reject
	15	14	Green	Reject	ACCEPT	ACCEPT	Reject	ACCEPT	-
	16	18	Yellow	-	ACCEPT	ACCEPT	-	ACCEPT	-
	17	12	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	18	1	Green	ACCEPT	Reject	ACCEPT	-	-	-
RUSSIAN RUBLE	1	0	Green	-	-	-	-	-	-
	2	2	Gree	ACCE	Reject	ACCE	Reje	-	Reject

			n	PT		PT	ct		
	3	7	Green	Reject	ACCEPT	ACCEPT	Reject	ACCEPT	Reject
	4	13	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	-
	5	5	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	Reject
	6	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	7	0	Green	-	-	-	-	-	-
	8	0	Green	-	-	-	-	-	-
	9	13	Green	ACCEPT	ACCEPT	ACCEPT	-	ACCEPT	-
	10	24	Yellow	ACCEPT	Reject	ACCEPT	-	Reject	-
	11	12	Green	ACCEPT	ACCEPT	ACCEPT	-	ACCEPT	-
	12	0	Green	-	-	-	-	-	-
	13	1	Green	ACCEPT	Reject	ACCEPT	-	-	-
	14	0	Green	-	-	-	-	-	-
	15	14	Green	ACCEPT	ACCEPT	Reject	Reject	ACCEPT	-
	16	28	Red	-	Reject	ACCEPT	-	Reject	-
	17	8	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	-

	18	1	Green	ACCEPT	Reject	ACCEPT	-	-	-
SWEDISH KRONA	1	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	2	6	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	3	7	Green	Reject	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	4	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	5	0	Green	-	-	-	-	-	-
	6	1	Green	Reject	Reject	ACCEPT	-	-	-
	7	0	Green	-	-	-	-	-	-
	8	0	Green	-	-	-	-	-	-
	9	15	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	-
	10	43	Red	-	Reject	ACCEPT	-	Reject	-
	11	17	Green	ACCEPT	ACCEPT	ACCEPT	Reject	ACCEPT	-
	12	19	Yellow	-	ACCEPT	ACCEPT	-	ACCEPT	-
	13	8	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	14	11	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT

	15	0	Green	-	-	-	-	-	-
	16	6	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	17	3	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	18	2	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
SWISS FRANK	1	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	2	7	Green	Reject	ACCEPT	Reject	Reject	Reject	-
	3	0	Green	-	-	-	-	-	-
	4	4	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	5	2	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	6	0	Green	-	-	-	-	-	-
	7	1	Green	Reject	Reject	ACCEPT	-	-	-
	8	0	Green	-	-	-	-	-	-
	9	14	Green	ACCEPT	ACCEPT	Reject	Reject	ACCEPT	-
	10	5	Yellow	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	11	30	Red	ACCEPT	Reject	ACCEPT	Reject	Reject	Reject

	12	33	Red	ACCEPT	Reject	ACCEPT	Reject	Reject	Reject
	13	0	Green	-	-	-	-	-	-
	14	2	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	15	3	Green	Reject	Reject	Reject	Reject	Reject	Reject
	16	7	Green	-	ACCEPT	ACCEPT	-	ACCEPT	-
	17	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	18	0	Green	-	-	-	-	-	-
TURKISH LIRA	1	0	Green	-	-	-	-	-	-
	2	35	Red	ACCEPT	Reject	ACCEPT	Reject	Reject	Reject
	3	11	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	4	20	Yellow	ACCEPT	ACCEPT	ACCEPT	Reject	Reject	-
	5	8	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	6	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	7	8	Green	Reject	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	8	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject

	9	10	Green	Reject	ACCEPT	Reject	Reject	Reject	-
	10	2	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	11	1	Green	ACCEPT	Reject	ACCEPT	-	-	-
	12	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	13	0	Green	-	-	-	-	-	-
	14	1	Green	Reject	Reject	ACCEPT	-	-	-
	15	0	Green	-	-	-	-	-	-
	16	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	17	0	Green	-	-	-	-	-	-
	18	0	Green	-	-	-	-	-	-
POUND STERLING	1	1	Green	ACCEPT	Reject	ACCEPT	-	-	-
	2	0	Green	-	-	-	-	-	-
	3	0	Green	-	-	-	-	-	-
	4	0	Green	-	-	-	-	-	-
	5	0	Green	-	-	-	-	-	-

	6	0	Green	-	-	-	-	-	-
	7	0	Green	-	-	-	-	-	-
	8	0	Green	-	-	-	-	-	-
	9	5	Green	Reject	Reject	Reject	Reject	Reject	Reject
	10	6	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	11	2	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	12	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	13	0	Green	-	-	-	-	-	-
	14	1	Green	ACCEPT	Reject	ACCEPT	-	-	-
	15	0	Green	-	-	-	-	-	-
	16	2	Green	-	Reject	ACCEPT	-	-	-
	17	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	18	0	Green	-	-	-	-	-	-
US DOLLAR	1	5	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	2	1	Green	Reject	Reject	ACCEPT	-	-	-

	3	7	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	Reject
	4	4	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	5	3	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	6	1	Green	Reject	Reject	ACCEPT	-	-	-
	7	0	Green	-	-	-	-	-	-
	8	0	Green	-	-	-	-	-	-
	9	18	Yellow	Reject	ACCEPT	ACCEPT	Reject	ACCEPT	-
	10	11	Green	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
	11	4	Green	ACCEPT	Reject	ACCEPT	Reject	-	Reject
	12	5	Green	ACCEPT	Reject	ACCEPT	ACCEPT	-	Reject
	13	0	Green	-	-	-	-	-	-
	14	0	Green	-	-	-	-	-	-
	15	0	Green	-	-	-	-	-	-
	16	2	Green	-	Reject	ACCEPT	-	-	-
	17	3	Green	Reject	Reject	ACCEPT	Reject	-	Reject
	18	0	Gree	-	-	-	-	-	-