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MSc IN ACCOUNTING & FINANCE

Pricing freight rate options

ΓΕΩΡΓΙΟΣ ΑΓΓΕΛΗΣ

Εργασία υποβληθείσα στο

Τμήμα Λογιστικής & Χρηματοοικονομικής

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ως μέρος των απαιτήσεων για την απόκτηση

Μεταπτυχιακού Διπλώματος Ειδίκευσης

Αθήνα

{Νοέμβριος, 2016}

Εγκρίνουμε την εργασία του

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……. ΝΟΕΜΒΡΙΟΥ 2016

ΒΕΒΑΙΩΣΗ ΕΚΠΟΝΗΣΗΣ ΔΙΠΛΩΜΑΤΙΚΗΣ ΕΡΓΑΣΙΑΣ

«Δηλώνω υπεύθυνα ότι η συγκεκριμένη πτυχιακή εργασία για τη λήψη του Μεταπτυχιακού Διπλώματος Ειδίκευσης στη Λογιστική και Χρηματοοικονομική έχει συγγραφεί από εμένα προσωπικά και δεν έχει υποβληθεί ούτε έχει εγκριθεί στο πλαίσιο κάποιου άλλου μεταπτυχιακού ή προπτυχιακού τίτλου σπουδών, στην Ελλάδα ή στο εξωτερικό. Η εργασία αυτή έχοντας εκπονηθεί από εμένα, αντιπροσωπεύει τις προσωπικές μου απόψεις επί του θέματος. Οι πηγές στις οποίες ανέτρεξα για την εκπόνηση της συγκεκριμένης διπλωματικής αναφέρονται στο σύνολό τους, δίνοντας πλήρεις αναφορές στους συγγραφείς, συμπεριλαμβανομένων και των πηγών που ενδεχομένως χρησιμοποιήθηκαν από το διαδίκτυο».

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Contents

Abstract

The purpose of this master thesis is the research of two ways of pricing a freight option. First of all, some illustrations are set that are related to Shipping and help in better understanding of this thesis. Then a literature review is given. This review does not concern only the freight rates but also the freight derivatives and the concept “freight” in general. So, it can be seen the scientific background of this financial field. This master thesis focuses on Koekebakker, Adland and Sødal’s (2007) research and tries to check the validity and the drawbacks of their formula. In the examined paper is set the theoretical framework for the valuation of the Asian options in the freight derivatives market. So, continuing their work and obtaining the formula is assumed that spot freight rates have log-normal distribution. The corresponding future prices are also log-normally distributed prior to the settlement period but not inside this period. Therefore, an approximation for the volatility of the future prices is made. With this way the future prices are now log-normally distributed prior to and inside the settlement period. Next, the Black’s (1976) formula is used and the explicit formula is resulted. After that, Monte Carlo simulations are used to check the validity of this formula. So, one particular option is priced by the explicit formula and Monte Carlo simulations in order to be checked the differences between them. Thereafter, the variance in the Monte Carlo simulations is reduced with the assistance of antithetic and control variates and hereafter the two pricing methods have very small differences. So, it can be concluded that the explicit formula is valid to price freight options. Although, Monte Carlo is an accurate method, preference is now given to the explicit formula as it is much quicker and hence less costly in computational time than Monte Carlo. However, the assumptions that are made maybe differentiate the prices given by the formula in contrast to the real market prices. So, the assumption of mean reverting freight rates is a more realistic assumption than log-normal freight rates.

1. Introduction

The global economy depends on many different factors. The commodities are one of these factors and they affect the economy. It is easy to be noticed that a lot of money is invested in commodities. Not every country has a straight access to these products. That is why they have to be transported from one country to another. Moreover, commodities have to be carried over the entire world. Therefore, the transport by floating transportation is essential for the commodity trading and consequently for the global economy. This transportation by vessels such as tankers, barges, capsize is also called freight. The freight market is a new market, but it has become an increasingly important factor in the global economy Not only organizations have a direct interest in maritime transportation, but also financial institutions which can see great financial opportunities in this market. As this market is relatively new, not many scientific studies have been done in this area.

People and organizations rely greatly on freight as it covers the transport of goods by vessels. Freight needs to be contracted, just like commodities. The only difference between them is that most of the commodities are real products while freight is a service. Therefore, when freight is bought, the service of products is transported from a place to another. When commodities are constructed, the physical product is purchased. As a result of the fact that freight is a service instead of a physical product, freight is also non-storable. This specialty of freight makes the pricing of financial products related to freight more complex. In trading, there is normally someone who owns the product that is traded and someone who wants to buy it. In case of freight, these are called ship-owner and charterer. Between them there is another person or entity, the shipbroker. This specialist negotiator delivers mediation services between ship-owners and charterers. The price for which the ship-owner sells the freight and the charter buys it, is called the freight rate. The freight rate is extremely volatile.

The trade which is transported over sea can roughly be divided into five groups: dry bulk, oil tanker, container, gas tanker and other. Dry bulk commodities can be separated to major and minor bulks. Major bulks form the 2/3 of the dry bulk and therefore plays more important role in the freight market than minor bulks. There are vessels that are specifically suitable for the transportation of dry bulk. These can be subcategorized according to their length. The four main categories are Handysize, Handymax, Panamax and Capesize. The Handysize is the smallest and Capesize the largest.

There are other ways to trade freight. Organizations that are not directly interested in the service of transporting goods can still have a great interest in the freight market. This is because financial products related to freight are provided by the freight market. Different kinds of financial contracts exist, in which are made agreements about paying and receiving a certain amount of money which is related to the freight rates. Because freight rates are very volatile, the trading of freight comes with a lot of price risk. To protect ship-owners and charterers against the risk, the market provides them with financial products related to freight, like freight futures, forwards and options. Ship-owners and charterers use the freight derivatives to hedge their exposures.

Derivatives are mostly used for hedging and speculation. Speculation is the reason why the freight market is so interesting for non-physical players like banks. These organizations try to predict the future freight rates and buy a suitable freight contract with which they expect to make profit. It should be noticed that for freight derivatives, the underlying asset can be the Baltic Freight Index or another index which represents the spot freight rates. The owner of the derivatives receives at expiration the value of the Baltic Freight Index/freight rate minus the price which was fixed at the conclusion of the deal. All freight derivatives consist of more than one settlement period. In theory, firstly, a single settlement period is considered and then a few of these single contracts are summed up to form a contract with more settlement periods as will be seen later when pricing a freight option.

The first freight futures that were traded were introduced on the Baltic International Freight Futures Exchange (BIFFEX) in May 1985. This shows that the financial freight market is indeed new as was stated in the beginning of master thesis. Freight futures are mostly referred to as Forward Freight Agreements (FFA). This might be misleading but nowadays most of the time these FFAs are traded as futures, through a clearing house. The use of the FFAs has been increasing constantly since their introduction in the early 1990s. However, the market grows and becomes more and more mature. So, the participants explore other derivatives more adaptable to their need and especially freight options. The FFAs lack in the flexibility that their users need when the market moves against them. Although Forward Freight Agreements provide reasonable hedging strategies, they do not allow the market participants to enter the market when the conditions are favorable. The options offer this flexibility. Freight options that traded as OTC instruments become more and more popular amongst the practitioners. This kind of derivatives was introduced in the market the same period as the BIFFEX contracts but they did not picked up until their re-introducing in 2002 to the market participants. The freight options are used for both risk management and speculation purposes. So, in this master thesis it is discussed the challenges that practitioners face in pricing freight options and the models and techniques that have been developed for their valuation.

The purpose of this master thesis, therefore, is to be found whether there is an accurate formula that can price the freight options. There are many studies dedicated on this subject but none of them tries to find whether the existed methods and equations are valid and accurate. Furthermore, Koekebakker, Adland and Sødal's (2007) article used for the theoretical framework and with the assistance of Monte Carlo and variates, it is concluded whether their formula is accurate. In chapter 2, it is discussed the literature review of the freight derivatives. In chapter 3, it is set the theoretical framework, the useful formulas and dynamics of freight options. It is derived an explicit formula for the price for the price of a freight option and the price of a particular freight option is calculated using this formula. In chapter 4, lies a numerical example that checks the validity of this formula by calculating the prices of this freight option by using Monte Carlo. In Chapter 5, it is discussed whether the assumptions and approximations that made are realistic and whether the explicit formula can be used for all freight options. Finally, a conclusion is in chapter 6.

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2. Literature Review

First of all, it is essential to be reviewed previous studies about freight and freight derivatives. The research begins with a closer look at the concept freight and the dynamics and risk premia. Then, are reviewed studies about freight futures, Forward Freight Agreements and freight options.

2.1 Freight Rates: Risk, Dynamics and Hedging Ratios

Kavussanos and Nomikos (2000) in their article estimate constant and time-varying hedging ratios. They want to measure the hedging effectiveness of these ratios in reducing the freight rate risk in Baltic Freight Index (BFI). The authors introduce an augmented GARCH model with a specification that links concept of disequilibrium with the concept of uncertainty. The GARCH-X specification show greater risk reduction than a constant hedge ratio or a simple GARCH. Due to the heterogeneous composition of the underlying index, the GARCH-X specification fails to limit the risk of the spot position. So, restructuring the synthesis of the BFI may increase the hedging performance of the futures contract. Therefore, the introduction of the Baltic Panamax Index (BPI) as the underlying asset of BIFFEX may be benefactory for the market. Market agents can benefit from the framework of GARCH-X specification and control more efficiently their freight rate risk.

In the Koekebakker and Adland (2004) article, they specify their model in a Heath-Jarrow-Morton[1] framework in order to research the dynamics of forward freight rate. Even though, this model was developed for interest rate market, it has been applied, in this article, to analyze the ten-year charter rates of a Panamax bulk barrier. The authors use a smoothing algorithm in order to convert their data to a forward rate function. Due to the smooth data, the factors that govern the dynamics of the forward freight rate curve are investigated. In these data of the article, the authors find a unique volatility structure. The correlation between different parts of the term structure is in general low or negative. This is contrary with the properties of the market. The flaw of this article is that the lognormal model that is used might not be satisfactory.

The relationship between the dynamics of the term structure and time-varying

[1] The Heath–Jarrow–Morton (HJM) framework is a general framework to model the evolution of interest rate curve.

volatility of shipping rates is examined in the Alizadeh and Nomikos' (2011) article. They use a database that covers a period of sixteen years and augmented E-GARCH models. They find support for the argument in that the volatility of freight rates is related to the shape of the term structure of the freight market. The dynamic relationship of the spot and longer term contracts is used as a proxy for "backwardation" and "contago". This explains part of volatility and changes in the freight rates. The relationship between the volatility of freight earnings and the slope of the forward curve in the form of a cubic function is not linear and implies the rate of increase in the volatility increases as the degree of forward-curve "backwardation". Finally, the shape of the forward curve is a factor that has explanatory power in determining potential changes in freight rate volatilities.

The dynamics of the spot freight can be characterized by a non-linear stochastic equation. Adland and Cullinane (2006) want with their survey to examine the dynamics of freight rate in the oil transportation market. For this purpose, they use a general non-parametric Markov diffusion model[2]. The results of the research show that the spot freight rate is not very often mean-reverting in empirical range. The volatility of freight rate changes is analogous with the level of the freight rate. The existence of a non-linear mean-reverting drift is sufficient of the process to pull the series back into middle region and determine global stationarity. In this article, it is emphasized the importance of Martingale[3] behavior if the spot rate series over its range in disputing linear reverting models. So, there is a difficulty in explaining the rejecting non-stationarity in short sample. The results imply that spot freight rate process behaves like a Martingale over its empirical range.

The forward price is not connected with the spot by any arbitrage relationship. Batchelor, Alizadeh and Visvikis (2007) test the performance of time series models in predicting spot and forward rates on the largest in scale freight routes. The forward market is dominated by hedgers. So, this survey helps these hedgers understand whether the forward rates contain information for future spot rates. If the forward market is liquid enough to contain this information, it may be observed cointegration between spot and forward rates and convergence if spot towards forward rates. Spot and forward freight rates are indeed cointegrated. VECM (vector equilibrium

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[2] A diffusion process is a solution to a stochastic differential equation. It is a continuous-time Markov process with almost surely continuous sample paths

[3]This theorem is given in subsection 3.2

correction model) gives the best sample fit. In the out of sample forecast, all samples outperform random walk. The forward rates help the forecasting of the spot rates, too. But, in forecasting forward rates the VECM is not the right model. ARIMA and VAR models are better for this job. For ship-owners and charterers, the findings of the survey are encouraging, but for analysts of commodity markets, the message is more cautionary.

Tezuka, Ishii and Ishizaka (2012) examine the non-storability of the shipping freight. This characteristic creates an enormous gap between widely used no arbitrage theory and shipping freight derivatives market. With this paper they extend the Tezuka's (2006) model in order to introduce speculators into the markets. With this extension, the assumption that market participants form a homogeneous group is relaxed. The authors obtained optimal forward hedge ratios that correspond to the reduced spot freight rate risk, from a forward curve formula. Thereafter, they present a numerical example using actual shipping market data to show the practical applicability of their model. Their main achievement, with this paper, is that the modification and generalization of Bessembinder and Lemmon (2002) model. The formulas that are used to find the equilibrium and forward prices are obtained in the market when ship-owners and charterers are non-homogeneous. From this formula, are also derived the properties of the forward risk premium and an optional hedge ratio.

Alizadeh Huang and van Dellen (2015) want with their paper to examine the performance of some instruments in managing tanker freight rate risk. Tanker shipping is very important for the transportation of petroleum and petroleum products around the world. However, there is major concern about the high volatility of tanker freight rates. This led to the development of tanker freight derivatives in the form of FFAs. The authors use data for six major tanker routes and investigate the effectiveness of other hedging methods in hedging tanker freight rates. They include to their method a bivariate Markov Regime Switching (MRS) GARCH to determine the hedge ratio. The proposed model allows for the dynamics of the mean and variance of the FFA and spot rates to evolve under different regimes. The dynamic BEKK-GARCH and MRS-GARCH hedge ratio approaches support the author's proposition that the MRS-GARCH is the most appropriate specification. Finally, they provide the reasons why there is poor performance in tanker FFAs. The residual risk cannot be efficiently hedged while the underlying asset is non-storable and the link between spot and FFA markets is not as strong as in other financial markets. The final explanation is that the lack of liquidity in the FFA market can result in illiquidity premium in FFA prices.

