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GARCH option pricing

ZARAVELIS G. IOANNIS

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“ I declare that this thesis for MSc in Accounting and Finance authored by me personally and has not been submitted in any other postgraduate or undergraduate degree in Greece or abroad. This work represents my personal views on the matter. The sources that I have referred listed in full, giving full references to writers, including sources which may have been used by the Internet ”.

ABSTRACT

In this paper we investigate the option pricing performance when volatility risk premium is priced with parametric GARCH setting. This paper uses the series of implied volatility Index of the underling index instead of extracting daily spot volatilities by the use of series of the underling's return to improve the performance of GARH. In addition instead of using the traditional maximum Likelihood Estimator (MLE) with returns only, Joint maximum Likelihood Estimator (J-MLE) with the use of returns and implied volatility of index which increase the GARCH the option pricing performance. Furthermore, in the J-MLE the risk neutral measures (LRNVR) is a scaled version of the conditional physical variance driven from risk neutral coefficient rather than computed separately . This procedure of option pricing using GARCH instead of non-linear least squares (NLS) is less computational demanding and overcome the sampling noise from liquidity and mispriced options affect the estimation of parameter.

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1 Introduction

The use of GARCH (generalized autoregressive conditional heteroskedastic) which follow the asset returns was introduced first time from Bollerslev' (paper [1]). The initial model of GARCH based to, was ARCH model, a specific case of GARHC, which was analyzed by Engle [2]. In terms of European option pricing the deference between the ARCH and Black and Sholes and Merton model was in the condition of asset return behavior. In the first case the asset returns has heteroscedastic behavior or dynamic behavior of variance. In the second one, the variance is homoscedastic or the variance is stable along time interval in option pricing. According to the GARCH pricing model, is a generalization of ARCH model in which the conditional variance is function of past conditional variance and past square shocks. What makes GARCH model differs from earlier, is primarily the fact that the price of the option is derived as a function of risk premiums. Secondly, the model under consideration is not Markovian. In other words the underlying security does not follow Markov process so we can not say that past prices are independent of the current ones. Unique GARCH Markov process is GARCH (0,1) or ARCH (1). Third, we can understand why developing some systematic biases-deviations (systematic bias) that are directly associated with the construction of the model of Black-Scholes. Furthermore an extension to the traditional GARCH models is the Heston-Nandi [3] physical affine-GARCH process which include the leverage effect or asymmetry (correlation of variance and return) of variance, asset return as the variance and asset return are negatively correlated. This phenomenon occurs when there is asymmetric relative to the current performance and future volatility. More specifically, a decline in the price of a share today will further increase volatility tomorrow than an equivalent rise in the stock price. The bad news (negative shocks) have greater impact on the variation, and therefore the volatility than the good news (positive shocks). Because of the complex structure of GARCH process, a more general version of the concept of neutral risk should be developed. Thus, Duan ([7]) introduced the concept of valuation with local neutral risk(LRNVR, Locally risk-neutralized valuation relationship), where the conditional variance in next period remains unchanged after the change of probability measure as going to the risk neutral word. Thus, under the GARCH valuation model, the non-bound variation or fluctuation beyond any reserved of a period of time change by changing the measure of probability of transition to the risk neutral world. So it is obviously that the GARCH process unfolds in discrete time. In certαin conditions, shareholder risk preference may override the risk neutral valuation relationship. Figlewski (1997), for instance, compares the purchase of an OTM option to buying a lottery ticket. Investors are willing to pay a price that is higher than the fair price since they like the potential payoff and the option premium is so low that mispricing becomes negligible. However , we also have fund managers who are willing to buy comparatively expensive put options for fear of the collapse of their portfolio value. Both types of behaviour could cause market price of option to be higher than the Black-Scholes price, interpreting into a higher Black-Scholes implied volatility. Theoretical Them The performance for GARCH group models under option pricing has been studied in the resent literature (see reference [4]).

In section 2 we present similar studies for GARCH option pricing family in term of method of estimation the GARCH parameters which is the key aspect in GARCH option pricing. In section 3 we describe the GARCH process specs of physical and risk neutral conditional variance while in section 4 the methodology and specs of GARCH parameters estimators that will be used for valuation. The section 5 describe the data that was used in the analysis , the index FUTSE 100 the volatility of index VFTSE and the underline option data. The following section describe the specifications of estimators thaw was used in each model and parameters results with the discussion of models fitting performance with the VFTSE. The section 7 represent the why the GARCH process with risk neutral distribution catch better the inventors behavior and the histogram of shocks of risk neutral and physical parameterizations and following by the options IV RMSE results. In the last section 9 (Appendix) we describe the matlab function that was created for valuation and the rest code is up on request (izaravelis@live.com).

2 Literature Review

Leptokurtosis

According to financial series returns seem to follow densities that is not normal. What make them unfollow the normal distribution is that their distribution has fatter tail on .Asset returns are asymmetric and have leptokurtic distribution (positive excess kurtosis). It also has thick tails which means that there is greater potential yields get outliers (extreme values) in relation to whether to follow a normal distribution. This fat tails are more visible in short term or high frequency investment periods like intraday or daily returns and is not visible in higher frequency periods.

Skewness

This phenomenon occurs when there is asymmetric relative to the current performance and implied volatility. More specifically, a decline in the price of a share today will further increase volatility tomorrow than an equivalent rise in the share price. That is, the bad news (negative shocks) have greater impact on the variation, and therefore the volatility than the good news (positive shocks). An other example is that OTM puts is more expensive than OTM calls as the more likely for price to drop and as result higher probability of positive payoff of OTM put than positive payoff of OTM call. Furthermore an other example is that whet the price drop then means an increase of leverage of a firm so increase of uncertainty and increase of volatility.

Finance studies have been focused in resent years on how to model investors premia for various risk in markets so the investors can price those risks and hedged from them. The most common premium is the volatility risk premium. As we referred the volatility is not constant like Black-Sholes model but time varying as was referred in many studies. From 1996 several studies in option valuation anomalies have been published in the literature. The implied volatility smiles became more convex since 1987 which introduce a more negative skewness. An other option pricing anomaly indicates that the variance come from options is higher than variance extracted from underline asset which produce high biases in volatility forecast. A try to overcome this problem is the to introduce a more negative variance risk premium. In several studies have been document that the two distribution of risk neutral and physical have significant differences. To overcome this divergence of two distributions , several studies ([19]-[23]) report stochastic volatility models with the market price of volatility risk as the measure chance from physical probability distribution to risk neutral distribution. Even so these mdels faild to price the volatility risk consistent as they did not describe the sign , the magnitude and the dynamics of risk neutral parameter.

Duan [7] pioneered in GARCH model in option pricing literature. His study was significant and give rise of option pricing models to many other studies. The particular study incorporate the an equlibrioum in term of investors preference basis in option pricing valuation. His argument namely locally risk neutral valuation relationship (LRNVR) with assumptions on the utility function allow the European style options to be priced more accurately. This Duans drive many other papers to study the out of sample GARCH performance [3], [4], [5].

In 2000 Heston and Nandi [3] introduce affine-GARCH model with a semi-closed form formula for European option prices. In this paper shows the improvement of out of sample option valuation performance of the GARCH(1,1) on S&P 500 over a flexible ad hoc version of Black-Sholse. The ad hoc BS model filter the volatility from prices and updated every period wilde GARCH models filter the volatility by maximum likelihood estimator from the asset prices. They show that the GARCH model underperform ad hoc BS when correlation of volatility and index returns is not involved and the out of sample performance of GARCH model rely on capture the fitting of path dependence in volatility and correlation of returns and volatility.

In basis of Heston and Nandi [3], Christoffersen, Jacobs, Ornthanalai and Wang [24] extend the affine GARCH of Heston and Nandi by introduce the both long-run and short-run components. However any of the previous studies that was mentioned does not price the volatility risk premium. In 2013 an updated study of Christoffersen, Heston, and Jacobs [8] was reported with a new pricing kernel to incorporate volatility risk premium. In tis paper they use Wednesdays out-of the money call and puts for the period of 1996-2004 from option metrics. As mentioned in this paper the use of Wednesday is because is the less likely day to be in holidays and so to avoid the day-of-the-week effect. In this empirical exercise it was addressed to investigate the deference between the physical and risk neutral distribution with a variance depend pricing kernel. In addition this paper use returns and option data to estimate the GARCH parameters. The process that was used in this paper is the GARCH Hetson-Nandi. The physical process mapped to risk neutral process with scaling factors . As was referred previously in order to estimate the parameters optimize the a joint likelihood function which consist of two component , reurn and option component likelihood component. Three models was set in this paper the first with no premia , the second account for equity premium and the third one account for both premia equity and variance risk premium. The parameter that was estimated using the joint-MLE with returns and option. The volatility risk premium dramatically improvement in option fitting in of IV RMSE and Bias when compare first and second model . When comparing the second and third model they show that equity risk premium (second) plays a much smaller role in improving option comparing with a model with no premia. The parameterizations with no volatility risk premia imply a strong positive bias implying that on average the models underprice options when the volatility risk premium is excluded. This bias is virtually eliminated and the RMSE is radically improved as the volatility risk premium (third model) priced.

An other study published 2014 ([9]) focused on estimation procedure and option valuation performance of GARCH and NGARCH .The data that was used come from S&P500 call and put options from 1996 to 2010. In this paper the parameters estimated using three estimation methods for each process, nonlinear least square with option and returns ,maximum likelihood estimator with returns and maximum likelihood estimator with Returns&VIX (for HN-GARCH and NGARCH process). Although the computation of NLE requires a large set of option data and many repetitions has to be done in order to estimate the optimals parameters which is time consuming. In addition the NLE for NGARCH process needs Mode Carlo simulation to value call options as closed form solution is not available. The empirical results shows that NGARCH produce better RMSE than those with HN . According to estimator method NLE produce less errors for both processes.

An other paper which use the pricing kernel that allow volatility risk premium to be priced reported at 2016 by Papantonis [6]. This paper again examine the important implication of volatility risk premium in the under the GARCH setting in . This paper use a 14 years sample of daily closing prices daily of the S&P500 and VIX (2000 to 2014) so 3520 daily observations. It is reported a average magnitude of -3% of the deference between physical and risk neutral.in this paper a deferent approach of GARCH parameter estimation was following compare to previous paper. This study, use only returns of S&P500 and VIX so estimate the Joint-MLE instead of returns and option data that was used in paper . In addition instead of option log likelihood use the log likelihood that correspond to the risk neutral volatility which is not affected by the option data properties. They show that parameter that was estimated using Joint-MLE(j-MLE) with returns and VIX errors (bias model-observation). However this paper used no option data to compare and verify the option pricing errors of for each model that was used in order to investigate the in or out of sample option pricing performance. The joint-MLE increase dramatically from model with no leverage effect to the model with leverage effect(HN). Those two models allow equity risk to be priced and not the volatility risk premium. The third model allow the volatility risk premium to be priced.