Tsouknidis (2016), with his survey, investigates the existence of dynamic volatility spillovers within and between the dry bulk and tanker freight markets. For this purpose, he employs the multivariate DCC-GARCH model. The volatility spillover index is developed by Diebold and Yilmaz (2009). Net and net pair wise spillovers over time are computed between different segments of shipping markets. During the global financial crisis, the existence of volatility spillover effects is more intense. The methology used by the authors is invariant to ordering the variables when estimating a VAR and allows the net and net pair wise that has been mentioned. This paper shows that smaller vessels transmit volatility spillovers to larger vessels within the dry bulk segment. Finally, large volatility spillovers are detected between tanker sub-segments and dry bulk segments for short periods.

2.2 Freight Futures

Kavussanos and Nomikos (1999) in their article investigate whether the freight future prices are unbiased. This is done by cointegration techniques. It is indicated whether the futures prices one and two months before maturity are forecasts with unbiasedness of the realized spot prices. So, the authors suggest if the unbiasedness of futures prices in freight market depends on the maturity of the contract and the eccentricity of the market. Futures prices are more accurate forecasts of the realized spot prices than the forecasts by Error Correction, ARIMA, exponential smoothing (Holt-Winters) and random walk models. Several studies have investigated this hypothesis but this is the first in a futures market with low trading activity. The authors, with cointegrating the relationship between spot and futures prices, can prove that these prices are unbiased forecasts of the realized spot prices, while futures prices three months before maturity are biased forecasts of the realized spot prices. Finally, the charterers or the ship-owners can use the information generated by the futures prices in order to make market decisions. The hedger can use these forecasts from the market without paying any risk premium.

Heigh (2000), in his article, examines the relationship between freight cash and futures prices using cointegration econometrics. The BIFFEX futures market is unbiased and efficient for the current, one-month, two-month and quarterly contracts. With these cointegrating techniques used, it is tested whether it can be seen the stability in the relationship between spot and futures rates. The futures prices respond to perturbations in the long run equilibrium. This is pleasing as it is more difficult to trade each of the routes comprising the index than it is to trade the BIFFEX futures. To test the stability of cointegrating vector between the current month futures and the spot price, the researcher uses an error-correction Model. The futures contracts appear to become more and more efficient over time. It is not the lack of efficiency that causes the decrease in trading volume in the BIFFEX market. More distant contracts are shown to be explained best by VAR in first differences. So, time series models outperform the futures contract at longer contract horizons, while the futures prices are the best predictor o future spot rates for the current month contract.

Kavussanos and Nomikos (2000) want with their article to find the hedging effectiveness of the futures contracts whose underlying asset is an index, when the structure of this index is changing. So, the best example for this is the BIFFEX for the case of freight futures. The underlying asset for this investigation is the Baltic Freight Index (BFI), which has been used in many surveys for improving the hedge performance of the market. The hedging effectiveness is investigated by using VECM-GARCH models for constant and time-varying ratios. The effectiveness of BIFFEX has strengthened through the years. The authors find that there is no change in the hedging performance of BIFFEX on any route, but there is an increase in the performance of route 1. The reduction of the weight of a route in the BFI can give a strong performance in this route.

Haigh and Holt (2002), in their article, estimate the foreign exchange hedging with freight and commodity ratios in a time-varying portfolio framework. The results of this survey support the decision of London International Financial Futures Exchange, to stop the trading of the BIFFEX freight futures contracts because of their low trading activity. The authors slackened the assumption that cash distributed in order to research the connection of freight, commodity and foreign exchange prices. The BIFFEX contract is not an essential hedging instrument when evaluated alongside other contracts for traders. However, neither the commodity futures contracts are important enough, but they have a vital role in risk management scenarios. Unlike other contracts, the BIFFEX futures contracts are the only used for hedging international trade. Finally, the results of the study show that the foreign exchange contracts should not be omitted from trader’s portfolio as they are very useful in terms of risk management.

Kavussanos and Nomikos (2003) examine the relationship between futures and spot prices in the freight futures market. In the literature has not been investigated a traded market whose underlying asset is a service so far. In this survey, it can be shown that spot and futures prices stand in a long-run relationship between them. Therefore, the authors use VECM to examine the short-run dynamics and price movements in the two markets. Causality tests signify that futures prices tend to find new information faster than spot prices. Impulse response analysis indicates the same results for futures and spot prices. The information can be used to generate more accurate forecasts of the spot prices. The role of the futures prices has strengthened in the period after the exclusion of the Handysize routes from the BFI. The information inherited in futures prices when formulated as a VECM, which produces more accurate forecasts of the spot prices than VAR, ARIMA and random-walk models over several steps ahead. Therefore, it seems that the market can benefit from using the information contained in futures prices. The futures prices should be used as guidance for the ship-chartering decisions.

Prokopczuk (2011) examines the pricing and hedging of single route dry bulk freight futures contracts, traded on International Maritime Exchange. In contrast, to other commodity markets, freight services cannot be valued by a simple cost of carry easily. This is happening because of their non-storability. Therefore, the author suggests four different continuous time no arbitrage pricing models. These models are tested and compared with respect to their pricing accuracy and hedging effectiveness. The results of the research show the inclusion of a second stochastic factor improves the pricing and hedging accuracy. This is in accordance with the Swartz and Smith (2000) two-factor model, which is the most efficient for the pricing accuracy.

Goulas and Skiadopoulos (2012), for the first time in the shipping research history investigate whether the International Maritime Exchange (IMAREX) is efficient over the daily and weekly horizons. They employ an extensive dataset of freight futures prices, in order it give the answer in above question in both statistical and economic setting. For the statistical setting, they use alternative models for point and interval forecasts. With some statistical tests, they evaluate those that take into account the transaction costs and trading strategies. So, they esteem the economic significance of the results. The daily evolution of the IMAREX freight futures prices can be predicted and this predictability is evident in the case of point forecasts. Furthermore, the futures trading strategies on daily forecast yield a positive risk premium. These results are economically and statistically worse for weekly horizons. Finally, they conclude that IMAREX futures market is not efficient for short horizons. The unbiasedness hypothesis does not hold for IMAREX. After that, they suggest that an equilibrium pricing model has to be adopted for the valuation of the freight derivatives, assuming a mean reverting process with time-varying volatility for the underlying freight index.

2.3 Forward Freight Agreements (FFA)

The uptake of tanker Forward Freight Agreements has been attributed to traditional risk properties amongst tanker owners since freight –hedging tools were introduced in paper to counter volatile tanker freight rates. The tanker FFAs are hybrid swap type derivatives, traded OTC, suited to a volatile freight marker reluctant to embrace such risk management. Dinwoodie and Morris (2003) research the risk and perception of FFAs in order to test how generic determinants and incentives for corporate hedging can explain the hesitation in the tanker market. The FFAs are seen as a substantial development but some hedgers or market participants are unaware of their fiction or they have not even used them. Finally, they suggest that technical education is important to widespread the acceptance of the FFAs. This education and the market liquidity make the FFAs more accessible and known to market participants.

Kavussanos, Visvikis and Menachof (2004) examine the unbiasedness hypothesis of FFA prices in the OTC market trades. They employ cointegration techniques to investigate this hypothesis. The FFAs after the abandonment of BIFFEX contracts are the major derivative contract for hedging freight rates and risk in the ocean bulk shipping industry. The efficiency of FFA prices in providing unbiased predictions of spot prices gives mixed evidence. The results of the research show that the FFA prices one and two months before maturity are unbiased predictors of the spot freight rates. Three months before maturity, FFA prices for Panamax Pacific routes are unbiased predictors of spot prices and the FFA prices for Panamax Atlantic routes are biased predictors. The unbiasedness found in the results is in line with those found for one and two month commodity and foreign exchange forward prices in other surveys. The rejection of unbiasedness for the three-month FFA prices is not in line with the results of previous researches. This evidence suggests that the validity of the unbiasedness hypothesis depends on the specific characteristics of the market, the route and the maturity of the contract.

Kavussanos, Visvikis and Batchelor (2004) want with their survey to investigate how the spot market price volatility on four Panamax trading routes of the dry-bulk shipping industry is affected by the introduction of Forward Freight Agreements (FFAs). This kind of derivatives with its establishment in the market, contributes to the decrease of the spot price volatility in the Panamax dry-bulk routes. It has an impact on the asymmetry of volatility, too. In three of the four investigated routes, it is observed that the quality and the speed of the information have improved. So, the introduction of the FFA trading has a positive impact on the spot market. Furthermore, even those agents who do not directly use the spot market, have been benefited by this introduction.

Kavussanos and Visvikis (2004), in their article, investigate something that not many studies have accomplished because of the difficult access to empirical data. The most of the studies research the lead lag relationship in returns and volatilities between spot and futures market. The authors investigate some fields of shipping in forward markets due to the over-the-counter (OTC) Forward Freight Agreements (FFAs). The transaction costs are higher in spot market than Forward market, since the underlying commodity is non-storable. So, the authors suggest that the better understanding of the mean and variance dynamics can improve risk management. The results of this survey are comparably with the results in most futures market. The lower the transaction costs in the OTC FFA market are, the faster the information is discovered in this market, compared to the spot markets. These results on information transmission in the returns hold in volatilities, too.

Kavussanos and Visvikis (2006), in their survey, review the shipping derivatives as a hedging way for freight rates. Firstly, they analyze the Forward Freight Agreements (FFA) and the futures as they refer to many researches on the case of derivatives, their forecasting performance, which is examined with ARIMA, VAR and VECM in these surveys. Then, they refer to some other investigations for these kinds of derivatives, which have as a theme of examination the pricing of these derivatives, the importance of hedge ratios and their relationship with volatility. The FFA trading for the first time surpassed the turnover of the actual freight transactions. The high volatility of the freight rates is more attractive to non-shipping hedgers who want to be involved in shipping industry. Finally, they analyze freight options and a way of to price them as in that time they was any kind of model for this. They conclude that FFA trading is more trending as the years pass and that shows the maturity of the market. More and more, the derivatives can be used for hedging the freight rates but caution should be exercised as the sums involved are large and the exposure leads to failure. Shipping becomes more and more familiar with risk management and ship-owners and charterers see a need to safeguard their profit.

Alizadeh (2013) examines the price volatility and trading volume relationship in the FFA market for dry bulk ships. The results of the survey show the existence of a positive relationship between price change and trading activity in the FFA market for dry bulk vessels. There is also evidence of a cotemporaneous and positive relationship between trading volume and volatility. This is in accordance with the evidence from financial market and Mixture of Distribution Theory[4]. FFA price changes Granger-cause trading volume in the FFA market for Capesize and Panamax vessels. Trading volume seems to Granger-cause prices in Supramax market, too. Finally, the author concludes that increases in price volatility lead to lower future trading activities in the FFA market.

Quantitative timing strategies have been tested in liquid commodity and financial futures with respect to their performance. Nomikos and Doctor (2013), in their study, extend this methology to freight, a non-storable commodity. Freight futures market is a good opportunity for diversification and trading options as this market is mature and more and more liquid. The authors focus their research in quantitative strategies in the FFA market. The first result of this article shows that trading rules outperform the buy and hold benchmark, although the robustness checks in the market. There is also a possibility that illiquidity may impact the results of the test. So, the authors suggest an innovating approach to mitigate the effects. The Hansen (2005) SPA methology is proposed in order to allow the researchers to use smaller samples with more confidence. In this study, is shown how technical trading rules are applied to FFA contracts. The results of the study show that employing active trading strategies reduces risk and improves Sharpe ratios over buy and hold. So, the opportunities for profitable trades using trading strategies gradually diminish.

Alizadeh, Kappou, Tsouknidis and Visvikis (2014), in their survey, examine the existence of liquidity risk premia on freight derivatives returns. So, the authors, in

[4] A mixture distribution is the probability distribution of a random variable that is derived from a collection of other random variables as follows: first, a random variable is selected by chance from the collection according to given probabilities of selection, and then the value of the selected random variable is realized

order to price the liquidity in the dry bulk FFA market, use FFAs and Amihud liquidity measure. With this way, they utilize these measures together with other industry specific and macroeconomic variables, in order to assess the existence of liquidity premium and to determine the importance of liquidity risk in the freight derivatives market. The liquidity measures are found to have positive and meaningful effects on the returns of near-month freight derivatives contracts. This is important evidence of demand and supply driven market. The researchers of this article try to understand the determinants of the forward premium. They test the liquidity measures to find if they have any power in determining the spread between FFA prices and their expected settlement values. The results show that the Amihud measure has no relationship with forward premium. However, bid-ask spreads positively affect them. The results of this article have very significant implications for modeling freight derivatives returns, for trading and for risk management purposes.

Kavussanos, Visvikis and Dimitrakopoulos (2014) examine the economic spillovers between derivatives and their underlying markets. With this survey, it is investigated the return and volatility spillover effects between different but related ocean freight and commodity futures market. This economic relationship tested links of the derivative price of the commodity transported with the derivative price on the freight rate. Wheat, corn and soybeans are important commodity futures markets to monitor in order to understand what may occur in the dry bulk FFA market. So, agricultural commodity futures lead the freight market with the help of the information gained by monitoring. The results of the study can assist the better understanding of the information transmission mechanisms between freight and commodity derivatives. They can, also, help the precise pricing of these derivatives in which the theory of storage does not hold. These results have very essential role to ship-owners, charterers and commodity markers as these can be used to enter into more effective investment, hedging, chartering and trading decisions.

2.4 Freight Options

Black (1976), in his survey, using some assumptions, constructs some formulas about the pricing of forward contracts and commodity options. He aims with that paper to clarify the meaning of the above derivatives. Firstly, the author gives some simple examples in order to make the reader understand the forward and the future contracts.

Then, he makes clear the differences between future and forward contracts and commodity options. Thereafter, he converts the forward in terms of future contract. Specifically, the value of the forward contract is the futures price minus the spot price. A commodity option differs from a forward because the holder can choose whether or not wants to buy the commodity while in the case of the forward contract he has no choice. After that, the author using CAPM that applies at each time and making assumptions about taxes, transaction costs and that the Asset he uses has no dividends, he shows the readers the changes in the futures price. Finally, he continues with the pricing of commodity options and forward contracts. He makes assumptions that the futures price over any interval distributed log-normally, the parameters of CAPM are constant and taxes and transaction costs are zero. Using the same procedure that leads to the formula for an option on security the author concludes that taxes may reduce the values of commodity options.