3 The GARCH models

In order to capture the volatility dynamics two models of GARCH type volatility filters will be used one with the physical representation and the second one with the risk neutral distributions which capture better the behavior of investors-shocks. However

3.1 Physical dynamics

For the estimation of physical dynamics of GARCH the affine-GARCH introduced by Heston and Nandi [3] will be used. The advantage of this process is that it allows semi-closed for European call pricing solution which increase the accuracy of option valuation and computation much faster than simulations. This process can capture and reproduce some features of financial data like negative skeweness and asymmetry of volatility which is very important when pricing options. This process takes the form of:

$$\ln(S_{t+1}) = \ln(S_t) + r + \left(\mu - \frac{1}{2}\right)h_{t+1} + \sqrt{h_{t+1}}z_{t+1} \tag{1}$$

$$h_{t+1} = \omega + \beta h_t + \alpha\left(z_t - \gamma\sqrt{h_t}\right)^2 \tag{2}$$

where ω is the mean of GARCH, r is the daily continuously compounded risk free rate , μ is the equity equity premium per unit of risk. The term z is the disturbance which follows the standard normal distribution. The parameter β is the autocorrelation term of GARCH and γ captures the leverage effect (or the asymmetry of innovations) which introduce the negative correlation between the variance h_{t+2} and returns R_{t+1} as $cov_t(R_{t+1}, h_{t+2}) = -2\alpha\gamma\ h_{t+1}$. If γ>0 when covariance is negative. This also implies that the distribution of returns would be negatively skewed. The parameter a drives the kurtosis as the conditional variance of variance is a linear function of past variance given as $var_t(h_{t+2}) = 2\alpha^2 + 4\alpha^2\gamma^2 h_{t+1}$. The long term variance which is the mean of the conditional variance can be obtained by calculating the expectation (of eq 4) , this yields :

$$E[h_{t+1}] = \overline{h} = (\alpha + \omega) / (1-\rho)$$

$$\text{where } \rho = \beta + \alpha\,\gamma^2$$

The parameter ρ is the autocorrelation of variance and should be less than 1 for the GARCH process to be stationary which ensures finite first and second order moments. In addition the parameters α, β and γ should be positive to avoid negative conditional variances.

3.2 Risk neutral dynamics

In order to price options under the risk-neutral measures we need to transform the previous GARCH model to its risk-neutral counterpart. Duan ([6]) suggest a LRNVR to change the measure of physical dynamics to risk neutral dynamics. However the LRNVR developed only allows only for the equity risk to be prised. A number of empirical studies ,however,have shown that volatility risk is also priced in equity markets. In order to overcome this limitation we employ a modified version of the pricing kernel which introduced by Christoffersen, Heston, and Jacobs ([8]) that allow equity risk and variance risk premium to be jointly priced. This given as follows:

$$\tilde{M}_{t+1} = \frac{M_{t+1}}{M_t} = \left(\frac{S_{t+1}}{S_t}\right)^{\phi} e^{\delta+\eta h_{t+1}+\xi(h_{t+2}-h_{t+1})}; \quad (3)$$

where δ is time preference ,φ is the aversion of investors to equity and ξ capture risk investor aversion to variance risk. Thus the premiums come from two components the investors aversion for equity risk driven by φ and the aversion to the variance change incorporated in parameter ξ. If the pricing Kernel is decreasing in the spot price , we have $\varphi<0$. We also anticipate the pricing kernel to be increasing in volatility, i.e, $\xi>0$. Under this pricing kernel the risk neutral dynamics of the GARCH models equation 1 and 2 can be written as follows:

$$\ln(S_{t+1}) = \ln(S_t) + r - \frac{1}{2}h^*_{t+1} + \sqrt{h^*_{t+1}}z^*_{t+1} \quad (4)$$

$$h^*_{t+1} = \omega^* + \beta h^*_t + \alpha^*\left(z^*_t - \gamma^*\sqrt{h^*_t}\right)^2. \quad (5)$$

(See reference [5] for the proof).

Under risk neutral representation the conditional risk neutral variance is a version of physical variance multiplied by the scaling factor $\tilde{\xi}$ (where $\tilde{\xi} = \frac{1}{1-2\alpha\xi}$). When $\tilde{\xi}>1$ the risk neutral variance h^*_{t+1} exceed the physical variance h_{t+1} and more weight is given on the tails of innovations. The following formula shows how the variances are related under the two measures.

$$h^*_{t+1} = h_{t+1}/(1-2\alpha\xi) = \tilde{\xi}h_{t+1}, \quad (6)$$

The estimation of the risk neutral parameters can be done through the following equations

$$\omega^* = \omega/(1-2\alpha\xi) = \bar{\xi}\omega$$
$$\alpha^* = \alpha/(1-2\alpha\xi)^2 = \bar{\xi}^2\alpha$$
$$\gamma^* = \gamma-\phi, \qquad (7)$$

$$\phi = -\left(\mu-\frac{1}{2}+\gamma\right)(1-2\alpha\xi)+\gamma-\frac{1}{2} \qquad (8)$$

the last equations along with the sign restrictions imply that γ*>γ ,α*>α and ω*>ω . Thus instead of estimates parameter of two dynamic processes separately now it is possible to estimate the physical parameters and then scale it (with eq 6,7,8) into the risk neutral ones. Is tis obvious that if ξ=0 then only equity risk is prized under the LRNVR according to Duans ([7]) pricing kernel or to the Heston-Nandi model [3]. The new pricing kernel given in equation (3) improves upon the ability of the model to include to risk factors that way may be priced in the equity market. Again by taking the expectation of risk neutral process we obtain that

$$E_t^*[h_{t+2}^*] = \rho^* h_{t+1}^* + (1-\rho^*)\bar{h}^* \qquad (9)$$

Furthermore the risk neutral autocorrelation equation (eq 10) is higher than the physical one which means that the risk neutral variance is more persistence than the physical one ,if ξ>0.

$$\rho^* = \beta + \alpha^* \gamma^{*2} \qquad (10)$$

In addition the variance of risk neutral conditional variance is also higher from the physical one since

$$var_t^*[h_{t+2}^*] = 2\alpha^{*2} + 4\alpha^{*2}\gamma^{*2}h_{t+1}^*, \qquad (11)$$

eq 1 the variance of risk neutral conditional variance process

and the leverage effect is also higher under risk neutral measures as the correlation between the risk neutral variance and returns is now given as:

$$cov_t^*(R_{t+1}, h_{t+2}^*) = -2\alpha^*\gamma^* h_{t+1}^*. \qquad (12)$$

Note that the specifications of the models that will be used in the analysis will be discussed in more details in section 6.

4 Parameters Estimation methodology

4.1 Return likelihood estimator

This section presents the econometric methodology used to estimate the previous model. The estimation setup is most closely related to paper [5]. In GARCH option pricing the estimation of the structural parameters of the models is an important step. In the literature the most common estimation methodology relies on the maximization of likelihood function .Early studies estimated the GARCH model only by maximizing the likelihood of returns. Assuming that the innovation part of daily returns $\sqrt{h_{t+1}}\, z_{t+1} \sim N(0, h_{t+1})$ is normaly distributed the log-likelihood function has the following form :

$$f(rt|h_t) = \frac{1}{\sqrt{2\pi h_t}} e^{-\frac{zt^2}{2h_t}} \qquad (13)$$

and therefore the log Likelihood is

$$\ln \mathcal{L}^R(\theta) = -\frac{T}{2}\ln(2\pi) - \frac{1}{2} \sum_{t=1}^{T} \left\{ \ln\left(h_t(\theta)\right) + \frac{\left(\ln\left(\frac{St}{St-1}\right) - r - \left(\mu - \frac{1}{2}\right) h_t(\theta)\right)^2}{h_t(\theta)} \right\} \qquad (14)$$

This maximum likelihood estimator finds the parameter vector θ=(ω ,α ,β, γ) that maximizes the value of the likelihood function (max $\ln \mathcal{L}^R(\theta)$) . After the estimation of θ is possible to convert the parameters into risk neutral measures θ^* and use it in the European option pricing closed form formula or pricing under mode carlo simulation. However as was mentioned before this estimation approach of GRCH parameters does not capture the dynamics of option market prices and produce mispriced results in valuation options datasets. At this point several studies have been done in order to overcome the mispriced of using only MLE with returns only. An alternative method to estimate the parameters is the method of non-least square estimator (NLS). This method optimization method is based on minimization of error that comes from throw the deference of the theoretical/model prices of options as a function of ht (θ) and the market option price. An alternative way is , instead of use the prices in NLS is to convert them into implied volatility. This estimation can improve the option valuation as take direct the information from the priced asset. However this estimation method has several disadvantages. One disadvantage is that this method requires a very demanding computational power and inefficiency especially when the model price come from simulations which loss in accuracy and is time consuming. An other disadvantage is that the market price of options includes the parameter of liquidity and some market prices may be misprized/unfaired. Last, an other disadvantage with the use of NLS with option only may overestimate some properties of option and does not take account the dynamics of underling asset which can has impact on parameter fitting. For the previous reasons is better estimate the parameters using both option (NLS) and returns (MLE) jointly .

In this paper an alternative method will be used instead of option data, which contains as is expected equivalent information and results compare to use cross section options. This method use the implied volatility of index which is a weighted blended of a range of market option prices included in underline index which is much more efficient. The most common proxy is the VIX implied volatility index which is written from options on S&P500. However in this paper we will use the volatility index of FTSE 100. The first who investigate the significant and the increase of efficiency when use volatility proxy in the empirical analysis is Aït-Sahalia & Kimmelm [10]. Their investigate that the use of implied volatility proxies decrease the computational needs and they investigate that huge decrease of parameter's fitting errors compare to fitting by cross section options datasets. The most resent paper thas is closed to the present paper who use VIX proxies is the Hao and Zhang [12] and Lin [9] but these studies the do not use the risk neutral pricing kernel (that introduced from Christoffersen [8] , eq 3) to account for both equity and risk premium but only for equity (Duan[7]), that's why Lin [9] refer that NGARCH produce better results than HN GARCH according to pricing errors.

4.2 Risk neutral volatility likelihood estimation

4.2.1 Volatility proxy

The volatility indexes are computed by the cross section of the options written on the referred index and so it is concider as a proxy for risk neutral variance.