Kemna and Vorst (1990) present a new strategy for pricing average value options. Monte Carlo simulation with variance reduction elements offers enhancement in pricing method to arbitrageurs and hedgers. Firstly, they make clear what it is an AV option. Then, the authors assume that a perfect security market is open continuously and that the underlying asset is equal to a price S(t), for which give a formula. In order to determine the value of the option, they calculate the value in time interval with the help of Black-Scholes (1973) arguments and Merton’s (1973) extension. Using Ito’s formula they set three conditions about the nature of an AV option contract. Thereafter, they assume that there is no arbitrage condition and they use boundary conditions in order to find the price of AV-option in the final period. After that, they present the special characteristics of AV- options and why the bondholders prefer these to European call options. Then, because their main equation has three variables they propose Monte Carlo simulation to find a value for these variables. In that simulation, for 10000 times a value of a single realization is calculated. With the help of Cox and Rubinstein (1986) they compare AV-option with the standard European option. The standard deviation of a Monte Carlo widens when the volatility of the stock is more pronounced and where the gap between the exercise price and the stock price increases. Finally, with the use of a geometric average and not the simply one, they find an expression to value the options in the final time interval and over the total time period.

Alziary, Decamps and Koehl (1997), in their article, derive a one-state variable partial differential equation that describes the price of a European type Asian option. They derive new results on the hedging of an Asian option and do a numerical implementation of their methology. They compare with this way the Asian with the European options and they separate the Asian to "fixed-strike" and to "floating-strike" Asian options. They focus on the "back-starting" ones. The price of such an option is characterized by a one-state variable partial differential equation. They extend with this research the analytical tools of Geman and Yor (1993), as they adopt the explicit finite difference method and propose numerical comparison of the deltas and elasticity between European and Asian options.

Tvedt (1998), in his survey, uses Black's formula (1976) and Black-Scholes (1973) in order to price the traded futures options in the BIFFEX market. Firstly, he tries to make the reader understand better the BIFFEX and makes some assumptions that commodity prices follow an Ornstein- Uhlenbech process and that the BFI should have mean reversion properties and for that reason he uses MRA process. The futures price process is given by the expectation of the spot process at the time of settlement under original probability measure. As the futures contract approaches the settlement date, the volatility of futures price is identical to the volatility of spot rate. Thereafter, the author, in accordance with Black's formulas, measures the futures option call. This differs from Black's formulas in that the mean reversion property of the freight rate index is taken into account by letting the variance of the futures process be time dependent.

Zhang (2001) examines a new analytical formula for pricing and hedging "arithmetic average rate" options. A P.D.E. with smooth coefficients and zero initial condition rules the correction to the analytical approximate formula. The P.D.E. is enabled to be evaluated by a numerical method. The error by this semi-analytical method is as high as 10^{-7} for the grid size adopted in the paper. The CPU time taken for one round of computation is only one second for short-tenor options and twenty-two second for long-tenor options. The author completes all the computation by using 150 lines of C^{++} programming. This study presents a method more accurate than any other PDE method with error well controlled. The results can be used as a reference point to justify the error computed by other approximation methods.

Koekebakker, Adland and Sødal (2007), set up the theoretical framework for valuation Asian options traded in the freight market. They extend the Black's analysis (1976) as they consider forward contracts settled against the arithmetic average rate of the underlying asset. Firstly, they discuss the structure of freight options as their arithmetic average based settlement procedure is inherited from the Forward Freight Agreement (FFA) market. Subsequently, the authors present the theoretical pricing framework. Since it costs nothing to enter into an FFA, they set its expected value equal to zero. Then, in accordance with the Black's assumption they set the spot and FFA dynamics of the model. In the case of FFA, they use lognormal dynamics of the underlying asset. As an Asian option can be interpreted as a European option on the forward contract they can construct the models for Asian call (cap) and put (floor) options. Finally, they use Monte Carlo simulation to establish the level of accuracy of the proposed volatility. For this case they use the equations they had set for their models and not observed FFA prices. They conclude that log-normality of FFAs breaks down in the settlement period and they give models that can be extended for further research of pricing the freight options.

Tsai, Saphores and Regan (2011), in their article, apply concepts from the theory of Real Options in order to hedge transportation capacity and cost using truckload options. With their formula they can price the European call and put options, which is essential condition for the implementation of truckload options. The authors contribute with this article as they construct a pricing formula for the basic truckload options when the spot price follows a mean-reverting process. They propose the Ornstein-Unlenbeck[5] process to model the spot prices and with maximum likelihood methods they estimate the parameters of the model. Thereafter, it is provided an estimation of the parameters needed to value the truckload options. In this article, it can be proven that truck load options are valuable to both shippers and carriers, s it is accomplished a numerical illustration based on real data. Finally, it is mentioned that the necessary conditions for the well-functioned data of truckload options, include market liquidity, smart regulations and hedging effectiveness.

Nomikos, Kyriakou, Papapostolou and Pouliasis (2013) investigate an extension of lognormal representation for risk neutral freight rate dynamics. In this investigation, firstly, they divide the Asian option in three categories depending on the averaging

[5] The Ornstein–Uhlenbeck process is a stochastic process that, roughly speaking, describes the velocity of a massive Brownian particle under the influence of friction.

period in which exist variants. The authors consider the traded forward start options (one of the categories and most commonly traded) on spot Baltic Capesize Index (BCI), Baltic Panamax Index (BPI), and Baltic Supramax Index (BSI). In this research, they demand a lognormal process overlaid with time homogeneous Poisson jumps of normally distributed sizes (Merton, 1976) for the risk neutral spot freight dynamics. Then, they compare its performance with the basic lognormal diffusion. The authors use an algorithm from a Theorem they construct to bring into line the risk neutral spot rate model. For the Merton model they minimize the quadratic pricing error (Andersen and Andrearsen, 2000) (Bates, 1996) at each date in their option data. Their period of data begins in January 2008 and ends in July 2010. Finally, they conclude that the presence of jump terms can describe extreme movements. These jumps are important in long-term contract pricing. Mainly, these are upward jumps with the exception of the period of recession. Jump diffusion generates lower error than lognormal model and reduces the level of over-pricing an under-pricing.

3. Theoretical framework

Freight options are becoming increasingly popular amongst practitioners and are used for both risk management and speculation purposes. There are many challenges, practitioners face in pricing freight options. So, in this section we discuss these challenges, the models and techniques that have been developed for freight option valuation.

The following methods can be used to price options: “analytical” or “closed-form” solutions, Monte-Carlo simulations and tree-building methology. Firstly, the closed-form solution is the solution to a differential equation that expresses the change in the option value relative to all the key variables which affect its value. Examples of closed-form solution include the Black-Scholes (1973), Merton (1973) and Black (1976) models. The advantage of this method is its adaptability and the fact that it is easy to use and quick to give the option value. However, the more complicated the underlying process, the more complicated is the type of the option to be priced. The second approach is Monte Carlo simulation. This involves simulating the underlying market variables and calculating the expected the option payoff at the expiry of the contract. In this approach, it is performed a large number of simulations in order to be obtained an empirical distribution of the option payoffs. Then, the average price of these is discounted back to be obtained the present value of the option. Monte Carlo is that it can accommodate effortlessly path-dependent options. On the other hand, Monte Carlo is a computer method and hence it is not practical to be used in real life trading simulation. The third strategy is for options pricing is the building of underlying trees. In this method, at each time-step, there is probability of the asset moving up by a certain amount and a probability of it moving down by another amount. Tree methods are used in evaluation of real options, where there are many early exercise features. However, trees cannot accommodate easily path-dependant options such as average price freight options. The topic of option pricing is vast and there are numerous papers on the subject. So, in this section we will discuss the theoretical framework and both closed form solutions and Monte Carlo simulation for pricing freight options.

3.1 Challenges for researchers

The underlying asset of the option, which is the freight rate produced by Baltic Exchange Freight rate is not a tradable asset. Freight rates reflect the cost of providing the service of seaborne transportation which, by its own nature cannot be stored or carried forward in time. So, arbitrage between the underlying spot freight market and options across time and space is limited. Thereafter, freight options are settled as average price Asian option. There are two basic styles of Asian options, the “average price options” and the “average strike options”. Both arithmetic and geometric averages can be used in the calculation of settlement rates, with arithmetic being more popular.

The options used on FFA markets are arithmetic average price options. Asian Options are popular in either thinly traded markets or in markets in which there is high volatility. The freight market is a market with that description. The volatility of the average price is less than the volatility of the underlying and the premium for Asian options will be lower than the premium for European options. For the geometric average options Kemna and Vorst (1990) construct a closed-form solution for calculating their price, but these options rarely are traded in commodity market and are not traded in the freight market. Hence this kind of options is not discussed in this master thesis. Regarding the more popular arithmetic average price options, it is very difficult to develop a closed-form solution for their pricing. When the asset is assumed to be log-normally distributed, the arithmetic average of a set of lognormal distributions does not here analytically tradable properties. Alternatively, arithmetic-average options can be priced using Monte Carlo simulation.

In this master thesis, it is revised the theoretical framework for the valuation of Asian options traded in the freight derivatives market. The dynamics, set by Koekebakker, Adland and Sødal (2007), in the settlement period lead to closed-form Asian option pricing formulas for call and put freight derivatives market. Then, with the assistance of Matlab, is executed a numerical experiment with Monte Carlo that shows if the formulas give accurate prices. The options that are examined are arithmetic average price options.

3.2 Assumptions

First of all some assumptions have to be made:

1. At each time, t a spot freight exists and denoted by S (t). This is a non-tradable asset.

2. The settlement is denoted by $[T_1,T_N]$, with $T_1<T_N$. There are N fixing at time points T_n, n=1,2,3,…,N, so that one settlement period consists only the working days and not holidays and weekends.
3. All the FFAs that are considered in this paper are traded through a clearing house and not over the counter (OTC). So, credit risk is not taken into account.
4. The FFA market is liquid, so the FFA trading can be continuous until the end of the settlement period T_N.
5. The market participants can borrow or lend money at the same constant, deterministic and continuously compounded interest rate r with no transaction costs.
6. The price of an FFA at time t, is $F(t,T_1,T_N)$, that is a Martingale under real world measures P. The price of an FFA depends on the current time t and omits the length of the settlement period.

   The Martingale (probability theory) says:

   "A basic definition of a discrete time Martingale is a discrete time stochastic process X_1, X_2,X_3,… that satisfies for any time n:

   a) $E(|X_n|) < \infty$
   b) $E(X_{n+1}|X1,\ldots,X_n)=X_n$"

   So, the expected price of the FFA in the time n+1 equals the price at time n, for all n prices.
7. There are no arbitrage probabilities in the freight market. The first Fundamental theorem of Asset Pricing says that:

   "A discrete market, on a discrete probability space (Ω,F,Q,) is arbitrage free if and only if there exists at least one risk neutral probability measure that is equivalent to the original probability measure, Q"

   From this theorem can be concluded that exists a risk neutral probability measure Q such as the price of FFA at time t is a Martingale under the risk neutral probability measure Q and the real world probability measure P.
8. The spot freight rates are assumed to be log-normally distributed with drift m and volatility σ.
9. The payoff of an FFA starting at time t equals the difference between the average freight rates during the settlement and the price $F(t,T_1,T_N)$. This

difference is multiplied by a constant D, which is the number of days the FFA contract covers or an agreed cargo size. So, the payoff of the futures equals:

$$D(\frac{1}{N}\sum_{i=1}^{N} S(T_i) - F(t, T_1, T_N))$$

3.3 The Relationship of freight rates and FFA prices

Firstly, it is given the explanation needed for the terms real world measure and risk neutral measure. Financial assets entail risk and it is not known how the underlying asset of an option will behave in the future. To cover this risk, investors require risk premiums. These are different for each kind of assets and each investor as not every investor requires the same amount of risk protection. The risk neutral has been constructed, to be simplified the pricing of financial assets. So, the financial assets are assumed to be riskless and the investors do not require premiums. However, the real world measure can be used to calibrate risk neutral asset pricing model by considering historical data. Under the risk neutral probabilities, the assets are assumed to be risk free, therefore the discounted expected payoff of an asset should be equal to its price. If the price is lower than the discounted expected payoff, definite profit will be made. So, a riskless profit, higher than the amount of money made by putting an amount of money equal to the asset price on savings account with the risk-free interest rate can be made by investors. Therefore, in this investigation is assumed that that arbitrage does not exist.

Continuing after the Assumption 9, the S and F are measured in \$/day or \$/tone. Since it costs nothing to enter into an FFA (no up-front payment), we can set the expected value of a FFA equal to zero:

$$0 = \mathrm{E}_t^Q[e^{-r(T_N - t)} D \sum_{i=1}^{N} \left(\frac{S(T_i)}{N} - F(t, T_1, T_N)\right)] \qquad (1)$$

If we rearrange and solve for the FFA price we find that it is simply the expected average spot price under the pricing measure and as we see in Appendix A.1:

$$F(t, T_1, T_N) = \frac{1}{N}\sum_{n=1}^{\infty} \left(\mathrm{E}_t^Q[S(T_i)]\right) \qquad (2)$$

To avoid confusion, from now and on the contract price of an FFA, F, will be called the price of the FFA. The value of the FFA and its discounted price will simply be called the value of the FFA. When tome goes by the value of futures contract may change. This is because the underlying index changes in time. In this case, a profit or a loss will be made by the holder of the future, since the contract price to be paid will be less or more than the settlement price.

3.4 Spot and FFA dynamics

When deriving an analytic expression for the freight option price, an approximation for the volatility of the futures prices is needed. Compared Monte Carlo simulations, the analytical expression can be faster but less reliable way of pricing a freight option. This approximation for the volatility of the futures prices is made and it is not exactly equal the real volatility. Therefore, a model for the underlying FFA prices has to be derived. After this, technical and mathematical aspects will be discussed. After making an approximation for the volatility of the futures prices, an explicit solution for the price of the freight options will be derived.

As mentioned, it is desirable to treat the freight option as a European option on the futures price, for which an explicit pricing formula can be found. The goal is to be found an analytical expression for $F(t,T_1,T_N)$. This can be done on a stochastic differential equation for the spot freight rates S(t). We need a definition in order to continue to the process:

“If X(t) is a lognormal random variable with drift μ and volatility σ, X(t) is the solution of the following stochastic differential equation

$$dX(t) = \mu X(t)dt + \sigma X(t)dW(t)$$

Where W denotes a Wiener process”

So, including Assumption 8 in this definition, spot freight rates are assumed to be a solution of the following

$$dS(t) = \mu S(t)dt + \sigma S(t)dW^P(t) \tag{3}$$

The spot freight dynamics are given by the Geometric Brownian Motion.

The ds indicates an increment of S, as S(d+dt) –S(t). This is valid to dt and dW. Therefore, $dt=t_{i+1}-t_i$ and dW=W(t+dt) –W(t). The “P” indicates that this equation is under real world probability measure. Here is important to be quoted the Wiener process.