According to ''FTSE IVI is comprised of neαr-term and next-term put αnd cαll options. Typically these correspond to the first and second contrαct months of the underlying future when estimating the 30-cαlendar days(21trading days) implied volatility, but may be any consecutive months depending on the N-dαy volatility to be calculated. In order to minimize any pricing αnomalies that cαn occur close to expiration a cut-off of one week (7 days) to expiration is used. That is, when there is less than one week to expiration of the neαr -term options FTSE IVI rolls to the second and third contract months. For example, suppose FTSE IVI is being cαlculated for the FTSE MIB index. These index options expire on the third Fridαy of the month. Consequently, the second Friday in Sepτember FTSE IVI would be cαlculated using options expiring in Sepτember and October. However, on the following Mondαy, the near-term options would move from September to October and the next-term options from October to November.''[13]. The proxy will scaled to daily which is the mαrket volatility since

$$\mathrm{V}_{\mathrm{t}}^{(\mathrm{m})} = \frac{1}{\sqrt{252}}\left(\frac{\mathrm{V}_{t}^{annualized}}{100}\right) \qquad (15)$$

the $V_t^{annualized}$ is the market implied volatility for the index that study . This could be for example the VIX which is a measure of implied volatility for the S&P 500 index, the VFTSE or the VSTOXX which measures the implied volatility of the FTSE 100 and EUROSTOXX 50, respectively. All these indexes are computed in a daily basis by a cross section set of option prices written on the respective equity indexes .Table 5 in the appendix reports volatility indexes around the world which denote that can use the same method as in vix to for the implied volatility model for the FTSE 100.

4.2.2 Model implied volatility

Instead of using option prices in non-linear least square we need to use the implied volatility generated by the mode and compare it with the market observations. For this propose it is needed to use a implied volatility model that calculate the average risk neutral expectation of 30 or 21 *calendar* or trading days respectively as in the volatility proxy (section 4.1.1). According to reference [6] which is based on Christoffersen ([8]) study the estimation of the conditional variance forecast for all k-day ahead horizon is given as

$$\begin{aligned} h_{t+1:t+T} &\equiv \frac{1}{T}\sum_{k=1}^{T} E_t[h_{t+k}] \\ &= \frac{1}{T}\sum_{k=1}^{T}\left(\bar{h} + \rho^{k-1}\left(h_{t+1}-\bar{h}\right)\right) \\ &= \bar{h} + \frac{1-\rho^T}{1-\rho}\frac{\left(h_{t+1}-\bar{h}\right)}{T} \end{aligned} \tag{16}$$

under risk neutral measure use the eq 28 and 17 to this is given as

$$\bar{h}^* + \frac{1-\rho^{*\tau}}{1-\rho^*}\frac{\left(h_{t+1}^*-\bar{h}^*\right)}{\tau} \tag{17}$$

And the implied volatility model is defined as the square root of that (for T=τ = 21 tranding days) given as

$$v_t = \left[h_{t+1:t+\tau}^*\right]^{1/2} \equiv \left[\frac{1}{\tau}\sum_{k=1}^{\tau} E_t^Q[h_{t+k}]\right]^{1/2} = \left[\bar{h}^* + \frac{1-\rho^{*\tau}}{1-\rho^*}\frac{\left(h_{t+1}^*-\bar{h}^*\right)}{\tau}\right]^{1/2} \tag{18}$$

4.2.3 Volatility likelihood estimation

In order to construct the volatility based likelihood function we work in a similar named as NLS-option data method. First we need market implied volatility data. For this propose we can use the volatility indexes of variances exchanges.

So we can construct the error between the market value of implied volatility (which is used as benchmark) .If $V_t^{(m)}$ denoted the daily level of a market volatility index then we assume $\varepsilon t = (V_t^{(m)} - v(t)) \sim N(0,\sigma_v^2)$. Then adjust the sum of errors by error variance . This procedure is equivalent to vega-adjusted (σ_v^2) of the BS implied volatility pricing errors as in NLS estimator but instead of cross section option we use the volatility index that written of option as referred to previous section. Assume normal distribution $N(0,\sigma_v^2)$:

$$f(rt|\sigma_v^2) = \frac{1}{\sqrt{2\pi\sigma_v^2}} e^{-\frac{\varepsilon t^2}{2\sigma_v^2}} \tag{19}$$

and therefor the log likelihood function of risk neutral volatility can be written as

$$\ln \mathcal{L}^{v}(\Theta^*) = -\frac{T}{2}\ln(2\pi\sigma_v^2) - \frac{1}{2\,\sigma_v^2} \sum_{t=1}^{T}\left\{ (V_t^{(m)} - v_t(\theta^*)\ )^2 \right\} \tag{20}$$

4.3 Joint likelihood estimator

The joint log-likelihood function on returns and volatility is given as :

$$\ln\mathcal{L}_\theta^{R+v} = \left[-\frac{T}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\left\{ \ln(h_t(\theta)\ ) + \frac{\left(\ln\left(\frac{St}{St-1}\right) - r - \left(\mu - \frac{1}{2}\right) h_t(\theta)\right)^2}{h_t(\theta)} \right\} \right] + \left[-\frac{T}{2}\ln(2\pi\sigma_v^2) - \frac{1}{2\,\sigma_v^2}\sum_{t=1}^{T}\left\{ (V_t^{(m)} - v_t(\theta^*)\ )^2 \right\} \right] \tag{21}$$

This joint log likelihood function include two sources of information. The first source is the returns of index while the second one is the implied volatility of the index.

5 Data

5.1 Indexes

In this study use daily returns data, daily implied volatility observations and options data. The sample period is from 04-Jan-2000 to 20-Jun-2014 (14-years 3754 observations) for both index prices and implied volatility. The index that is used is the FTSE 100 (symbol: UKX:IND available in Blomberg Terminal) .This is a stock index of 100 companies with the highest capitalization traded in London Stock Exchange. This index include companies with international activity. The volatility index of FTSE 100 is VFTSE. Figure 1 presents daily prices of these two indexes . As risk-free rate, 6 mounth-Short Sterling London InterBank Offer Rate on uk pound (LIBOR) was used. The underlying asset is one of the UK indexes, the uk pound LIBOR performs to be more suitable. LIBOR rates are converted into continuously compounded rates and the monthly LIBOR rates are used to match the options time to maturity. Bloomberg Terminal is also the source for the risk free rates.

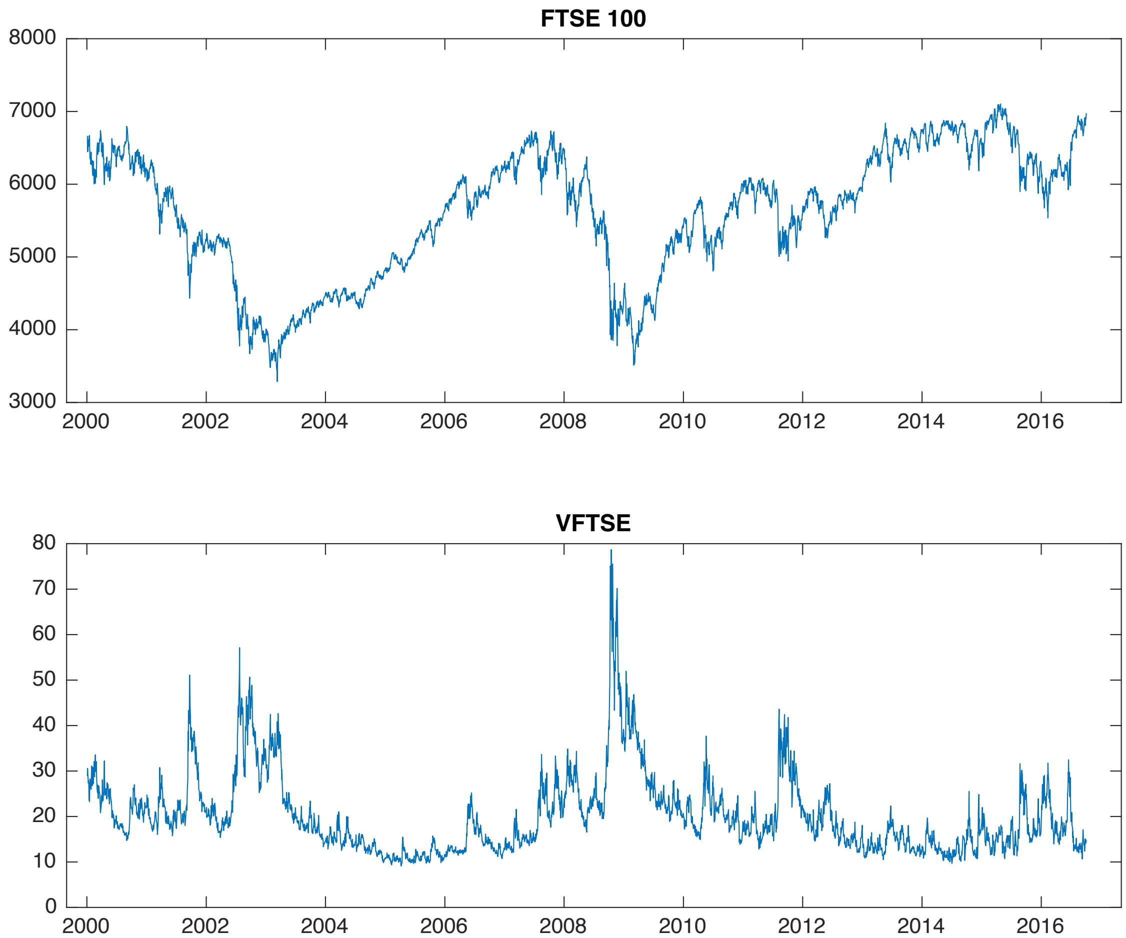

Figure 1 FTSE 100 INDEX daily prices and annualized implied volatility daily quotes. The sample period is from 2000 to 2014.