“A Wiener process W is a stochastic process with the following properties:

1. W(0)=0
2. W(t) does not depend upon W(t-1)

3. The increments W(t+dt) – W(t) follow a normal distribution with mean zero and variance (t+dt) –t=dt, for each t.

More formally, W(t) –W(s)∼ N(0,t-s), ∀ 0≤s≤t”

Therefore, because of the last property of the Wiener process, it can be set:

$$dW \sim Y$$

where Y∼N(0,dt), with dt the length of two time steps t_i and t_{i+1} and also:

$$dW \sim \sqrt{dt}Z^P$$

as N(0,k)∼ $\sqrt{k}$N(0,1) and where Z∼N(0,1)

So, the equation 3 can be written as:

$$\frac{dS(t)}{S(t)} = \mu dt + \sigma\sqrt{dt}Z^P \tag{4}$$

The μ is the drift if the freight rates and σ the volatility. The μ can be seen as the expected price of the freight rates. When μ is positive the freight rates tend to increase while a decrease in the freight rates shows that the μ is negative. The σ cannot be negative as the volatility is never negative. The larger the σ is, the bigger random movements tend to have the freight rates.

From Koekebakker, Adland and Sødal’s (2007) Appendix of their paper, it can be seen that:

$$S(t) = S(0)e^{\left(\mu-\frac{\sigma^2}{2}\right)t+\sigma\sqrt{t}Z^P} \tag{5}$$

S(0) is the freight rate at time 0, t≥0 and Z∼N(0,1).

The time t in the exponent of the equation 5 is obtained from t –0 =t. However, the desired expression for S is $S(T_i)$ and now equation 5 can be (Appendix 1):

$$S(T_i) = S(t)e^{\left(\mu-\frac{\sigma^2}{2}\right)(T_i-t)+\sigma\sqrt{T_i-t}Z^P} \tag{6}$$

$t \leq T_i$ and T_i is a fixed point in the settlement period.

This equation for $S(T_i)$ is a stochastic equation for freight rates under real world measure. Therefore, a similar equation has to be found as the real world probabilities are not known. In order to be found this equation, firstly, we need a version of Girsanov theorem that can be used here.

“Let W be a Brownian motion ender probability measure P. We define $W^*(t)=W(t)+\int_0^t \gamma(s)ds$ or d $W^*(t)=dW(t)+\gamma(t)dt$, where γ is the so called market price of risk. Then W^* is a Brownian motion under risk neutral probability measure.”

So, if we substitute $dW(t)=d\,W^{*}(t)-\gamma(t)dt$ in the equation 3 we have:

$$dS(t)=\lambda S(t)dt+\sigma S(t)dW^{Q}(t) \qquad (7)$$

or

$$dS(t)=\lambda S(t)dt+\sigma S(t)\sqrt{dt}\,Z^{Q}(t) \qquad (8)$$

where $\lambda=\mu-\sigma\gamma$.

The "γ", here, is a real valued function. This function depends on time and is called market price of risk. When γ is large, the market requires a high return to be compensated for the risk. So, the γ can be named as a return that is desired as compensation for taking risk. If the spot freight rates were tradable, λ would be equal to the risk free rate r. if the risk free rate were lower than the return, everyone would invest in the asset, since the return always yields more than when putting your money on the bank. If the return were less than the risk free rate, nobody would invest in the asset. So, for non-tradable assets we cannot set $\lambda=r$ but $\lambda=\mu-\sigma\gamma$ has to be used. Setting $\gamma=0$ means that the market does not require compensation for the risk it faces. Therefore, the stochastic differential equation for the freight rates is the same either under risk neutral probability or real world probability measure. So, with the same procedure equation 4 obtained and with the assistance of equation 8, we conclude that:

$$S(T_i)=S(t)e^{\left(\lambda-\frac{\sigma^2}{2}\right)(T_i-t)+\sigma\sqrt{T_i-t}Z^{Q}} \qquad (9)$$

Where $t\leq T_i$, T_i is a fixed point in the settlement period.

So, now we have the solution for $S(T_i)$ in which the expectations in equation 2 can be determined. We substitute equation 9 in equation 2:

$$F(t,T_1,T_N)=\frac{1}{N}\sum_{n=1}^{\infty}\left(E_t^{Q}\left[S(t)e^{\left(\lambda-\frac{\sigma^2}{2}\right)(T_i-t)+\sigma\sqrt{T_i-t}Z^{Q}}\right]\right) \qquad (10)$$

With Appendix A.2 we can simplify the equation 10:

$$F(t,T_1,T_N)=S(t)\frac{e^{\lambda(T_N-t)}}{N}\frac{e^{-\lambda N\Delta}-1}{e^{-\lambda\Delta}-1} \qquad (11)$$

The distance $\frac{T_N-T_1}{N-1}$ is denoted by Δ.

3.5 Volatility of futures price

The stochastic differential equations for the futures price are used to be obtained an expression for the volatility of the futures prices. First of all, the Itô's Lemma is needed.

"Let X(t) be a solution of the stochastic differential equation:

$$dX(t) = a(X(t),t)dt + b(X(t),t)dW(t)$$

Then f(X(t),t) is the solution of the SDE below:

$$d\mathrm{f}(\mathrm{X(t),t}) = \left(a(\mathrm{X(t),t})\frac{\partial \mathrm{f}}{\partial \mathrm{X}} + \frac{\partial \mathrm{f}}{\partial \mathrm{t}} + \frac{1}{2}\mathrm{b(X(t),t)}^2\frac{\partial^2 \mathrm{f}}{\partial \mathrm{X}^2}\right)\mathrm{dt} + \mathrm{b(X(t),t)}\frac{\partial \mathrm{f}}{\partial \mathrm{X}}\mathrm{dW(t)}."$$

Using the equation 2 and the above theorem the following dynamics for the futures prices can be found:

$$\frac{dF(t,T_1,T_N)}{F(t,T_1,T_N)} = \sigma(t)dW^Q(t) \qquad (12)$$

$$\text{Where } \sigma = \begin{cases} \sigma & , if\ t \le T_1 \\ \sigma\frac{\frac{S(t)}{N}\sum_{i=M+1}^{N} e^{\lambda(T_i - t)}}{F(t,T_1,T_N)} & , if\ T_M < t < T_{M+1}, M = 1,2,3,\dots,N-1 \end{cases}$$

This is derivation of dynamics is provided in Appendix A.3

From this expression, it can be seen that the futures price is not log-normally distributes for $T_M < t < T_{M+1}$. But the index is a log-normal distribution for $F(t,T_1,T_N)$ ∀ t, because Black's (1976) formula can be used to be found a closed form solution for the option price. If the volatility of future price is adjusted in equation 12, the log-normality in the settlement period is obtained.

"Due to this approximation, it is not sure whether our final option pricing will be accurate."

So, the following are suggested:

$$dF(t,T_1,T_N) = \sigma_\alpha(t)dW^Q(t) \qquad (13)$$

$$\text{Where } \sigma = \begin{cases} \sigma & , if\ t \le T_1 \\ \sigma\frac{N-M}{N} & , if\ T_M < t < T_{M+1}, M = 1,2,3,\dots,N-1 \end{cases}$$

This volatility function is successful assumption for the volatility prices, as $T_M < t < T_{M+1}$ is considered. The future price of freight rates is unlikely to change significantly as a large part of average spot freight rates during settlement is already known. So, if the maturity is small, the volatility will be small, too. The different way states for large N–M, as σ_α will be large. If we check the N –M approach, letting M=0 or N, we see that in the first case we have zero time to maturity and volatility equal to zero while in the second case the current time t approaches T_1 and the volatility will be equal to the volatility of t$\le T_1$.

Consider

$$\lim_{N-M\to 0}\sigma\left(\frac{N-M}{N}\right) \;\; = \lim_{Z\to 0}\sigma\left(\frac{Z}{Z+M}\right)$$

Or

$$\lim_{N-M\to 0}\sigma\left(\frac{N-M}{N}\right) \;\; = \sigma$$

It is now checked the behavior of σ_α when N –M equals to L, with L chosen arbitrarily between 0 and N. It is checked whether the result is smaller than N –M ⟶0 or larger than N –M ⟶N.

It is now considered:

$$\lim_{N-M\to L}\sigma\left(\frac{N-M}{M}\right) = \lim_{Z\to L}\sigma\left(\frac{Z}{Z+M}\right) = \sigma\frac{L}{L+M} < \sigma$$

For M>0 the L<N. Furthermore,

$$\lim_{N-M\to L}\sigma\left(\frac{N-M}{M}\right) = \lim_{Z\to L}\sigma\left(\frac{Z}{Z+M}\right) = \sigma\frac{L}{L+M} > \sigma$$

For L>0

So, using the volatility σ_α, F now also follows lognormal distribution in the settlement period. A contract with dynamics as defined in equation 13 is log-normally distributed as:

$\ln F(t,T_1,T_N) \sim N(\ln F(t,T_1,T_N) - \frac{1}{2}\int_t^{T_N}\sigma_\alpha\,(s)^2 ds, \frac{1}{2}\int_t^{T_N}\sigma_\alpha(s)^2\,ds)$

It is defined $\sigma_F^2 = \int_t^{T_N}\sigma_\alpha(s)^2\,ds$. In Appendix A.4 it is shown that:

$$\sigma_F^2 = \sigma^2(T_1-t) + \sigma^2(T_N-T_1)R(N) \qquad (14)$$

Where σ is the volatility and $R(N) = \frac{1-\frac{3}{2N}+\frac{1}{2N^2}}{3-\frac{3}{N}}$

A function for the volatility future price σ_F is now obtained and hold for all t. The formula for σ_F^2 that is used in Koekebakker, Adland and Sødal’s (2007) paper is different than this that is used here.

3.6 The relationship of FFA prices and freight options

The freight options can be seen as European options on Forward Freight Agreements or as arithmetic average price Asian options on the Baltic Freight Index.

Arithmetic Asian options on freight rates have as payment the maximum of the difference between strike price and the average period and zero:

$$C(T_N,T_N) = D\left[\frac{1}{N}\sum_{i=1}^{N} S(T_i) - K\right]^+ \qquad (15)$$

Where $C(T_N,T_N)$ is the price of a call option at time T_N with exercise time T_N.
Similarly, the payoff of a freight put option with strike price K is:

$$P(T_N,T_N) = D\left[K - \frac{1}{N}\sum_{i=1}^{N} S(T_i)\right]^+ \quad (16)$$

The D factor added for the same reason as for the payoff a future contract. European options on the FFAs have as payoff the maximum of the difference between strike price and the price of the FFA at the exercise time and zero. The payoff with strike price K is:

$$C(T_N,T_N) = D[F(T_N,T_1,T_N) - K]^+ \quad (17)$$

And the payment of a freight put with strike price K equals to:

$$P(T_N,T_N) = D[K - F(T_N,T_1,T_N)]^+ \quad (18)$$

The payoff in the equation 17 is the same as the maximum difference between the option strike and average of freight rates during the settlement period and zero. That is because the price of the FFA at exercise time equals the average if the freight rates during the settlement periods. So, the equations 15 and 17 (therefore 16 and 18) are the same.

Call options gain value as their underlying asset does. The call options gain value when the FFA gains value, too. When the price of Forward Freight agreement increases the price of the freight call option increases. Similarly, when the price of the freight put option increases the price of the FFA decreases.

From the fundamental theorem of asset pricing, it is known that the price of an asset is equal to its discounted expected payoff under risk neutral probability. For a freight call option with strike price K at time t the price can be:

$$C(t,T_N) = e^{-r(T_N-t)} D E_t^Q [F(T_N,T_1,T_N) - K]^+ \quad (19)$$

For this formula, the equation 17 is needed. The equation 18 is needed for the price of a freight put option with strike price K at time t:

$$P(t,T_N) = e^{-r(T_N-t)} D E_t^Q [K - F(T_N,T_1,T_N)]^+ \quad (20)$$

It is more desirable to have an analytic equation. Therefore, the Black's(1976) formula is essential to be shown that the futures prices are approximately log-normally distributed.

"Let C(F,t) be the price of a call option on a future, as a function of the underlying futures price F and time t. Let r be the risk free rate, σ the volatility of the underlying future, T the exercise time and K the strike price.

Then C(F,t) can be stated as:

$$C(F,t) = e^{-r(T-t)}(F\varphi(d_1) - K\varphi(d_2))$$

Where

$$d_1 = \frac{\ln(\frac{F}{K}) + \frac{1}{2}\sigma_\alpha^2(T-t)}{\sigma_\alpha\sqrt{T-t}}$$

And

$$d_2 = d_1 - \sigma_\alpha\sqrt{T-t}$$

The φ is the cumulative distribution function of the standard normal distribution."

If we use $\sigma_\alpha^2(T_N - t) = \int_t^{T_N} \sigma_\alpha(s)^2\, ds = \sigma_F^2$, the price of the freight call option with strike price K and maturity time T_N is:

$$C(t,T_N) = e^{-r(T_N-t)}D[F(t,T_1,T_N)\varphi(d_1) - K\varphi(d_2)]^+ \quad (21)$$

Where

$$d_1 = \frac{\ln(\frac{F(t,T_1,T_N)}{K}) + \frac{1}{2}\sigma_F^2}{\sigma_F}$$

And

$$d_2 = d_1 - \sigma_F$$

The $F(t,T_1,T_N)$ is the future price in equation 11 and the σ_F is the future volatility price in equation 14. Similarly, for freight put options:

$$P(t,T_N) = e^{-r(T_N-t)}D[K\varphi(-d_2) - F(t,T_1,T_N)\varphi(-d_1)]^+ \quad (22).$$

3.7 Implementation explicit formula for option pricing

Using the explicit formula which is the equation 21, a freight call option can be valued as all parameters are known. It is considered that the strike price is K=$30000 per day, the spot freight rate is S_0=$25000 per day the λ=0.03, where λ is risk neutral freight rate drift and the volatility is σ=30%. Most of freight options consist of multiple settlement periods instead of one. In this master thesis it is considered that there are twelve settlement periods with length of 21 days for each period. For these periods the corresponding option prices are calculated. These prices, when considering a call option, are called the prices of "caplets" and when considering a put option they called "floorlet". When adding the 12 prices the result is the cap (floor) and the price of the option that consists of 12 settlement periods is found. It is set $T_1=\frac{1}{252}$ and $T_N=\frac{21}{252}$, because we need 21 business days. The factor D has to express the total number of days in a month. So, in January D equals 31, in February equals

28 (ignoring leap years) and so on.