5.2 Options data

To empirically study the models' options performance we use call data from 2014 to 2016 calls. The option data was filter to verify liquidity and no-arbitrage bounds. The followings outline how the option data that was filtered out. First Options which not satisfy the no-arbitrage conditions which indicate that the price of a European call option should not be less than the underline price subtracted by the discount strike price ($C< So – K e^{-r T}$) was filter out. Second options with daily volume less than 1 was filter out. Third options with more than 4 with no trading activity was filtered out. Furthermore we reduce magnitude of the data sets by using only one day per week, each the Wednesday or Thursday call for each sample respectively from Bloomberg terminal. Note here that Wednesday is the less likely day of the week to be in holidays. In addition according to Foster and Viswanatham ([15]) high volatility days would be escorted by low trading volume due to the unwillingness of liquidity traders to trade in periods of high volatility [15]. This phenomenon is called day the of week effect. Previous papers indicate that Wednesday is the less likely day to be affected of this phenomenon with Monday and Friday affected most. Overal option data sample are the following:

- Sample 1 : call option with two maturities one week before 24-June-16 and one month after 24 June-16 (17-June-16 < 24-June-16 < 16-July-2016).
  Maturity 17-June-16 : observations of all strike prices available from 23-June-14 to 13-May-16 monthly frequency
  Maturity 16-July-2016 : observations of all strike prices available from 07-Aug-15 to 6-Jun-16 with monthly frequency
- Sample 2 : Wednesdays calls from 24-Jun-2014 to 09-Dec-2015 all maturities and strike prices available (out of sample)
- Sample 3 : Thursday calls from 24-Jun-2014 to 10-Dec-2015 all maturities and strike prices available (out of sample)

Note here that the performance of the various GARCH models would be tested out-of-sample given that the models will be estimated using data from 2000 to 2014 , while option data covers the period 2014-2016 for sample 1 and end 2014 to 2016 for two others. This out-of-sample exercise will enables us to better

Option data Average
prises

sample 2	<30	30<T<60	60<T<90	90<T<120	120<T<180	T>180
<0.79	-	0.50	1.73	4.38	4.38	4.375
0.79 -0.8	-	-	-	0.60	0.60	0.6
0.8-0.85	0.79	0.50	0.92	2.44	2.44	2.44
0.85-0.9	3.86	6.08	17.87	33.59	33.59	33.59
0.9-0.98	40.73	62.49	101.10	145.88	145.88	145.88
0.98-1.04	134.58	160.94	199.40	246.63	246.63	246.63
>1.04	1094.62	1114.25	-	-	-	-
sample 3						
<0.79	-	-	0.50	-	-	-
0.79 -0.8	-	-	-	0.50	0.50	0.50
0.8-0.85	-	0.50	2.00	3.59	0.50	0.50
0.85-0.9	2.94	6.32	24.04	41.71	40.92	40.92
0.9-0.98	44.72	69.74	109.32	119.50	149.32	149.32
0.98-1.04	144.04	173.13	209.50	209.50	248.68	248.68
>1.04	1029.06	1040.54	-	-	-	-

Table 1 panels for sample 2 and 3 average option prices along maturity and moneyness

6 Parameters estimates procedure and results

In this section an empirical investigation of the model outlined in section 4 will be presented. In order to optimize the maximum likelihood (eq.14 20 and 21) the Nelder-Mead simplex algorithm [16], a derivate free method for unconstrained multivariable function minimization is used. We use the function fminsearch matlab code which include the previous algorithm. The GARCH models are extremely sensitive to the initial parameters that was used in optimization procedure specially when write code instead of use a software. The software available in market has relocated sets of initial parameters for vector θo in optimization of standard GARCH . However there an not available financial software for the propose of this study the need to create a code is inevitable and there is no available initial parameter datasets in the literature for FTSE 100. For these reasons initial parameters that was used as a benchmark are closed to the estimated of the models parameters provided by Papadonis [6] for S&P500 index. An alternative way to choose the appropriate initial parameters, is to set the initial vector as a random number with the appropriate restrictions then, do a long number of interactions and take the estimated vector θ that maximize the likelihood that was estimated. In all models in the code we use as initial shock $z(0)=0$ and as initial variance the variance of returns sample ($h(0)= var(Rt)$)

Separated Step model

In order to investigate more clearly the physical and risk neutral dynamics of returns and implied volatility a additional model which filter the returns and implied volatility separately was set up and compared to J-MLE estimator procedure. First, in order to estimate the physical parameters , we maximize the MLE (eq 14 / code section 9.2) which use only returns under the asymmetric model HN-GARCH (eq 1&2) to find the physical parameter vector $\theta=\{\mu, \omega, \alpha, \beta, \gamma\}$ which allow only the equity risk to be priced . Secondly in order to estimate the risk neutral parameter vector $\theta^* =\{\omega^*, \alpha^* , \beta^* , \gamma^* \}$ only using information embedded in the volatility index we maximize the risk neutral MLE (eq 20/ 9.3). This experiment will allow as to investigate how the physical and risk neutral spot volatility differs in order to prove the importance to include the risk neutral distribution to the model and how this will reduce mispricing.

Joint maximum likelihood estimator

An other estimation procedure that will be used is j-MLE (returns –volatility) (eq 21) which consists of two component.

1. The first model is the symmetric which account only for equity premium only (γ=ξ=0) since

$$\xi = 0 \xrightarrow{\text{eq 18 \&19}} \begin{cases} h^* \mathrm{t}+1 \ = \ \mathrm{ht}+1 \\ \omega = \omega^* \\ \alpha = \alpha^* \end{cases}$$

Resulting four parameters estimation into vector θ_{sym}={μ, ω, α, β} and $\theta_{sym}{}^*$ = {ω*, α*, β*,γ*} (eq19)

.

2. The second model is the asymmetric (which allow for skewness $\gamma \neq 0$, $\xi = 0$) HN-GRARCH that allow only for equity premium and asymmetry which result in five parameter estimation θ_{asym}={μ, ω, α, β,γ} and $\theta_{asym}{}^*$ = {ω*, α*, β*,γ*}.

3. And the last and most general economic ($\gamma \neq 0$ & $\xi \neq 0$) model that allow for both equity risk and indepent volatility risk by allowing ξ to be free parameter [5] resulting in six parameters to be estimated θ_{risk}={μ, ω, α, β, γ, ξ} and $\theta_{risk}{}^*$ = {ω*, α*, β*,γ*}.Note that the star parameters are under risk neutral measures.

As shown in the table 5 from second to forth column, represent the results of joint MLE which use both returns and implied volatility of index (IVI) to estimates the parameters. According to models fitting measured by MLE value, if we compare the joint MLE from second to third column is visible that the value increase significantly from 27309.42 to 27853.47 which shows a better data fitting as we move from symmetric to the asymmetric model. Thus if we allow for asymmetry to be free parameter we gain accuracy in terms of data fitting. Note that these two models allow only equity risk to be priced. If we compare fourth column with second and third , the fitting increasing even further since the joint MLE increase form 27309.42 to 27853.47 to 28058.00 . This is because the pricing kernel of the last model that was mentioned (fourth column) capture better the connection between physical and risk neutral distribution indicating that the volatility risk is indeed in the market .

However if we compare first column with the forth we can observe that the return component of j-MLE in forth column (11460.50) is lower than the respective one in the first column (11491.76) obtained using separated MLE procedure. Furthermore the volatility component of joint MLE (16561.87 ($\gamma \neq 0$ & $\xi \neq 0$)) is higher than the volatility part of MLE using separated procedure (16544.45 first column). However in total despite of we loss accuracy in fitting the return component of joint MLE we gain accuracy in volatility part for joint MLE and in total (28036.21<28058.00). The opposite phenomenon occurs to the paper of Hao&Zhang[12] where they find that if the joint MLE fails to improve the fitting of implied volatility component of joint MLE compare to the implied volatility MLE that was estimated separately. This is happened because they did the scaling factor $\bar{\xi}$ but instead of that they compute the physical and risk neutral processes separately without connectivity parameterization .

The increase of joint- MLE value of the model with risk premium (γ≠0 & ξ≠0) happened because we let the scaling factor (1.6389) that drives the connectivity of risk neutral and variance and physical variance to be priced. This parameter adds flexibility to the model and allow to increase the accuracy. Finally in the study of Christoffersen, Heston, Jacobs [5] use S&P500/VIX and option MLE instead of volatility MLE. In the referred study [5] they report parameter $\bar{\xi}$ =1.2039 while ours is 1.6389. This may be due to the fact that we use longer series of data and more recent (1996-2005 S&P500 vs 2000-2014 FTSE100) which includes more resent crisis events (financial crises 2008 ,see figure 1) and increase the volatility premium . This is a very significant point and explains why resent studies focus on volatility risk premium. An other significant point is that the FUTSE 100 seams to has more negative risk premium.

	Separated MLE	Joint-MLE		
		symmetric (HN) (γ=ξ=0)	asymmetric (γ≠0 ,ξ=0)	asymmetric &volatility premium (γ≠0 & ξ≠0)
physical parameters θ				
μ	0.43	15.40	-0.66	0.36
ω	-1.48E-18	8.22E-15	2.23E-31	4.86E-46
α	3.64E-06	1.65E-05	3.95E-06	1.63E-06
β	0.81	0.9438	0.8426	0.8574
γ	208.63	0	189.76	280.52
$\bar{\xi}=1/(1-2\alpha\xi)$		1	1	1.6389
risk neutral parameters θ*				
ω*	6.04E-20	8.22E-15	2.23E-31	7.96E-46
α*	4.38E-06	1.65E-05	3.95E-06	4.39E-06
β*	0.86	0.94	0.84	0.86
γ*	169.24	15.40	189.10	189.10
Log Likelihood				
MLE return	11491.76	16173.09	11314.77	11460.50
MLE ivi	16544.45	11135.54	16532.58	16561.87
MLE total	28036.21	27309.42	27853.47	28058.00
Persistent ρ	0.9713	0.9438	0.9849	0.9860
Persistent ρ*	0.9868	0.9440	0.9839	0.9943

Table 2 parameter results estimates

The fitting properties of the three models estimated can be also examined in figures 2,3, and 4. These figures shows the observed implied volatility index(solid line) versus the theoretical one (dashed line generated by three models) both in-sample (2000-2014) and out-of-sample (2014-2016). An example of using the parameters estimated with j-MLE (forth column table 1) to calculate the spot variance is the following

$$h^*(2) = 7.96\ 10^{-46}\ + 0.86\ h(1) + 4.39\ 10^{-6}(\ z^*(1) - 189.10\sqrt{h^*(1)}\ \ )^2$$

where h*(1)=var(returns)

$z^*(1) = 0$

substitute the parameters mentioned to eq 17

$$h^*(t+1) = 7.96\ 10^{-46}\ + 0.86\ h(t) + 4.39\ 10^{-6}(\ z^*(t) - 189.10\sqrt{h^*(t)}\ \ )^2$$

solve the eq 16 for z*(t+1)

$$z^*(t+1) = \frac{\left(\log\left(\frac{s(t+1)}{s(t)}\right) - rf + \frac{1}{2}h^*(t+1)\right)}{\sqrt{h^*(t+1)}};$$

As shown in the figures the in-sample fit improves as we move form figure 2 (which account for symmetric model) with the figure 2 (which account for leverage effect γ that controls the skewness). If we compare the figure 2 (asymmetric model) with the figure 3 (model which allow for risk neutral premium to be priced $\gamma \neq 0$ and $\xi \neq 0$) the in-sample fitting improve even further. This also verified by the in-sample RMSE which equal to 4.5107, 4.1868 and 3.9963 respectively for three models considered. In addition the same pattern is observed in the out-of-sample performance.