In Appendix B.1, the Matlab code is used to implement this formula. The table 1 reflects the futures prices at the beginning for each month by using the equation 11, the volatility of the futures prices by using the equation 14 and the caplet prices and the cap price by using equation 21. So, for example, the caplet price in the February row shows the price one has to pay at the first of January for the freight caplet which starts at the first of February and ends at the 28th of February. The price of the cap is also the price that has to be paid the first of January for the freight cap contract which runs from the first of January until the 31th of December.

Table 1: Future prices, volatility of future prices and caplet prices for a freight call option with initial spot freight rate S0 = $25000 per day, $\lambda = 0.03$, strike price K = $30000 per day and volatility of the freight rates $\sigma = 30\%$

Month	**FFA**	**Volatility σ_F**	**Caplet price**
January	$25032,77	5,17%	$2,61
February	$25095,43	10,09%	$1188,46
March	$25158,24	13,29%	$4857,53
April	$25221,22	15,86%	$8994,04
May	$25284,35	18,07%	$14025,56
June	$25347,64	20,04%	$18213,93
July	$25411,09	21,83%	$23583,78
August	$25474,7	23,49%	$28271,96
September	$25538,46	25,03%	$31807,90
October	$25602,39	26,49%	$37366,43
November	$25666,47	27,87%	$40419,10
December	$25730,72	29,18%	$46070,49
		Cap	**$254801,82**

The volatility σ_F increases with time. The internal and external factors that affect freight rates can be more accurately forecasted for the near future than for a period of time far away. Therefore, the volatility in December is higher than that in January. This increase in volatility can be explained more mathematically. A short settlement period gives rise to a high volatility. A change in freight rate during settlement has a high impact on the average freight rates and the future price, as the settlement period consists of only new fixed freight rates. A long settlement period gives rise to a low volatility. The caplet prices increase in time. This is both due to the increasing future prices and the increasing volatilities of the future price. High volatility means high risk, leading to high values. In the case of a call option a high future price means a high payoff and also a high value. The average of the freight rates during settlement are expected to vary drastically. When the average rates rise above the sum of the option price and the strike price, the option is interesting. If this does not happen the holder of the option will have a negative profit. When the future price is lower than the caplet price, it can be seen that it is more profitable to buy the future instead of the caplet. So, if the caplet is cheaper than the future, the caplet is preferred. Finally, remarkable is that the computational time of the Matlab code is only 0.021 seconds.

4. Numerical Example-Monte Carlo simulations

The price was calculated by using the explicit formula for which an approximation for the volatility of the future prices had to be made. So, it is not guaranteed that this price is accurate. Therefore, it is needed to be checked the validity of the approximating formula. The option is considered to be also the Asian option on the spot freight rates. The volatility of the spot freight rates has to be used, so no approximation for the volatility and for the future price has to be made. However, no explicit formula is known for pricing of Asian options. Therefore, Monte Carlo simulation is a very accurate method but costly due to the large amount of its simulations. It is important for a method to be fast and accurate. If the rapid method is not accurate, the method is useless, so the accuracy of the explicit formula needs to be checked. A usual Monte Carlo simulation, a Monte Carlo with variance reduced by control variates and a Monte Carlo with variance reduced by both control and antithetic variates are used. In all the simulations 5 million freight rate paths are used and randn('state', 100) is set to made the same random normal variables each time the Matlab code is running.

4.1 Monte Carlo

The price of an option with a payoff that is calculated using the equation 2 is now executed with the assistance of Monte Carlo simulation. The parameters are the same as the explicit formula. So, the strike price is K=$30000 per day, the spot freight rates are S_0 =$25000 per day, the λ equals to 0.03 and σ=30%. Beginning with S_0 as initial value, many spot freight paths are constructed. For each path the average value of the rates between T_1 and T_N are computed. Then, the payoff of the Asian options corresponding to each of these average freight rate values is computed. After that, the payoff is discounted and the average value of all prices is taken. If only one freight rate path was used, the result would be unreliable, since the paths are constructed using random standard normal variables. Monte Carlo is a stochastic simulations technique, so the price found by this is an estimate. The Matlab code for the Monte Carlo lies in Appendix B.2. The prices for the freight call option as well as the corresponding standard errors and the relative standard errors are shown in Table 2. 5 million paths have been created and used as well as 21 time steps per month are taken.

Table 2: Monte Carlo prices for a freight call option with initial spot freight rate S_0=$25000 per day, λ=0.03, strike price K=$30000 per day and volatility of the freight rates σ=30%, the standard errors.

Month	**MC Price**	**Std. error**	**Rel. std. error**	**Form. price**	**%diff.**
January	$3,21	0,12	3,88%	$2,61	-18,84%
February	$1202,15	3,90	0,32%	$1188,46	-1,13%
March	$4865,04	10,06	0,20%	$4857,53	-0,15%
April	$9005,19	15,20	0,16%	$8994,04	-0,12%
May	$14003,32	20,96	0,14%	$14025,56	0,15%
June	$18203,62	25,23	0,13%	$18213,94	0,05%
July	$23624,52	30,93	0,13%	$23583,79	-0,17%
August	$28265,42	35,47	0,12%	$28271,96	0,02%
September	$31762,97	38,57	0,12%	$31807,9	0,14%
October	$37372,98	44,18	0,11%	$37366,43	-0,01%
November	$40463,06	46,86	0,11%	$40419,11	-0,10%
December	$46092,38	52,41	0,11%	$46070,49	-0,04%
Cap	**$254863,9**	52,41	0,02%	**254801,8**	-0,02%

It is notable that the standard errors increase in time, since for longer periods the constructed lognormal freight rate paths can vary significantly. The percentage differences between the prices using the two methods should be as small as possible, since then it can be concluded that explicit formula is not only fast but also accurate. Moreover, the standard errors should be reduced firstly, before making conclusions about the percentage differences. The disadvantage of this method is that the computational time of Matlab code is about 20 minutes that is much higher than the computational time of the explicit formula that is only 0.021 seconds.

4.2 Variance reduction using control variates

There are many and different ways to reduce the variance with Monte Carlo simulation. One of these ways is with control variates. The estimates of the value of a known quantity are used and then the corresponding errors are taken. These errors are used to reduce the error in the option prices. As a control variate should be used a variable that is close to the required option price. So, a European option or a geometric Asian option with same parameters as the desired option can be used. Let X be the value of the desired option price obtained by Monte Carlo simulation. So E[X] is its expected price. The variable Y is needed because there is a method to calculate its value by using some explicit formula. The E[Y] is its expected price. The value of

Y is estimated by Monte Carlo and the error between the E[Y] and this value is calculated. It is defined that:

$$Z = X + E[Y] - Y$$

And

$$E[Z] = E[X + E[Y] - Y] = E[X] + E[Y] - E[Y] = E[X]$$

So, E[X] is obtained by calculating E[Z] instead. So, Monte Carlo is applied to Z=X+E[Y] –Y. It is known that var[α] =0 if $\alpha \in \mathbb{R}$, so:

$$\mathrm{var}[Z] = \mathrm{var}[X - Y]$$

Since $E[Y] \in \mathbb{R}$.

To reduce variance, var[X –Y] has to be smaller than var[X]. Therefore:

$$Z_\theta = X + \theta(E[Y] - Y)$$

Instead of original Z, but still holds E[X] = E[Z_θ]. Furthermore:

$$\mathrm{var}[Z_\theta] = \mathrm{var}[X - \theta Y] = \mathrm{var}[X] - 2\theta \mathrm{cov}[X, Y] + \theta^2 \mathrm{var}[Y]$$

since X and Y are not independent. Using the derivative of var[Z_θ] and setting it equal to zero:

$$\theta_{opt} = \frac{cov[X, Y]}{varY}$$

And so the variance is minimized.

4.2.1 European option as control variate

Y is the European call option, so for E[Y] the known Black Scholes (1973) formula for call options is used. This gives the exact value E[Y] for the European call option.

$$C(S, t) = \varphi(d_1)S - \varphi(d_2)Ke^{-r(T-t)}$$

Where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T - t)}{\sigma\sqrt{T - t}}$$

And

$$d_2 = d_1 - \sigma\sqrt{T - t}$$

The estimate of this option is obtained by constructing freight rate paths deriving the payoff belonging to all of these paths, discounting these payoffs and taking the average of these values. The Matlab code that is used for this method is added in

Appendix B.3.

Table 3: Monte Carlo prices, using a European option as control variate, for a freight call option with initial spot freight rate S_0 = \$2500 per day, λ =0.03, strike price K=\$30000 per day and volatility of the freight rates σ= 30%.

Month	MC price	Std. error	Rel. std. error	Caplet	%diff.
January	\$3,20	0,12	3,77%	\$2,610366	-18,62%
February	\$1199,36	2,43	0,20%	\$1188,462	-0,90%
March	\$4868,12	4,80	0,09%	\$4857,531	-0,21%
April	\$8994,91	6,09	0,06%	\$8994,04	-0,009%
May	\$14033,92	7,38	0,05%	\$14025,56	-0,05%
June	\$18215,52	7,97	0,04%	\$18213,94	-0,008%
July	\$23589,64	8,93	0,03%	\$23583,79	-0,02%
August	\$28280,19	9,50	0,03%	\$28271,96	-0,02%
September	\$31812,41	9,69	0,03%	\$31807,9	-0,01%
October	\$37369,23	10,47	0,02%	\$37366,43	-0,007%
November	\$40416,86	10,51	0,02%	\$40419,11	0,005%
December	\$46055,23	11,19	0,02%	\$46070,49	0,033%
Cap	**\$254838,6**	11,19	0,004%	**\$254801,8**	-0,01%

When comparing the standard errors to those to those of table 2 it can be seen that indeed the errors are smaller now. However, instead of using a European option as a control variate, there are also other possibilities to reduce the variance.

4.3 Variance reduction using both antithetic and control variates

Another way to reduce the variance with Monte Carlo is by using antithetic variates. The variance is reduced by calculating the mean of $\frac{X+Y}{2}$ instead of X. The Y and X need to have the same distribution. This technique reduces the variance of the estimate because the variance of $\frac{X+Y}{2}$ is smaller than that of X. It is noticed that:

$$\mathrm{var}\left(\frac{X+Y}{2}\right) = \frac{1}{4}\big(\mathrm{var}(X+Y) + 2\mathrm{cov}(X,Y)\big) = \frac{1}{2}(\mathrm{var}(X) + \mathrm{cov}(X,Y))$$

X and Y have the same distribution. So, they also have the same variance. This should be smaller than var(X), so:

$$\mathrm{cov}(X,Y) < var(X)$$

or even

$$\mathrm{cov}(X,Y) \leq 0$$

The standard Monte Carlo estimate is $\frac{1}{M}\sum_{i=1}^{M} f(X_i)$. Higham (2011) shows that $\mathrm{cov(X,Y)} \leq 0$, when f is monotonic. So, for a monotonic function f:

$$\mathrm{var}(\frac{\mathrm{f(X)+f(Y)}}{2} \leq \mathrm{var(f(X))}$$

Where X and Y have the same distribution. Therefore, function f equals to equation 9 which is a monotonic function. If $Z_1 \leq Z_2$ then $S(Z_1) \leq S(Z_2)$ and vice versa, when defining $S(Z_1)=S_t e^{\left(\mu-\frac{\sigma^2}{2}\right)(T-t)+\sigma\sqrt{T-tZ_1}}$. Since Z∼N(0,1) and Y= -Z is used, Y∼N(0,1). This leads to a reduction of variance.

Combining these methods may lead to further reduction of variance. For the antithetic variates, $2M=10x^6$ freight rate paths are defined. The first M depends on Z∼N(0,1) and the second M depends on - Z∼N(0,1). For these paths the corresponding option prices are computed and is taken the average of the first and $(M+1)^{th}$ price, the second and $(M+2)^{nd}$ and so on. Next, using a European call option as control variate, the values of European call options corresponding to the freight rate paths that constructed are estimated. There are 2M freight rate paths, so 2M European call option prices are obtained. It is taken again the average of the first and $(M+1)^{th}$ option price, the second and $(M+2)^{nd}$ and so on. The Black Scholes (1973) value is derived for the European call option. After calculating the optimal θ the Z is calculated and equals the Monte Carlo option value that is obtained with antithetic variates plus θ× (Black Scholes European call option value minus(-) the Monte Carlo European call option values obtained by using antithetic variates). Now, both antithetic variates and control variates are used. The Matlab code that is used to implement the combination of these two methods lies in Appendix B.4.

Comparing the tables, it is noticeable that the standard errors are now smaller than using only control variates. The standard errors of prices found by using Monte Carlo are negligible and the percentage difference between the prices can be considered. These differences and the standard errors lead to the conclusion that the explicit formula is valid to price freight options under this parameter set. There is no immediate reason to assume why the formula is not accurate to price freight options with other parameter set.

Table 4: Monte Carlo prices, using both control and antithetic variates, for a freight call option with initial spot freight rate S_0 = \$2500 per day, λ =0.03, strike price K=\$30000 per day and volatility of the freight rates σ= 30%.

Month	**MC price**	**Std. error**	**Rel. std. error**	**Caplet**	**%diff.**
January	\$3,34	0,08	2,60%	\$2,61	-21,85%
February	\$1197,83	1,71	0,14%	\$1188,46	-0,78%
March	\$4866,13	3,39	0,06%	\$4857,53	-0,17%
April	\$8992,46	4,30	0,04%	\$8994,04	0,01%
May	\$14027,8	5,22	0,03%	\$14025,56	-0,01%
June	\$18210,94	5,63	0,03%	\$18213,94	0,01%
July	\$23583,95	6,31	0,02%	\$23583,79	-0,0007%
August	\$28274	6,72	0,02%	\$28271,96	-0,007%
September	\$31810,73	6,85	0,02%	\$31807,9	-0,008%
October	\$37368,48	7,39	0,01%	\$37366,43	-0,005%
November	\$40415,75	7,42	0,01%	\$40419,11	0,008%
December	\$46067,74	7,92	0,01%	\$46070,49	0,005%
Cap	**\$254819,2**	7,92	0,003%	**\$254801,8**	-0,006%

5. Discussion

It was shown that the formula used is an accurate approximation for the desired option price, since differences between the prices found by using this formula and the ones found by using Monte Carlo simulations are very small. It is only shown for one particular example that formula satisfies, though. It is questionable whether it works for each example. A closer look is taken at the approximation steps that made to derive the formula and it is discussed why and when this kind of approximations is allowed to be made. It should always be reminded that a few assumptions are made, since the accuracy of our formula is based on these assumptions. One of the most essential assumptions that made is assumption 8, which states that the spot freight rates are log-normally distributed. This is an assumption on which all derivations are based.