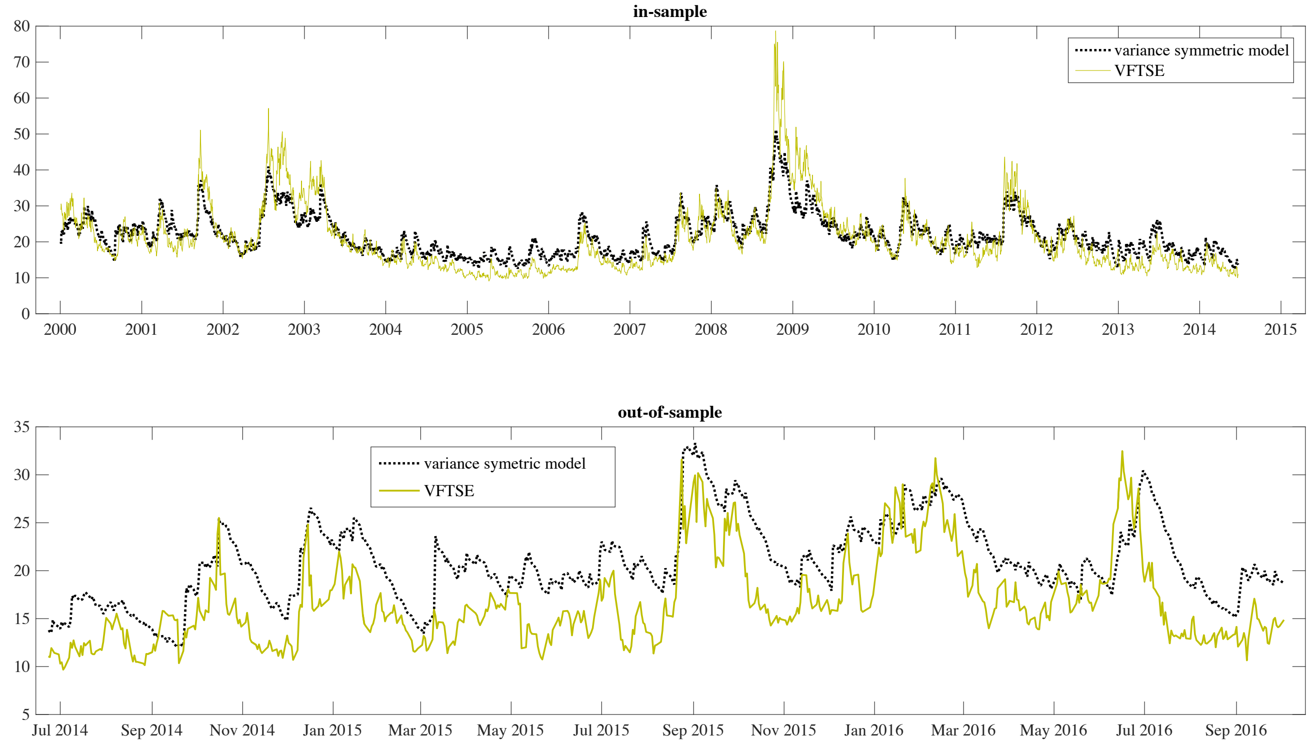

Figure 2 annualized variance dotted line compared with the implied volatility of index VFTSE for symmetric model estimated with J-MLE method for in sample and out of sample

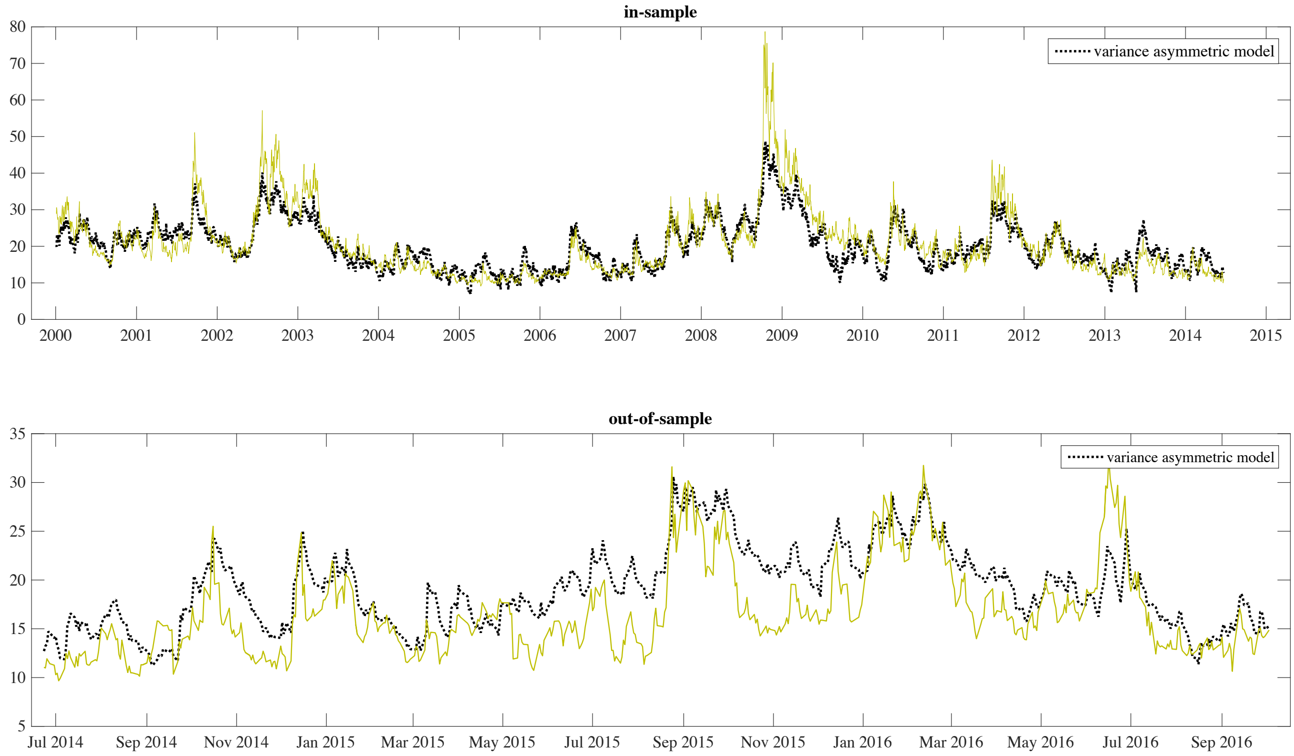

Figure 3 annualized variance dotted line compared with the implied volatility of index VFTSE for asymmetric model estimated with J-MLE method for in sample and out of sample

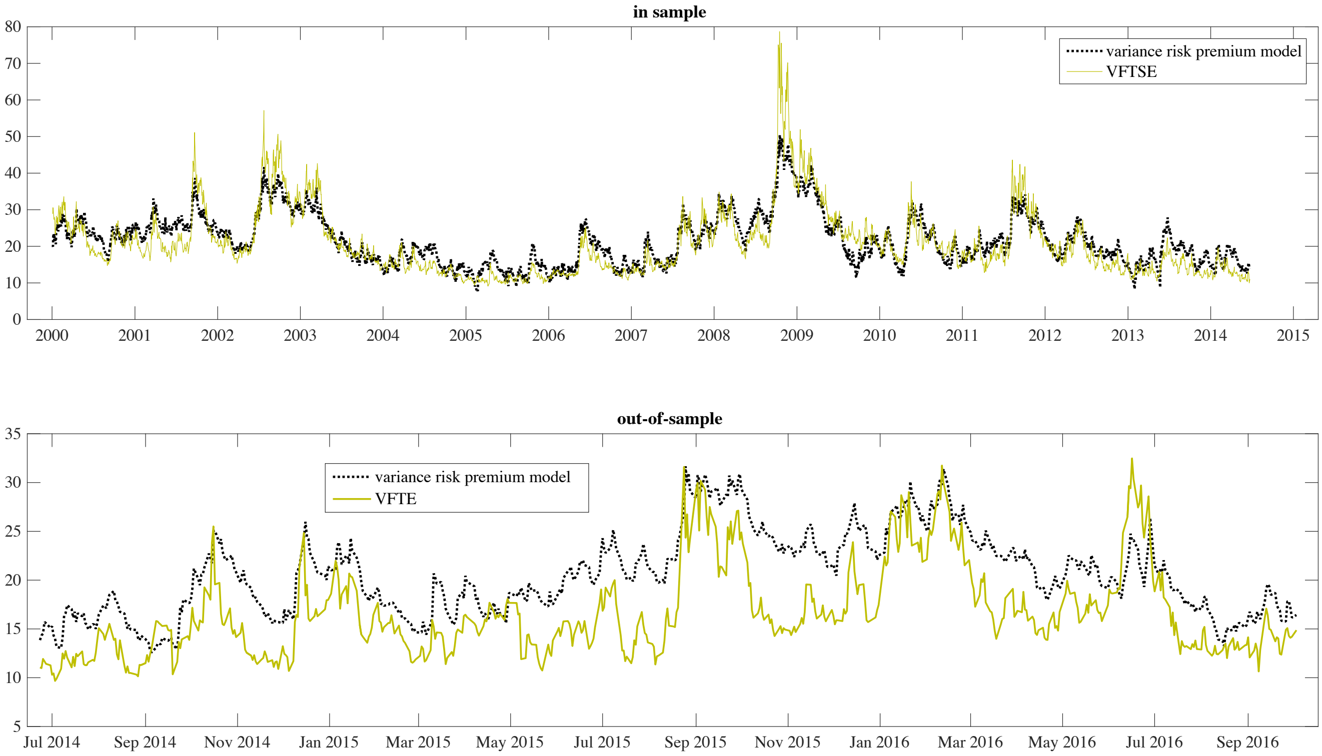

Figure 4 annualized variance dotted line compared with the implied volatility of index VFTSE for asymmetric and vol. risk premium model estimated with J-MLE method for in sample and out of sample.

7 Results and discussion

7.1 Economic implications

7.1.1 Volatility risk premium

In order to investigate the performance of j-MLE approach to draw volatility risk premium we define the volatility risk premium as

$$VRP_t = E_t\left[var_t(R_{t+1})\right]^{1/2} - E_t^Q\left[var_t(R_{t+1})\right]^{1/2} = \sqrt{h} - \sqrt{h^*} \tag{21}$$

which denote the deference between the physical conditional volatility and the risk neutral conditional volatility. In order to produce the volatility premium we use the parameter that was estimated with the separated procedure. Then we calculate the variance time series of h_{t+1} and h^*_{t+1} from the historic daily prices of the index (using the eq 1&2 and 4&5 for physical and risk neutral measures respectively) .The deference of the square root of these two processes defines the VRP_t (see eq 21) .This is shown in solid line in figure 5. The average VRP_t is the dash-dotted line figure 5 . The same procedure followed to calculate the average volatility premium under the Joint-MLE model accounting for asymmetry and risk premium ($\gamma \neq 0$ & $\xi \neq 0$) . This is shown as the dash line in figure 5.

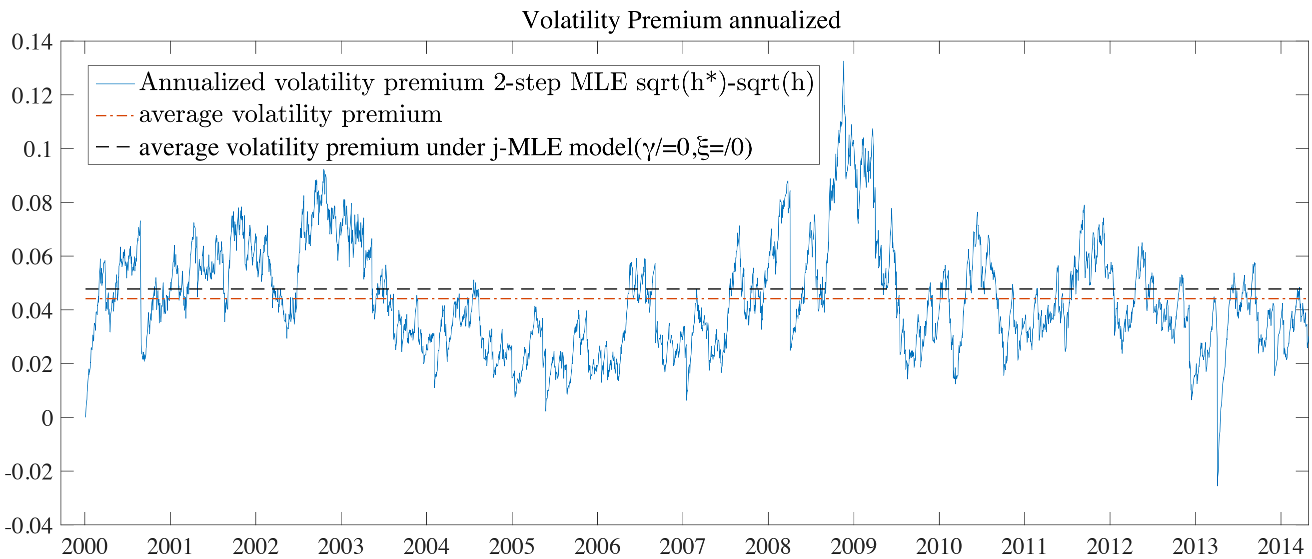

Figure 5 in-sample annualized square route volatility risk premium (solid line) average annualized volatility under joint MLE model (risk premium model) (dashed line) average annualized volatility premium under 2 step model (dashed dotted line).

As shown in figure 5 the VRP drops bellow zero just one time imitating that the risk neutral variance in higher than the physical variance almost in all sample points. The dash-dotted line indicates the annual average volatility risk premium of -2.8%. According to the j-MLE model (with the pricing kernel which allow variance risk ξ to be priced) the average annual volatility risk premium is -3.01% (dash line figure 5) which is close to the value obtained by separated procedure (-2.8%). This negative value is also very close to other studies (see reference [17] [18]) . These results indicate that the byers of market volatility want to pay a premium to hedge their position. This motive is equivalent to negative volatility premium.

7.1.2 Persistence effect

In order to investigate how options behave when news related to the underling index come (shock) and produce a vibration of volatility , an impulse response function of the conditional volatility should be considered. Furthermore is also important to compare the risk neutral and physical variance impulse responses. To do so we convert equation (2) into an ARCH process as follows :

$$h_{t+1} = \bar{h} + \alpha \sum_{i=0}^{\infty} \rho^i \left(z_{t-i}^2 - 1 - 2\gamma \sqrt{h_{t-i}} z_{t-i} \right) \quad (22)$$

then the impulse response function of k-day ahead horizon is obtained as follows

$$\mathrm{irf(k)} = \alpha \rho^{\kappa} \frac{\left(z^2 - 1 - 2\gamma \sqrt{\overline{h^*}}\, z \right)}{\overline{h^*}} \quad (23)$$

under risk neutral measures $\overline{h}$ is substituted by $\overline{h^*} = (\alpha^* + \omega^*) / (1-\rho^*)$) with $\rho^* = \beta + \alpha^* \gamma^{*2}$. In figure 6 we present the irf under the risk neutral measure for a specific shock $z = \pm 1, \pm 3, \pm 7$ and the parameters account for the j-MLE modes with both equity and risk premium ($\gamma \neq 0$ & $\xi \neq 0$). As shown in figure 6 the model has an approximated 250 k-day ahead horizon of forecast. Now if we set k=0 we can plot the sudden return innovation impact.

$$\Delta h^*(\mathrm{z})\% = \alpha\rho \frac{\left(z^2 - 1 - 2\gamma \sqrt{\overline{h^*}}\, z \right)}{\overline{h^*}} \quad (24)$$

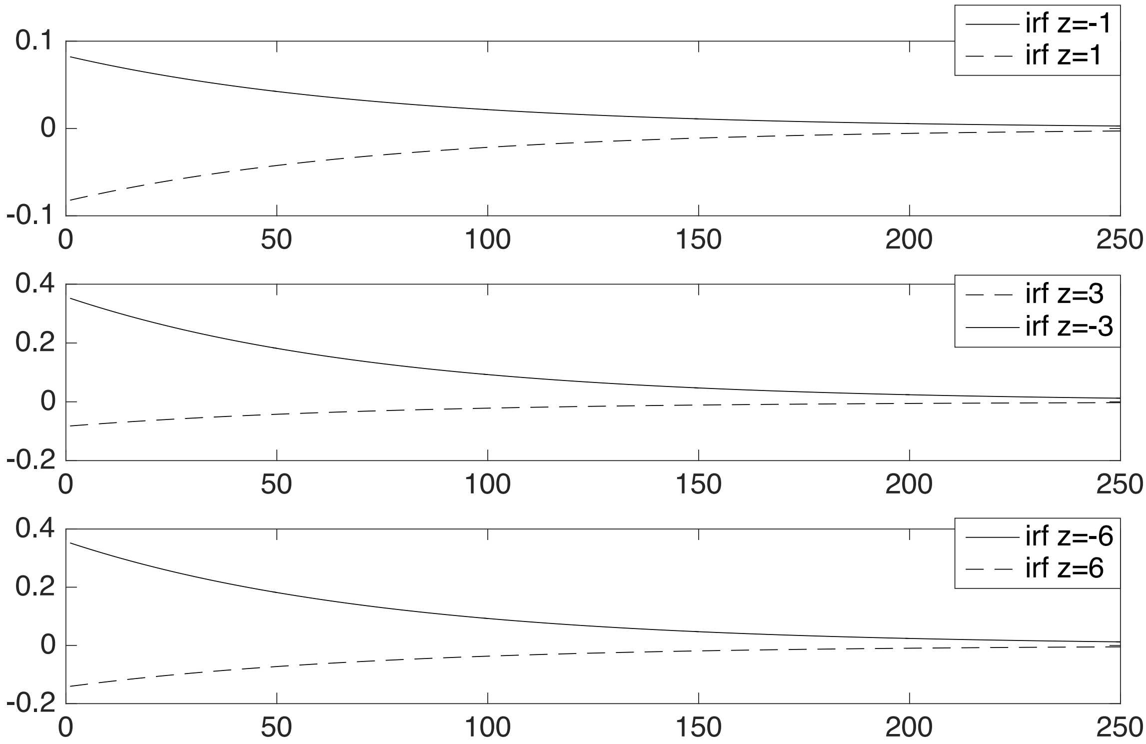

Figure 6 Impulse response of variance of k-day horizon for shocks $= \pm 1, \pm 3, \pm 7$ of risk neutral variance for parameters of asymmetric &volatility premium model (table 1 column 4)

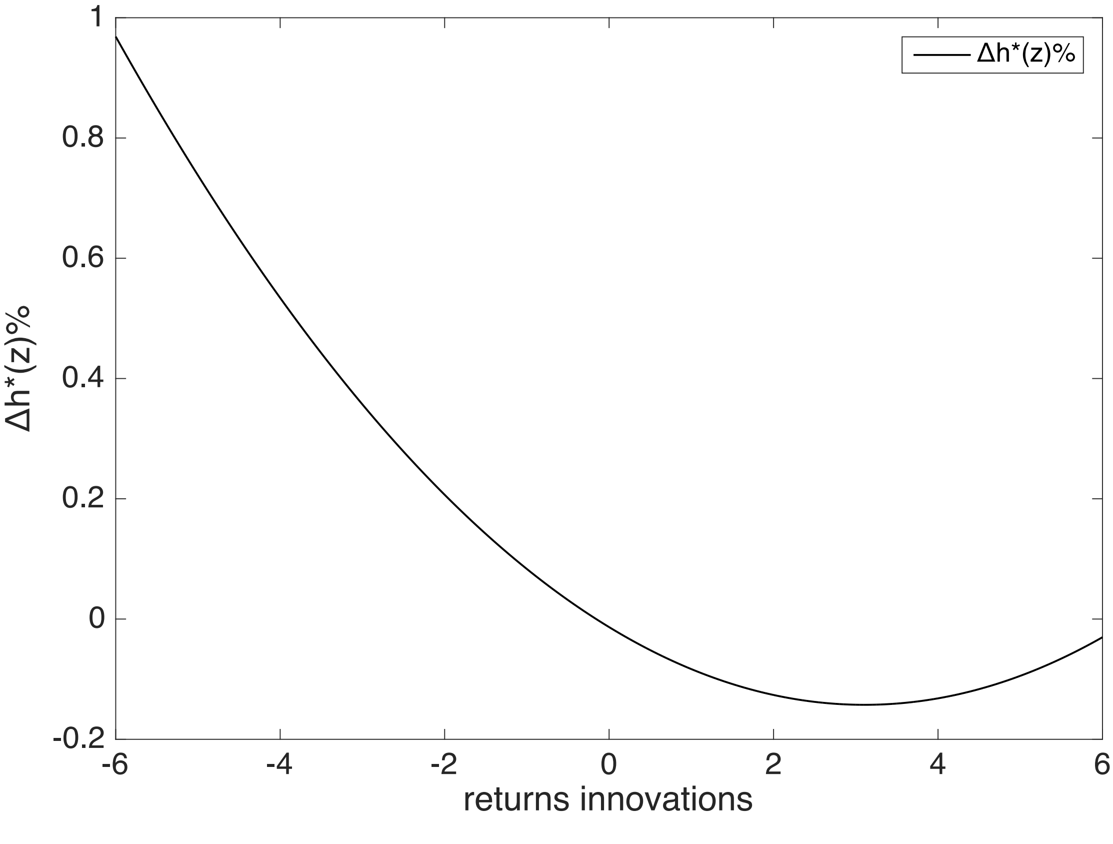

Figure 7 returns innovations shocks impact for risk neutral variance for j-MLE model with risk premium

7.1.3 Histogram

In order to extract in-sample histogram of innovations under the risk neutral and physical measure for the asymmetric and equity and volatility risk premium model we filter the series of innovation z and z*.
As initial innovation and variance z(1) =z*(1)=0 and h(0)=h(0)=var (returns of index).