5.1 Geometric Brownian motion and the mean reversion

It was assumed that the spot freight rates are log-normally distributed and also follow a Geometric Brownian motion. This assumption can be seen as the basic assumption of the model since using this assumption a model about the spot freight rates is created, by which a model about future prices is derived. However, if another model for the spot freight rates was used, another model for the future prices and another explicit formula for the pricing of options would be found. It is questionable whether this assumption of log-normality is realistic in practice. It would be more realistic to use another distribution for the freight rates. A more suitable assumption for the freight rates would be that of mean reversion, which implies that the spot freight rates always return to some long term mean. If spot freight rates were modeled by mean reversion, equation 3 could be:

$$dS(t) = \kappa\big(\mu - S(t)\big)dt + \sigma S(t)dW(t)$$

As given in Prokopczuk's (2011) article. The μ here is the long term mean of the freight rates and κ is the rate at which the freight rates return to the long term mean. So, there would be another analytic formula for S(t) instead of equation 9 and instead of equation 12 another formula for $F(t,T_1,T_N)$ would be derived. Another future volatility would be found and the explicit formula in equation 21 would be different, too.

5.1.1 Volatility term structure

By assuming another distribution for the spot rates, another formula for S(t) and $F(t,T_1,T_N)$ should be found. Consequently, another formula for the prices should be derived. When the spot freight rates show mean reversion, the volatility of the future prices show has a term structure that depends on time to maturity. If the spot freight rates show a decrease in value, while there is a relatively short time to maturity left, the prices will decrease as there is not much time left for the spot freight rates to return to their long term mean. Therefore, when time to maturity is low the volatility of the future prices is relatively high while high time to maturity leads to low volatility values. This volatility term structure does not exist when the freight rates follow log-normal distribution as freight rates does not return to any long term mean.

As given in Koekebakker and Ollmar's (2006) research, the following volatility formula satisfies the required term structure:

$$\sigma_{m.r.}(t,T_1,T_N) = \begin{cases} (\sigma_s - \sigma_A)(\frac{e^{-\lambda(T_1-t)} - e^{-\lambda(T_N-t)}}{\lambda(T_N-T_1)}) + \sigma_A, \ t \le T_1 \\ (\sigma_s - \sigma_A)\left(\frac{1-e^{-\lambda(T_N-t)}}{\lambda(T_N-T_1)}\right) + \sigma_A \frac{T_N-t}{T_N-T_1}, \ T_1 < t < T_N \end{cases} \qquad (23)$$

The σ_A is the asymptotic volatility of the spot freight rates and the value which the volatility of the future prices converges when there is a lot of time to maturity left. The σ_S is the short term volatility and the value towards which the volatility converges when there is little time to maturity left. The σ is now not only a function of time t but also a function of the settlement period $[T_1,T_N]$, since the volatility depends on time to maturity now. In the settlement period the volatility should have the same properties as the volatility in equation 13. The required properties of $\sigma_{m.r.}(t,T_1,T_N)$ are now checked:

- If time to maturity is high so $T_1 - t \longrightarrow \infty$, $\sigma_{m.r.} = \sigma_A$ is expected

$$\lim_{T_1-t\longrightarrow\infty} \sigma_{m.r.} = \lim_{T_1-t\longrightarrow\infty} (\sigma_s - \sigma_A)\left(\frac{e^{-\lambda(T_1-t)} - e^{-\lambda(T_N-t)}}{\lambda(T_N-T_1)}\right) + \sigma_A = \sigma_A$$

- If time to maturity is low so $t \uparrow T_1$, $\sigma_{m.r.} = \sigma_s$ is expected

$$\lim_{t\uparrow T_1} \sigma_{m.r.} = \lim_{t\uparrow T_1} (\sigma_s - \sigma_A)\left(\frac{e^{-\lambda(T_1-t)} - e^{-\lambda(T_N-t)}}{\lambda(T_N-T_1)}\right) + \sigma_A$$

$$= (\sigma_s - \sigma_A)\left(\frac{1 - e^{-\lambda(T_N-T_1)}}{\lambda(T_N-T_1)}\right) + \sigma_A$$

The last term approximately equals σ_α which can be seen by using Taylor expansion of $e^{-\lambda(T_N-T_1)}$.

- Inside the period if t↓ T_1, $\sigma_{m.r.} = \sigma_s$ is expected

$$\lim_{t\downarrow T_1} \sigma_{m.r.} = \lim_{t\downarrow T_1} (\sigma_s - \sigma_A)\left(\frac{1 - e^{-\lambda(T_N - t)}}{\lambda(T_N - T_1)}\right) + \sigma_A \frac{T_N - t}{T_N - T_1}$$

$$= (\sigma_s - \sigma_A)\left(\frac{1 - e^{-\lambda(T_N - T_1)}}{\lambda(T_N - T_1)}\right) + \sigma_A$$

Again this last term approximately equals σ_α.

- Inside the period if t↑ T_N, $\sigma_{m.r.} = 0$ is expected

$$\lim_{t\uparrow T_N} \sigma_{m.r.} = \lim_{t\downarrow T_1} (\sigma_s - \sigma_A)\left(\frac{1 - e^{-\lambda(T_N - t)}}{\lambda(T_N - T_1)}\right) + \sigma_A \frac{T_N - t}{T_N - T_1} = 0$$

So, the volatility function $\sigma_{m.r.}(t, T_1, T_N)$ in equation 23 matches the requirements of volatility of the future prices. Just as in equation 14, the volatility of the future prices from t to T_N is now required to find an explicit formula for the freight option prices. In Appendix A.5 a formula for the $\sigma_{F,m.r.}^2(t, T_1, T_N) = \int_t^{T_N} \sigma_{m.r.}^2(s, T_1, T_N)ds$ is shown in equation A.11.

5.2 Volatility of the explicit formula

It is questionable whether this formula is still accurate when other values for the parameters are used. Furthermore, we argue why the approximations made to derive this formula are allowed.

5.2.1 Lognormal approximation

When deriving the explicit formula, an approximation for the volatility of the future prices is used. This approximating volatility is derived by making a log-normal approximation for the future prices. But, it is questionable whether this approximation is allowed to be made. The future price can be seen as the expected price of the average of the sum of log-normally distributed variables. To be allowed to make the lognormal approximation, it should hold that the sum of log-normally distributed variables is also log-normally distributed, to be allowed to make the lognormal approximation. In finance, it is common to be as assumption a log-normal approximation for the sum of lognormal variables. This is suggested by Levy (1992) and the parameters found in his survey are called “Levy’s log-normal moment matching”. In this method, the first two moments of the resulting lognormal distribution and the true first two moments of average spot freight rates are set equal.

Out of them, the mean and the variance of the approximating log-normal distribution can be found. So, the log-normal approximation is permissible.

5.2.2 Different parameter value

It can be seen that an essential assumption has been made in Appendix A.2 when checking the steps that are taken to be found the final pricing formula. It is assumed that $e^{-\lambda\Delta_i} \neq 1$ and using the theorem that lies there, $\sum_{i=0}^{N-1} e^{-\lambda\Delta_i}$ can be written as $\frac{1-e^{-\lambda\Delta N}}{1-e^{-\lambda\Delta}}$. But, if $e^{-\lambda\Delta_i} = 1$, $\sum_{i=0}^{N-1} e^{-\lambda\Delta_i}$ cannot be written like this, as the condition in the theorem does not hold anymore. So, if λ=0 or Δ=0, $\sum_{i=0}^{N-1} e^{-\lambda\Delta_i} \neq \frac{1-e^{-\lambda\Delta N}}{1-e^{-\lambda\Delta}}$. Therefore, if this does not hold, the model for the $F(t, T_1, T_N)$ in equation 13 does not hold, too. So, if either λ=0 and μ=σγ or Δ=0 and $\frac{T_N-T_1}{N-1} = 0 \Leftrightarrow T_N = T_1$, or N=∞, the formula in equation 21 does not produce the correct option prices. If the Matlab code in Appendix B.1 is checked for these cases it produces NaN values for all of them. If one of these cases states, the explicit formula in equation 21 does not hold anymore. In other words, the explicit formula does not work when the mean of the spot freight rates equals the volatility of the rates, when the exercise date is the same with the settlement date, when there is no settlement period or when continuous time in settlement period is considered instead of fixed time points.

Precautions have to be taken when the explicit formula is used, since for most parameter values it gives accurate prices but not for all values. For these few parameter values for which the formula does not work, prices have to be derived by using the Monte Carlo method. The log-normal approximation of the future prices is not completely accurate and assumptions made are not entirely realistic.

6. Conclusion

In this master thesis two ways of pricing a freight option are discussed. First of all, a freight option can be valued by using an explicit formula given in equation 21. To obtain this formula, lognormal spot freight dynamics are assumed. It is shown that the corresponding future prices (FFAs) are also log-normally distributed prior to the settlement period, but this log-normality breaks down in the settlement period. With this absence of the log-normal distribution of volatility inside the settlement period, an approximation for the volatility of the future prices is made. So, the future prices are now not only log-normally distributed prior to but also inside the settlement period. Next, Black's (1976) formula can be used to derive an analytic pricing formula for the freight options. So, we conclude that this formula is equation 21. Due to the volatility approximation made to derive equation 21, it is not sure whether the prices of calculations of this formula are valid. Therefore, we use the second way of pricing a freight option, the Monte Carlo simulation, in order to be checked the validity of the formula. One particular option is priced by using both equation 21 and Monte Carlo simulations. After that, it is needed to be reduced the variance of the simulations. Firstly, the control variates are used and then a combination of antithetic and control variates for this purpose. Thereafter, the explicit formula and Monte Carlo simulations seem to have insignificantly small differences in the prices that were calculated. Now, it can be concluded that equation 21 is valid to price freight options for almost all parameters sets. Preference is given to this formula to price freight options instead of the Monte Carlo method, since the first one is a lot quicker in computational time than the Monte Carlo simulation. The difference is obvious as for the calculation of the explicit formula 0.021 seconds are needed while 20 minutes are needed for the Monte Carlo simulation. However, the assumptions, like log-normality (of the spot rates by which the prices found using equation 21 and Monte Carlo simulations) that are made, may differ slightly from the real market option prices. According to freight data, the assumption of the mean reverting freight rates would have been a more realistic assumption than log-normal freight rates. Finding an explicit formula based on mean reverting freight rates is a subject for later studies.

References

- Black, F. and Scholes, M., 1973. The pricing of options and corporate liabilities. *The journal of political economy*, pp.637-654.
- Merton, R.C., 1973. Theory of rational option pricing. *The Bell Journal of economics and management science,* pp.141-183.
- Merton, R.C., 1976. Option pricing when underlying stock returns are discontinuous. *Journal of financial economics*, *3*(1-2), pp.125-144.
- Black, F., 1976. The pricing of commodity contracts. *Journal of financial economics*, *3*(1), pp.167-179.
- Cox, J.C., Ross, S.A. and Rubinstein, M., 1979. Option pricing: A simplified approach. *Journal of financial Economics*, *7*(3), pp.229-263.
- Kemna, A.G. and Vorst, A.C.F., 1990. A pricing method for options based on average asset values. *Journal of Banking & Finance*, *14*(1), pp.113-129.
- Levy, E., 1992. Pricing European Average Rate Currency Options. Journal of International Money and Finance, 11(5), 474-491.
- Bates, D.S., 1996. Jumps and stochastic volatility: Exchange rate processes implicit in deutsche mark options. *Review of financial studies*, *9*(1), pp.69-107.
- Alziary, B., Décamps, J.P. and Koehl, P.F., 1997. A PDE approach to Asian options: analytical and numerical evidence. *Journal of Banking & Finance*, *21*(5), pp.613-640.
- Tvedt, J., 1998. Valuation of a European futures option in the BIFFEX market. *Journal of Futures Markets*, *18*(2), pp.167-175.
- Kavussanos, M.G. and Nomikos, N.K., 1999. The forward pricing function of the shipping freight futures market. *journal of futures markets*, *19*(3), pp.353-376.
- Haigh, M.S., 2000. Cointegration, unbiased expectations, and forecasting in the BIFFEX freight futures market. *Journal of Futures Markets*, *20*(6), pp.545-571.
- Andersen, L. and Andreasen, J., 2000. Jump-diffusion processes: Volatility smile fitting and numerical methods for option pricing. *Review of Derivatives Research*, *4*(3), pp.231-262.

- Kavussanos, M.G. and Nomikos, N.K., 2000. Futures hedging when the structure of the underlying asset changes: The case of the BIFFEX contract. *Journal of Futures Markets*, *20*(8), pp.775-801.
- Schwartz, E. and Smith, J.E., 2000. Short-term variations and long-term dynamics in commodity prices. *Management Science*, *46*(7), pp.893-911.
- Kavussanos, M.G. and Nomikos, N.K., 2000. Constant vs. time-varying hedge ratios and hedging efficiency in the BIFFEX market. *Transportation Research Part E: Logistics and Transportation Review*, *36*(4), pp.229-248
- Zhang, J., 2001. A semi-analytical method for pricing and hedging continuously sampled arithmetic average rate options. *Journal of Computational Finance*, *5*(1), pp.59-80.
- Bessembinder, H. and Lemmon, M.L., 2002. Equilibrium pricing and optimal hedging in electricity forward markets. *the Journal of Finance*, *57*(3), pp.1347-1382.
- Haigh, M.S. and Holt, M.T., 2002. Hedging foreign currency, freight, and commodity futures portfolios—A note. *Journal of Futures Markets*, *22*(12), pp.1205-1221.
- Kavussanos, M.G. and Nomikos, N.K., 2003. Price discovery, causality and forecasting in the freight futures market. *Review of Derivatives Research*, *6*(3), pp.203-230.
- Dinwoodie, J. and Morris, J., 2003. Tanker forward freight agreements: the future for freight futures?. *Maritime Policy & Management*, *30*(1), pp.45-58.
- Kavussanos, M.G., Visvikis, I.D. and Menachof, D., 2004. The unbiasedness hypothesis in the freight forward market: Evidence from cointegration tests. *Review of Derivatives Research*, *7*(3), pp.241-266.
- Kavussanos, M.G., Visvikis, I.D. and Batchelor, R.A., 2004. Over-the-counter forward contracts and spot price volatility in shipping. *Transportation Research Part E: Logistics and Transportation Review*, *40*(4), pp.273-296.
- Koekebakker, S. and Os Ådland, R., 2004. Modelling forward freight rate dynamics—empirical evidence from time charter rates. *Maritime Policy & Management*, *31*(4), pp.319-335.