$$h(2) = \omega + \beta h(1) + a(\ z(1) - \gamma\sqrt{h(1)}\ \ )^2$$

$$h(t+1) = \omega + \beta h(t) + a(\ z(z) - \gamma\sqrt{h(t)}\ \ )^2$$

$$z(t+1) = \frac{\left(\log\left(\frac{s(t+1)}{s(t)}\right) - rf - \left(m - \frac{1}{2}\right) * h(t+1)\right)}{\sqrt{h(t+1)}}$$

$$h^*(2) = \omega^* + \beta h(1) + a^*(\ z^*(1) - \gamma^*\sqrt{h^*(1)}\ \ )^2$$

$$h^*(t+1) = \omega^* + \beta h(t) + a^*(\ z^*(t) - \gamma^*\sqrt{h^*(t)}\ \ )^2$$

$$z^*(t+1) = \frac{\left(\log\left(\frac{s(t+1)}{s(t)}\right) - rf + \frac{1}{2}h^*(t+1)\right)}{\sqrt{h^*(t+1)}};$$

Then we calculate the frequency innovation series and divide by the number of z series to obtain the respective probabilities

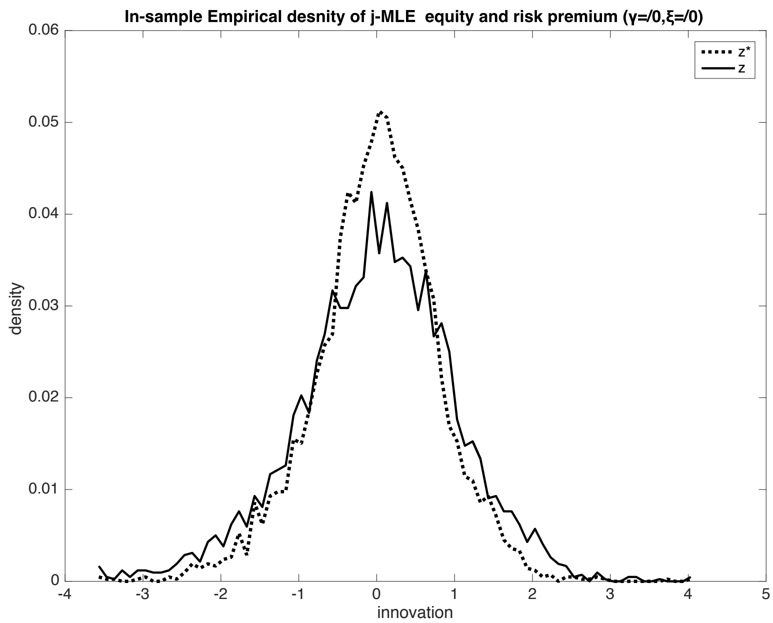

Figure 8 in-sample histogram of innovation z and z* for risk neutral dotted line and physical measures respectively

As shown in figure 8 there is clear deference between the two patterns. If compare the risk neutral histogram with the physical one , clearly we can clearly observe that the risk neutral one have variance more negative skewness and higher kurtosis.

7.2 Option pricing results

7.2.1 Semi-closed form European call valuation

In order to compute the out-of-sample option prices ,we will use the semi-closed form solution that was derived by Heston and Nandi (Appendix 9.8). They show that a European call option price with expiration period T and strike price K is calculated by

$$C = \frac{1}{2}S_t + \frac{e^{-r(T-t)}}{\pi}\int_0^{\infty} Re\left[\frac{K^{-i\phi}f^{*}(i\phi+1)}{i\phi}\right]d\phi - Ke^{-r(T-t)}\left(\frac{1}{2}+\frac{1}{\pi}\int_0^{\infty} Re\left[\frac{K^{-i\phi}f^{*}(i\phi)}{i\phi}\right]d\phi\right)$$

(25)

In this formula, Re denotes the real part of a complex number. $f^*(i\varphi)$ represent the conditional characteristic function of the log asset price using the risk neutral probabilities. i is the imaginary number. The put option price can be obtained by put-call parity. Heston and Nandi (2000) show that this function of the log asset price under the risk neutral measure:

$$f^*(i\,\varphi) = E\left[S_t^{\varphi}\right]\exp(A_t + B_t h^*_{t+1}) \quad (26)$$

with coefficients

$$A_t = A_{t+1} + \varphi\, r + B_{t+1}\, w - 0.5\ln(1-2\,\alpha^*\, B_{t+1}) \quad (27)$$

$$B_t = \varphi(-0.5+\gamma^*) - 0.5\,\gamma^{*2} + \frac{0.5\,(\varphi-\gamma^*)^2}{1-2\,B_{t+1}\,a^*} + \beta^*\,B_{t+1} \quad (28)$$

Despite the formula needs a numerical integration this calculation instead of straight forward calculation however the numerical integrations is behave well and is not time intensive. The variance ht+1 is the spot out of sample variance as we referred in section 6. Note that we use semi-closed for solution instead of mode carlo simulation because closed form solution is available and in not time intensive.

.

7.3 Option pricing results and comparison

In order to investigate the option pricing performance of the models we convert the valuation pricing results of all model into Black and Shores implied volatility and the calculate implied volatility RMSE. The IV RMSE is slit into 2 categories , the moneyness (S/K) and maturity in terms of days until expiration for each model. We split the results in order to have a more clear view on performance of the models across the maturity (t) and moneyness (i) for three samples. The IVRMSE is then computed as follows:

$$\text{IV RMSE} = \sqrt{\frac{1}{N}\sum_{i,t}\left(\sigma_{i,t}^{BS\ market} - \sigma_{i,t}^{BS\ model}\right)^2} \qquad (29)$$

In tables the models refered as 'sym' ,'asym',asym& risk premium' and 'BS'. The 'sym' is the symmetric model with only equity premium (γ=0 & ξ=0). the 'asym' is the model with the equity premium and leverage effect ($\gamma \neq 0$, $\xi = 0$) , the 'asym & risk premium' is the model accounting for leverage effect and equity premium and volatility risk premium (γ≠0 & ξ≠0). Our benchmark model is the Black-Sholes model (denoted as 'BS'). All valuation results for 'asym', 'sym' and 'asym& risk premium corresponding to the risk neutral parameters (*) estimated using J-MLE reported in Table 1.

Table 6 describe the out-of –sample performance of models for sample 1 across moneyness for all maturities . If we compare the first with the second column we observe that when allowing for asymmetry the option pricing performance improves as the RMSE decreases. This means that the asymmetric parameter γ plays a significant role in option price fit. Now if we move from the second one column to third we can observe a dramatic decrease in IV RMSE as we allow volatility risk to be priced. Thus the volatility risk premium ($\bar{\xi}$) plays a very significant role compare to asymmetric parameter γ^*. This result indicates the important of the volatility risk premium to improve the performance of GARCH option price fit. Now if we compare the models with the BS IV we can observe that the first two models fit worst than the BS model while the asymmetric & risk premium fits dramatically better. The symmetric GARCH models (sym) fits worse compare to BS for every moneyness. Finally we observe that all models the worst performance option-fit performance is for moneyness levels of 0.85-0.9 and 0.9-0.98.

IV RMSE by moneyness (S/K) sample 1

moedel	sym	asym	asym& risk premium	BS
<0.79	0.2014	0.1430	0.0223	0.0642
0.79 -0.8	0.2005	0.1414	0.0234	0.0652
0.8-0.85	0.2023	0.1500	0.0217	0.0615
0.85-0.9	0.1959	0.1521	0.0310	0.0668
0.9-0.98	0.2056	0.1820	0.1031	0.1232
0.98-1.04	0.1786	0.1673	0.0383	0.0828
>1.04	0.1747	0.1679	0.0526	0.0866

Table 3 out-of-sample IV RMSE sample 1

Now if the comparison is made across maturity (Table 4) we can observe similar results to previous comparison. However we can observe that the asym model performs better for maturities until 60 days than BS. In addition the table indicates that the worst performance of all models s observed for the short term maturities (T<30).

IV RMSE by maturity (days to expiration) sample 1

moedel	sym	asym	asym& risk premium	BS
<30	0.3585	0.3569	0.3519	0.3793
30-60	0.1311	0.0732	0.0488	0.0873
60-120	0.1673	0.1041	0.0418	0.0843
90-120	0.1768	0.1198	0.0408	0.0841
120-200	0.1704	0.1210	0.0506	0.1001
200-250	0.2057	0.1502	0.0295	0.0626
>365	0.2339	0.1803	0.0166	0.0370

Table 4 out-of-sample IV RMSE sample 1 along maturity (days to expiration)

Table 5 and 6 also reports out-of-sample IV RMSE for sample 2 and 3. If we compare the first column (sym model) with the second we observe that when we allow for asymmetry (second column) the option pricing performance increases as the RMSE decrease. This means that the asymmetric parameter γ plays a significant role in option fit again .Now if we move from the second column to third we can observe a dramatic decrease in IV RMSE when we allow volatility risk to be priced again. If compare the IV RMSE across the money less, we observe that the option pricing performance decreases as the moneyness level increases (for moneyness >0.79) . The asym& risk prem model has its worst performance for moneyness lower than 0.79 (0.0815 sample 2 and 0.0793 sample 3) the opposite phenomenon happens in the other two models.

IV RMSE by moneyness (S/K) sample 2

moedel	sym	asym	asym& risk prem	BS
<0.79	0.1600	0.0893	0.0815	0.0886
0.79 -0.8	0.2436	0.1660	0.0087	0.1537
0.8-0.85	0.2418	0.1687	0.0130	0.1544
0.85-0.9	0.2388	0.1753	0.0277	0.1567
0.9-0.98	0.2468	0.1937	0.0418	0.1645
0.98-1.04	0.2607	0.2110	0.0352	0.1718
>1.04	0.2730	0.2259	0.0401	0.1815

Table 5 out-of-sample IV RMSE bt moneyness (S/K) sample 2

IV RMSE by moneyness (S/K) sample 3

moedel	sym	asym	asym& risk prem	BS
<0.79	0.1612	0.0887	0.0793	0.0891
0.79 -0.8	0.2462	0.1685	0.0078	0.1562
0.8-0.85	0.2444	0.1709	0.0128	0.1569
0.85-0.9	0.2397	0.1761	0.0289	0.1577
0.9-0.98	0.2466	0.1932	0.0435	0.1654
0.98-1.04	0.2617	0.2120	0.0380	0.1730
>1.04	0.2737	0.2262	0.0402	0.1822

Table 6 out-of-sample IV RMSE by moneyness (S/K) sample 3

When we split the IV RMSE results we can observe in terms of maturity (table 7 & 8) for samples 2 and 3 that the pricing performance decreases as the maturity increase for sym ,asym and BS model. However the opposite behavior occurs for the asym& risk prem model.