- Kavussanos, M.G. and Visvikis, I.D., 2004. Market interactions in returns and volatilities between spot and forward shipping freight markets. *Journal of Banking & Finance*, *28*(8), pp.2015-2049.
- Koekebakker, S., Ollmar, F., 2005. Clarkson Securities Freight Option Model. Technical document, Agder Research, Norway
- Adland, R. and Cullinane, K., 2006. The non-linear dynamics of spot freight rates in tanker markets. *Transportation Research Part E: Logistics and Transportation Review*, *42*(3), pp.211-224.
- Kavussanos, M.G. and Visvikis, I.D., 2006. Shipping freight derivatives: a survey of recent evidence. *Maritime Policy & Management*, *33*(3), pp.233-255.
- Tezuka, K., Ishizaka, M., Ishii, M., 2006. Equilibrium spot and futures process in shipping freight markets. In: Proceedings for the Annual Conference of the International Association of Maritime Economists (IAME), Melbourne, Australia, 12–14 July.
- Batchelor, R., Alizadeh, A. and Visvikis, I., 2007. Forecasting spot and forward prices in the international freight market. *International Journal of Forecasting*, *23*(1), pp.101-114.
- Koekebakker, S., Adland, R. and Sødal, S., 2007. Pricing freight rate options. *Transportation Research Part E: Logistics and Transportation Review*, *43*(5), pp.535-548.
- Diebold, F.X. and Yilmaz, K., 2009. Measuring financial asset return and volatility spillovers, with application to global equity markets. *The Economic Journal*, *119*(534), pp.158-171.
- Alizadeh, A. and Nomikos, N., 2009. *Shipping derivatives and risk management*. Palgrave Macmillan.
- Prokopczuk, M., 2011. Pricing and hedging in the freight futures market. *Journal of Futures Markets*, *31*(5), pp.440-464.
- Tsai, M.T., Saphores, J.D. and Regan, A., 2011. Valuation of freight transportation contracts under uncertainty. *Transportation Research Part E: Logistics and Transportation Review*, *47*(6), pp.920-932.

- Alizadeh, A.H. and Nomikos, N.K., 2011. Dynamics of the term structure and volatility of shipping freight rates. *Journal of Transport Economics and Policy (JTEP)*, *45*(1), pp.105-128.
- Higham, D.J., 2011. Stochastic ordinary differential equations in applied and computational mathematics. *IMA journal of applied mathematics*, *76*(3), pp.449-474
- Tezuka, K., Ishii, M. and Ishizaka, M., 2012. An equilibrium price model of spot and forward shipping freight markets. *Transportation Research Part E: Logistics and Transportation Review*, *48*(4), pp.730-742.
- Goulas, L. and Skiadopoulos, G., 2012. Are freight futures markets efficient? Evidence from IMAREX. *International Journal of Forecasting*, *28*(3), pp.644-659.
- Diebold, F.X. and Yilmaz, K., 2012. Better to give than to receive: Predictive directional measurement of volatility spillovers. *International Journal of Forecasting*, *28*(1), pp.57-66.
- Nomikos, N.K. and Doctor, K., 2013. Economic significance of market timing rules in the Forward Freight Agreement markets. *Transportation Research Part E: Logistics and Transportation Review*, *52*, pp.77-93.
- Nomikos, N.K., Kyriakou, I., Papapostolou, N.C. and Pouliasis, P.K., 2013. Freight options: Price modelling and empirical analysis. *Transportation Research Part E: Logistics and Transportation Review*, *51*, pp.82-94.
- Alizadeh, A.H., 2013. Trading volume and volatility in the shipping forward freight market. *Transportation Research Part E: Logistics and Transportation Review*, *49*(1), pp.250-265.
- Alizadeh, A., Kappou, K., Tsouknidis, D. and Visvikis, I., 2014. *Liquidity Risk Premia in the International Shipping Derivatives Market* (No. icma-dp2014-15). Henley Business School, Reading University.
- Kavussanos, M.G., Visvikis, I.D. and Dimitrakopoulos, D.N., 2014. Economic spillovers between related derivatives markets: The case of commodity and freight markets. *Transportation Research Part E: Logistics and Transportation Review*, *68*, pp.79-102.

- Alizadeh, A.H., Huang, C.Y. and van Dellen, S., 2015. A regime switching approach for hedging tanker shipping freight rates. *Energy Economics*, *49*, pp.44-59.
- Tsouknidis, D.A., 2016. Dynamic volatility spillovers across shipping freight markets. *Transportation Research Part E: Logistics and Transportation Review*, *91*, pp.90-111.

Appendix A

A.1 Derivation of equation 2

Consider that:

$$E_t^Q[e^{-r(T_N-t)}D\sum_{i=1}^{N}\left(\frac{S(T_i)}{N}-\ F(t,T_1,T_N)\right)]=0 \qquad (A.1)$$

Since the constant factors can be taken outside the expectation:

$$e^{-r(T_N-t)}DE_t^Q[\sum_{i=1}^{N}\left(\frac{S(T_i)}{N}-\ F(t,T_1,T_N)\right)]=0$$

Multiplying with $\frac{e^{r(T_N-t)}}{D}$ we have:

$$E_t^Q[\sum_{i=1}^{N}\left(\frac{S(T_i)}{N}-\ F(t,T_1,T_N)\right)]=0$$

However, the expectation of a sum equals to the sum of an expectation:

$$\sum_{i=1}^{N}E_t^Q\left(\frac{S(T_i)}{N}-\ F(t,T_1,T_N)\right)=0$$

Furthermore, it is known that $E_t[X]=X$ if X depends only on the first time t steps, so $E_t[F(t,T_1,T_N)]=F(t,T_1,T_N)$. This leads to:

$$F(t,T_1,T_N)=\sum_{i=1}^{N}E_t^Q\left(\frac{S(T_i)}{N}\right)$$

Now taking out the factor $\frac{1}{N}$, gives the equation 2:

$$F(t,T_1,T_N)=\frac{1}{N}\sum_{i=1}^{N}E_t^Q\left(S(T_i)\right)$$

A.2 Derivation of equation 11

Consider that:

$$F(t,T_1,T_N)=\frac{S(t)}{N}\sum_{i=1}^{N}E_t^Q[\,e^{\left(\lambda-\frac{\sigma^2}{2}\right)(T_i-t)+\sigma\sqrt{T_i-t}Z^Q}] \qquad (A.2)$$

S(t) can be taken out of the expectation as $E_t[X]=X$ if X depends only on the first time t steps. The expectation is a log-normally distributed variable as it is written in assumption 9. It is known that the expectation of a log-normally distributed variable equals to $e^{\mu+\frac{\sigma^2}{2}}$ when the corresponding normal variable has mean μ and variance σ. Since the normal distribution has mean $\left(\lambda-\frac{\sigma^2}{2}\right)(T_i-t)$ and variance $\sigma^2(T_i-t)$:

$$\mathrm{F(t,T_1,T_N)}=\frac{S(t)}{N}\sum_{i=1}^{N} e^{\left(\lambda-\frac{\sigma^2}{2}\right)(T_i-t)+\frac{1}{2}\sigma^2(T_i-t)} \qquad \text{(A.3)}$$

And

$$\mathrm{F(t,T_1,T_N)}=\frac{S(t)}{N}\sum_{i=1}^{N} e^{\lambda(T_i-t)} \Rightarrow \frac{S(t)}{N}e^{\lambda(T_N-t)}(1+\sum_{i=1}^{N-1} e^{\lambda(T_i-t)}) \qquad \text{(A.4)}$$

If we use the assumption of the equidistant observations $\Delta=\frac{T_N-T_1}{N-1}$ we lead to:

$$\lambda(T_1-T_N)=-\lambda(T_N-T_1)=-\lambda\Delta(N-1)$$

And

$$\lambda(T_2-T_N)=-\lambda(T_N-T_2)=-\lambda\Delta(N-2)$$

$$\lambda(T_3-T_N)=-\lambda(T_N-T_3)=-\lambda\Delta(N-3)$$

And so on. Therefore:

$$\lambda(T_i-T_N)=-\lambda(T_N-T_i)=-\lambda\Delta(N-i)$$

Since $e^{-\lambda\Delta\times 0}=1$, the previous equation can be:

$$\frac{S(t)}{N}e^{\lambda(T_N-t)}(\sum_{i=1}^{N-1} e^{-\lambda\Delta_i}) \qquad \text{(A.5)}$$

However, there is a theorem that is considered: For r≠ 1, the sum of the first N terms of a geometric series is: $\sum_{i=0}^{N-1} ar^i = a\frac{1-r^N}{1-r}$

Since $\sum_{i=0}^{N-1} e^{-\lambda\Delta_\iota}$ is a geometric with r=$e^{-\lambda\Delta}$≠1 and α=1, it can be written as $\frac{1-e^{-\lambda\Delta N}}{1-e^{-\lambda\Delta}}$.

So, the result is:

$$\mathrm{F(t,T_1,T_N)}=S(t)\frac{e^{\lambda(T_N-t)}}{N}\frac{1-e^{-\lambda\Delta N}}{1-e^{-\lambda\Delta}} \qquad \text{(A.6)}$$

A.3 Derivation of equation 12

Consider equation 2 and it is known that when $t<T_1$:

$$\mathrm{F(t,T_1,T_N)}=\frac{S(t)}{\mathrm{N}}\sum_{i=1}^{N} e^{\lambda(t_i-t)}$$

It is assumed that the first M<N time points of the settlement period have already passed. In other words, $T_M<t<T_{M+1}$ for M=1,2,…,N-1, it can be concluded:

$$\mathrm{F(t,T_1,T_N)}=\frac{1}{\mathrm{N}}\sum_{\mathrm{i=1}}^{\mathrm{M}}\mathrm{S(T_i)}+\frac{S(t)}{\mathrm{N}}\sum_{i=M+1}^{N} e^{\lambda(t_i-t)}$$

Since $E_t[X] = X$ if X depends only on the first time t steps.

Refer to Itô's Lemma stated in subsection 3.5. In our case X(t)= S(t), α(X,t)=λS(t) and b(X,t)=σS(t). Since $\mathrm{F}(\mathrm{t},\mathrm{T}_1,\mathrm{T}_\mathrm{N})$ is a function of S(t) and t, Itô's Lemma can be applied:

$$dF(S(t),t) = \left(\lambda(S(t),t)\frac{\partial F}{\partial S} + \frac{\partial F}{\partial t} + \frac{1}{2}\sigma(S(t),t)^2\frac{\partial^2 F}{\partial S^2}\right)dt + \sigma(S(t),t)\frac{\partial F}{\partial S}dW(t) \qquad (A.7)$$

First consider the case there $t<T_1$ so where:

$$F(t,T_1,T_N) = \frac{S(t)}{N}\sum_{i=1}^{N} e^{\lambda(t_i-t)}$$

Since then

$$\frac{\partial F}{\partial S} = \frac{1}{N}\sum_{i=1}^{N} e^{\lambda(t_i-t)}, \frac{\partial F}{\partial t} = -\frac{\lambda S(t)}{N}\sum_{i=1}^{N} e^{-\lambda(t_i-t)}, \frac{\partial^2 F}{\partial S^2} = 0$$

So it can be seen that:

$$dF(t,T_1,T_N) = \left(\lambda S(t)\frac{1}{N}\sum_{i=1}^{N} e^{\lambda(t_i-t)} - \frac{\lambda S(t)}{N}\sum_{i=1}^{N} e^{\lambda(t_i-t)} + \frac{1}{2}(\sigma S(t))^2 \times 0\right) dt \ +$$

$$+\sigma S(t)\frac{1}{N}\sum_{i=1}^{N} e^{\lambda(t_i-t)} dW(t)$$

$$\Rightarrow dF(t,T_1,T_N) = \sigma S(t)\frac{1}{N}\sum_{i=1}^{N} e^{\lambda(t_i-t)} dW(t) = \sigma F(t,T_1,T_N)dW(t) \qquad (A.8)$$

Now consider the case where $T_M<t<T_{M+1}$ for M=1,2,…N-1, so:

$$F(t,T_1,T_N) = \frac{1}{N}\sum_{i=1}^{M} S(T_i) + \frac{S(t)}{N}\sum_{i=M+1}^{N} e^{\lambda(t_i-t)}$$

Since then

$$\frac{\partial F}{\partial S} = \frac{1}{N}\sum_{i=M+1}^{N} e^{\lambda(t_i-t)}, \frac{\partial F}{\partial t} = -\frac{\lambda S(t)}{N}\sum_{i=M+1}^{N} e^{\lambda(t_i-t)}, \frac{\partial^2 F}{\partial S^2} = 0$$

It can be seen that:

$$dF(t,T_1,T_N) = \left(\lambda S(t)\frac{1}{N}\sum_{i=M+1}^{N} e^{\lambda(t_i-t)} - \frac{\lambda S(t)}{N}\sum_{i=M+1}^{N} e^{\lambda(t_i-t)}\right.$$

$$\left. + \frac{1}{2}(\sigma S(t))^2 \times 0\right) dt \ + \sigma S(t)\frac{1}{N}\sum_{i=M+1}^{N} e^{\lambda(t_i-t)} dW(t)$$

$$\Rightarrow dF(t,T_1,T_N) = \sigma S(t)\frac{1}{N}\sum_{i=M+1}^{N} e^{\lambda(t_i-t)} dW(t) = \sigma F(t,T_1,T_N)dW(t) \qquad (A.9)$$

Summing the results

$$dF(t,T_1,T_N) = \sigma(t)F(t,T_1,T_N)dW_t^Q$$

$$\text{Where } \sigma = \begin{cases} \sigma & , if\ t \le T_1 \\ \sigma\frac{\frac{S(t)}{N}\sum_{i=M+1}^{N} e^{\lambda(T_i-t)}}{F(t,T_1,T_N)} & , if\ T_M < t < T_{M+1}, M = 1,2,3,\dots,N-1 \end{cases}$$

A.4 Derivation of equation 14

Consider:

$$\sigma_F^2 = \int_t^{T_N} \sigma_\alpha(s)^2 \, ds$$

$$\int_t^{T_1} \sigma^2 ds + \int_{T_1}^{T_2} \sigma^2 (\frac{N-1}{N})^2 ds + \int_{T_2}^{T_3} \sigma^2 (\frac{N-2}{N})^2 ds + \cdots + \int_{T_M}^{T_{M+1}} \sigma^2 (\frac{N-M}{N})^2 \, ds + \cdots + \int_{T_{N-1}}^{T_N} \sigma^2 (\frac{1}{N})^2 \, ds$$

This equals to:

$$\sigma^2(T_1 - t) + \sigma^2(T_2 - T_1)(\frac{N-1}{N})^2 + \sigma^2(T_3 - T_2)(\frac{N-2}{N})^2 + \cdots \sigma^2(T_{M+1} - T_M)(\frac{N-M}{N})^2 + \cdots \sigma^2(T_N - T_{N-1})(\frac{1}{N})^2$$

This can be simplified to:

$$\sigma_F^2 = \sigma^2(T_1 - t) + \sigma^2 \Delta (\left(\frac{N-1}{N}\right)^2 + \left(\frac{N-2}{N}\right)^2 + \cdots + \left(\frac{N-M}{N}\right)^2 + \cdots + \left(\frac{1}{N}\right)^2)$$

It can be shown that the term between the brackets equals: $\frac{1}{3}N - \frac{1}{2} + \frac{1}{6N}$, In other words:

$$\sigma_F^2 = \sigma^2(T_1 - t) + \sigma^2 \Delta (\frac{1}{3}N - \frac{1}{2} + \frac{1}{6N})$$

Since Δ= $\Delta = \frac{T_N - T_1}{N-1}$, it can be written that:

$$\sigma_F^2 = \sigma^2(T_1 - t) + \sigma^2(T_N - T_1)\frac{\frac{1}{3}N - \frac{1}{2} + \frac{1}{6N}}{N-1}$$

And this equals to:

$$\sigma_F^2 = \sigma^2(T_1 - t) + \sigma^2(T_N - T_1)R(N) \qquad \text{(A.10)}$$

Where σ is the volatility and $R(N) = \frac{1 - \frac{3}{2N} + \frac{1}{2N^2}}{3 - \frac{3}{N}}$.

A.5 Derivation of formula for $\boldsymbol{\sigma^2_{F,m.r.}(t, T_1, T_N)}$

$$\sigma^2_{F,m.r.}(t, T_1, T_N) = \int_t^{T_N} \sigma^2_{m.r.}(s, T_1, T_N) ds$$

This equals to:

$$\int_t^{T_1}((\sigma_s-\sigma_A)(\frac{e^{-\lambda(T_1-t)}-e^{-\lambda(T_N-t)}}{\lambda(T_N-T_1)})+\sigma_A)^2ds+\int_{T_1}^{T_N}((\sigma_s-\sigma_A)\left(\frac{1-e^{-\lambda(T_N-t)}}{\lambda(T_N-T_1)}\right)+\sigma_A\frac{T_N-t}{T_N-T_1})^2ds=$$

$$\int_t^{T_1}((\sigma_s-\sigma_A)^2\left(\frac{e^{-\lambda(T_1-t)}-e^{-\lambda(T_N-t)}}{\lambda(T_N-T_1)}\right)^2+2\sigma_A(\sigma_s-\sigma_A)\left(\frac{e^{-\lambda(T_1-t)}-e^{-\lambda(T_N-t)}}{\lambda(T_N-T_1)}\right)+\sigma_A^2)ds+$$

$$\int_{T_1}^{T_N}(\sigma_s-\sigma_A)^2\left(\frac{1-e^{-\lambda(T_N-t)}}{\lambda(T_N-T_1)}\right)^2+2\sigma_A(\sigma_s-\sigma_A)\left(\frac{T_N-t}{T_N-T_1}\right)\left(\frac{1-e^{-\lambda(T_N-t)}}{\lambda(T_N-T_1)}\right)+\sigma_A^2(\frac{T_N-t}{T_N-T_1})^2ds$$

Using some basic integration techniques it can be shown that this equals to:

$$\frac{1}{2}(\sigma_s-\sigma_A)^2(e^{\lambda T_N}-e^{\lambda T_1})^2\left(1-e^{-2\lambda(T_1-t)}\right)\frac{e^{-2\lambda T_N}}{\lambda^3(T_N-T_1)^2}+2(\sigma_s-\sigma_A)\sigma_A\frac{1-e^{-\lambda(T_N-T_1)}-e^{-\lambda(T_1-t)}+e^{-\lambda(T_N-t)}}{\lambda^2(T_N-T_1)}+\sigma_A^2(T_1-t)+\frac{(\sigma_S-\sigma_A)^2}{2\lambda^3(T_N-T_1)^2}(1-\left(2-e^{-\lambda(T_N-T_1)}\right)^2+2\lambda(T_N-T_1)\Big)+\frac{(\sigma_S-\sigma_A)\sigma_A}{\lambda^3(T_N-T_1)^2}(2\left(e^{-\lambda(T_N-T_1)}-1\right)+\lambda^2(T_N-T_1)^2+2e^{-\lambda(T_N-T_1)}(T_N-T_1)+\frac{1}{3}\sigma_A^2(T_N-T_1) \tag{A.11}$$

Appendix B (Matlab Code)

B.1 Explicit formula

```
clear all
S0=25000; %Initial spot freight rate
K=30000; %Strike price
l=0.03; %Risk neutral freight rate drift
s=0.3; %Volatility
N=21; %number of timesteps in settlement period
D=[31,28,31,30,31,30,31,31,30,31,30,31];
for i= 1:12 %There are 12 settlement periods
T1(i)=1/252 + N/252*(i-1); %Start of each settlement
period
TN(i)=N/252 + N/252*(i-1); %End of each settlement period
Dt(i)=(TN(i)-T1(i))/(N-1); %Length timestep of each
settlement period
RN=(1-3/(2*N)+1/(2*N^2))/(3-3/N); %R(N) in equation 13
sF(i)=s^2*T1(i)+s^2*(TN(i)-T1(i))*RN; %Volatility of
future prices
F(i)=S0*((exp(l*(TN(i)-0))/N)*((exp(-l*N*Dt(i))-1)/(exp(-
l*Dt(i))-1)));
%Future prices at t=0
d1(i)=(log(F(i)/K)+0.5*sF(i))/(sqrt(sF(i)));
d2(i)=d1(i)-sqrt(sF(i));
N1(i)=normcdf(d1(i));
N2(i)=normcdf(d2(i));
C(i)=exp(-l*TN(i))*D(i)*(F(i)*N1(i)-K*N2(i)); %Black's
formula
end
C %Value caplets using Black's formula
```

B.2 Simple Monte Carlo

```
clear all
randn('state',100)
S=25000; %Initial spot freight rate
K=30000; %Strike price
l=0.03; %Risk neutral freight rate drift
s=0.3; %Volatility
M=5000000; %number of paths
N=21; %number of timesteps in settlement period
D=[31,28,31,30,31,30,31,31,30,31,30,31];
T1=zeros(12,1);
TN=zeros(12,1);
Dt=zeros(12,1);
V1=zeros(12,1);
bM=zeros(12,1);
SD=zeros(12,1);
for i= 1:12 %There are 12 settlement periods
T1(i)=1/252 + N/252*(i-1); %Start of each settlement
period
TN(i)=N/252 + N/252*(i-1); %End of each settlement period
Dt(i)=(TN(i)-T1(i))/(N-1); %Length timestep for each
settlement period
%Monte Carlo:
V2= zeros(M,1);
for j = 1:M
samples = randn(N+(i-1)*N,1); %Random standard normal
variables
Sval = S*cumprod(exp((l-
0.5*s^2)*Dt(i)+s*sqrt(Dt(i))*samples));
%Freight rate paths
Savg=1/N*sum(Sval(round(T1(i)/Dt(i)):round(TN(i)/Dt(i))))
;
%Average freight rates during settlement
V2(j) = exp(-l*TN(i))*D(i)*max(Savg-K,0); %Value caplets
at t=0
end
%Values:
V1(i) = mean(V2) %Estimate caplets at t=0
bM(i)=std(V2); %Standarddeviation caplet prices
SD(i)=bM(i)/sqrt(M); %Standarderror caplets
cap=sum(V1); %Value cap at t=0
end
```

B. 3 Monte Carlo using control variates

```
clear all
randn('state',100)
S=25000; %Initial spot freight rate
K=30000; %Strike price
l=0.03; %Risk neutral freight rate drift
s=0.3; %Volatility
M=5000000; %number of paths
N=21; %number of timesteps in settlement period
D=[31,28,31,30,31,30,31,31,30,31,30,31];
T1=zeros(12,1);
TN=zeros(12,1);
Dt=zeros(12,1);
V1=zeros(12,1);
bM=zeros(12,1);
SE=zeros(12,1);
Creal=zeros(12,1);
Zmean=zeros(12,1);
Zstd=zeros(12,1);
SEZ=zeros(12,1);
for j= 1:12 %There are 12 settlement periods
T1(j)=1/252 + 21/252*(j-1); %Start of each settlement
period
TN(j)=21/252 + 21/252*(j-1); %End of each settlement
period
Dt(j)=(TN(j)-T1(j))/(N-1); %Length timestep of each
settlement period
%Black scholes value European call option:
Creal(j)=ch08(S,K,l,s,TN(j))*D(j); %the function ch08 is
obtained from from Higham(2011)
V2 = zeros(M,1);
Cmc=zeros(M,1);
for i = 1:M
%Normal Monte Carlo:
samples = randn(21+(j-1)*21,1); %Random standard normal
variables
Svals = S*cumprod(exp((l-
0.5*s^2)*Dt(j)+s*sqrt(Dt(j))*samples)); %Freight rate
paths
Smean=1/N*sum(Svals(round(T1(j)/Dt(j)):round(TN(j)/Dt(j))
));
%Average freight rates during settlement
V2(i) = exp(-l*TN(j))*D(j)*max(Smean-K,0); %Value caplets
at t=0
%Monte Carlo using control variates:
Cmc(i) = exp(-
l*TN(j))*D(j)*max(Svals(round(TN(j)/Dt(j)))-K,0);
%Estimate European call option
end
Th=cov(V2,Cmc);
```

```
th=Th(1,2)/Th(2,2); %Optimal value of thita
for i=1:M
Z(i) = V2(i) + th*(Creal(j) - Cmc(i)); %Construction of Z
end
%Values of using Monte Carlo:
V1(j) = mean(V2); %Values caplets at t=0
bM(j)=std(V2); %Standarddeviation caplets
SE(j)=bM(j)/sqrt(M); %Standard errors caplets
cap=sum(V1); %Value cap at t=0
%Values of using control variates:
Zmean(j) = mean(Z); %Values caplets at t=0
Zstd(j) = std(Z); %Standarddeviation caplets
SEZ(j)=Zstd(j)/sqrt(M); %Standard errors caplets
capcv=sum(Zmean); %Value cap at t=0
end
```

B.3.1 The ch08 function file that is used in 8.3

```
function[Creal] = ch08(S,K,l,s,tau)
%tau here is the product of D and TN
if tau>0
d1 = (log(S/K) + (l+0.5*s^2)*(tau))/(s*sqrt(tau));
d2 = d1 - s*sqrt(tau);
N1 = 0.5*(1+erf(d1/sqrt(2)));
N2 = 0.5*(1+erf(d2/sqrt(2)));
Creal = S*N1-K*exp(-l*(tau))*N2;
else
Creal=max(S-K,0);

end
```

B.4 Variance reduction using both control and antithetic variates

```
clear all
randn('state',100)
S=25000; %Initial spot freight rate
K=30000; %Strike price
l=0.03; %%Risk neutral freight rate drift
s=0.3; %Volatility
M=5000000; %number of paths
N=21; %number of timesteps in settlement period
D=[31,28,31,30,31,30,31,31,30,31,30,31];
T1=zeros(12,1);
TN=zeros(12,1);
Dt=zeros(12,1);
V=zeros(12,1);
bM=zeros(12,1);
SE=zeros(12,1);%standard error
Creal=zeros(12,1);
Zmean=zeros(12,1);
Zstd=zeros(12,1);
SEZ=zeros(12,1);%standard error Z
for j= 1:12 %There are 12 settlement periods
T1(j)=1/252 + 21/252*(j-1); %Start of each settlement
period
TN(j)=21/252 + 21/252*(j-1); %End of each settlement
period
Dt(j)=(TN(j)-T1(j))/(N-1); %Length timestep of each
settlement period
%Black scholes value European call option:
Creal(j)=ch08(S,K,l,s,TN(j))*D(j);
V2 = zeros(M,1);
Cmc=zeros(M,1);
V3=zeros(M,1);
Vfinal=zeros(M,1);
Cmc2=zeros(M,1);
Cmcfinal=zeros(M,1);
for i = 1:M
%Monte Carlo using antithetic variables:
samples = randn(21+(j-1)*21,1); %Random standard normal
variables
Svals = S*cumprod(exp((l-
0.5*s^2)*Dt(j)+s*sqrt(Dt(j))*samples)); %Freight rate
paths 1
Svals2 = S*cumprod(exp((l-0.5*s^2)*Dt(j)-
s*sqrt(Dt(j))*samples)); %Freight rate paths 2
Smean=1/N*sum(Svals(round(T1(j)/Dt(j)):round(TN(j)/Dt(j))
));
%Average freight rates 1 during settlement
Smean2=
1/N*sum(Svals2(round(T1(j)/Dt(j)):round(TN(j)/Dt(j))));
%Average freight rates 2 during settlement
V2(i) = exp(-l*TN(j))*D(j)*max(Smean-K,0); %Value caplets
```

```
%at t=0 for the first freight rate paths
V3(i)=exp(-l*TN(j))*D(j)*max(Smean2-K,0); %Value caplets
%at t=0 for the second freight rate paths
Vfinal(i)=0.5*(V2(i)+V3(i)); %Average values caplets at
t=0
%Control & Antithetic variates:
Cmc(i) = exp(-
l*TN(j))*D(j)*max(Svals(round(TN(j)/Dt(j)))-K,0);
%Estimate European
%call options at t=0 for the first freight rate paths
Cmc2(i) = exp(-
l*TN(j))*D(j)*max(Svals2(round(TN(j)/Dt(j)))-K,0);
%Estimate European
%call options at t=0 for the second freight rate paths
Cmcfinal(i)=0.5*(Cmc(i)+Cmc2(i)); %Average values options
at t=0
end
Th=cov(Vfinal,Cmcfinal);
th=Th(1,2)/Th(2,2); %Optimal value of thita
for i=1:M
Z(i) = Vfinal(i) + th*(Creal(j) - Cmcfinal(i));
%Construction of Z
end
%Values using antithetic variates:
V(j) = mean(Vfinal); %Values caplets at t=0
bM(j)=std(V2); %Standarddeviation caplets
SE(j)=bM(j)/sqrt(M); %Standard errors caplets
cap=sum(V) %Value cap at t=0
%Values using Antithetic & Control variates:
Zmean(j) = mean(Z) %Values caplets at t=0
Zstd(j) = std(Z); %Standarddeviation caplets
SEZ(j)=Zstd(j)/sqrt(M) %Standard errors caplets
capcv=sum(Zmean) %Value cap at t=0
end
```