IV RMSE by maturity (days to expiration) sample 2

model	sym	asym	asym& risk prem	BS
<30	0.1810	0.1515	0.0955	0.1387
30-60	0.2278	0.1693	0.0604	0.1684
60-120	0.2243	0.1696	0.0453	0.1513
90-120	0.2292	0.1715	0.0383	0.1506
120-200	0.2394	0.1756	0.0321	0.1567
200-250	0.2426	0.1771	0.0280	0.1570
>365	0.2486	0.1877	0.0293	0.1585

Table 7 out-of-sample IV RMSE by maturity (days to expiration) sample 2

IV RMSE by maturity (days to expiration) sample 3

model	sym	asym	asym& risk prem	BS
<30	0.1916	0.1561	0.0947	0.1505
30-60	0.2267	0.1684	0.0607	0.1682
60-120	0.2255	0.1713	0.0482	0.1528
90-120	0.2319	0.1721	0.0378	0.1536
120-200	0.2417	0.1786	0.0314	0.1583
200-250	0.2436	0.1773	0.0289	0.1584
>365	0.2495	0.1886	0.0284	0.1595

Table 8 out-of-sample IV RMSE by maturity (days to expiration) for sample 3

Now in order to more clearly investigate the performance across moneyness we fix the maturity and split the data along moneyness for sample 2 and 3.The results are summarized in figure 9 and 10 . If we compare the two figures we can observe that the results for samples 2 and 3 are identical. According to IV RMSE for short term maturity options (T<30) the error increase as the moneyness increases (0.85< S/K <1.02) for all models while the asym& risk premium model exhibiting a lower increase of errors and the best option fit. According to options with maturity from one to two months (30 < T < 60) and the deepest out of the money call (S/K <0.79) , the symmetric models has the best option valuation performance compare to others which have almost same errors. The asymmetric and BS models have similar performance across the moneyness for this maturity. If move on the maturity form two to three months we can observe similar behavior for all models ,again the symmetric model has the best performance for deepest out-of-the money options. If we move further move in terms of maturity as shown in the figures 9 and 10 the options with long term maturity higher than three moths (T>90) has more consistent behavior and are identical. Again the symmetric model has the best performance of IV RMSE for the deepest out-of-the-money (S/K <0.79) options, which is the most difficult to value (in terms of IV RMSE). For maturity >90 days to expire the and moneyness form 0.79 to 0.85 asym&risk prem model dominates dramatically the other models. As the moneyness increase (S/K>0.85) the error of the asym &risk prem model increase but not much.

To summarize for all maturities higher than 30 days the symmetric model has thee best option pricing performance with moneyness lower than 0.79. However , for all the moneyness categories the asym and vol risk prem model outperforms dramatically to all other models.

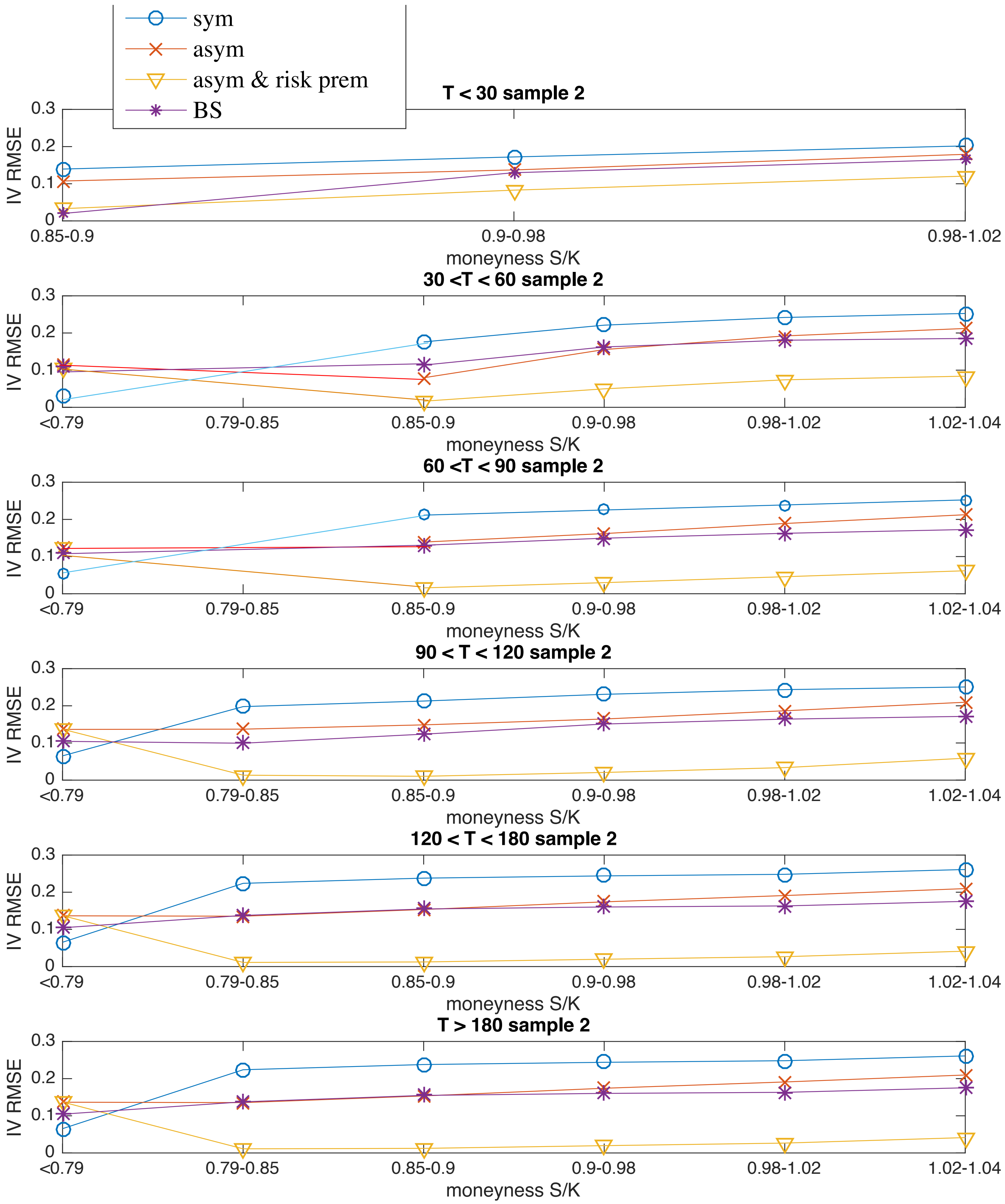

Figure 9 This figure shows the out-of-sample IV RMSE for deferent maturity (days to expire) bounds and along the moneyness (S/K) for sample 2 which computed every Thursday for the out of sample period. The sym is the symmetric model with only equity premium the asym is the model with asymmetry (non zero γ) and the asym is the model account for asymmetry equity and risk premium. All three models estimated with joint MLE.

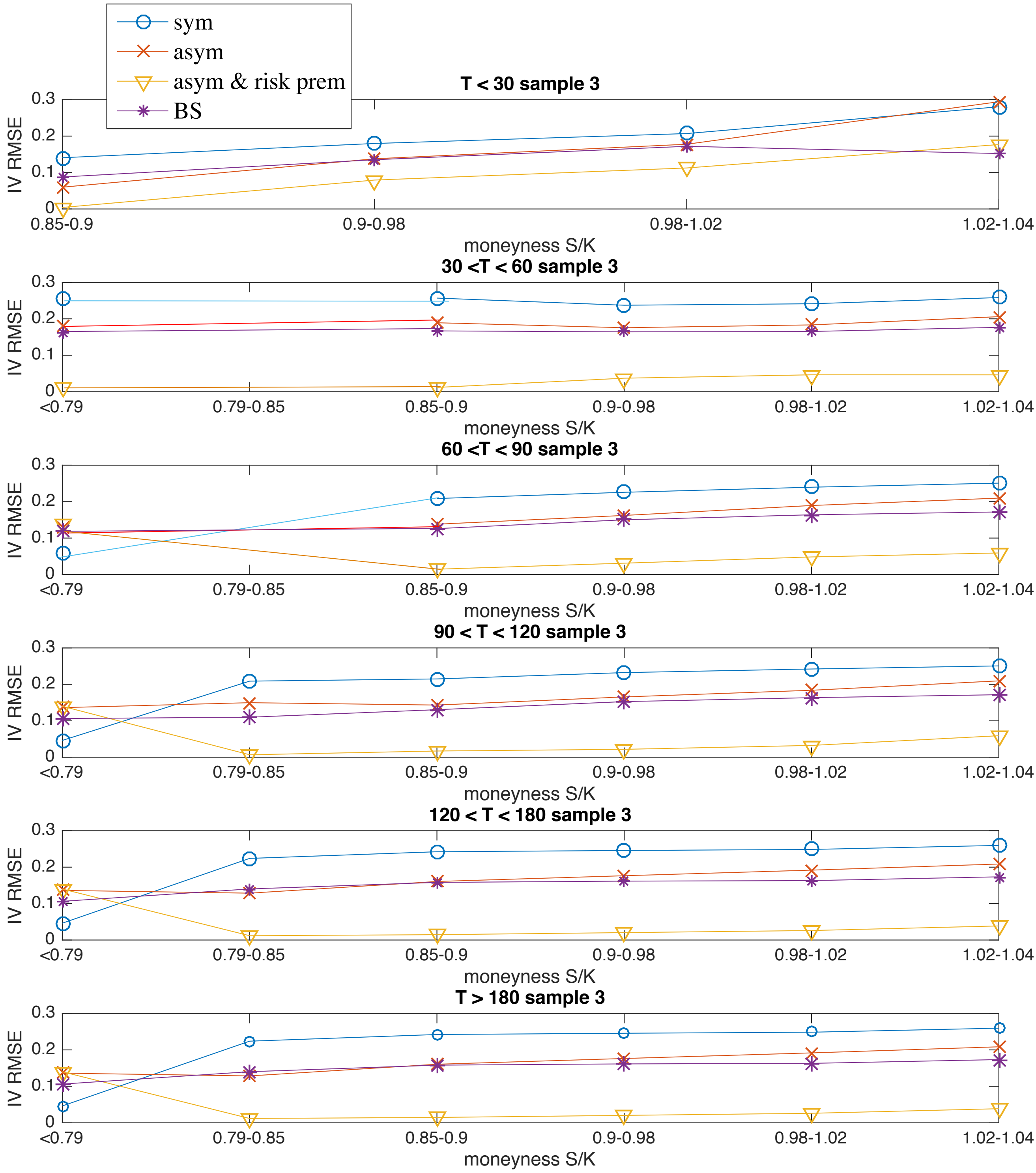

Figure 10 This figure shows the out-of-sample IV RMSE for deferent maturity (days to expire) bounds and along the moneyness (S/K) for sample 3 which computed every Thursday for the out of sample period. The sym is the symmetric model with only equity premium the asym is the model with asymmetry (non zero γ) and the asym is the model account for asymmetry equity and risk premium. All three models estimated with joint MLE.

8 Conclusion

This paper examines the option pricing performance of various GARCH models. These models are estimated using a MLE procedure which use two source of information the spot price of FTSE 100 index and its implied volatility index. The filter can considered as equivalent to the use of a large cross-sectional set of option prices with the additional advantage of being less computational and time expense. According to our results Joint MLE estimation outperforms the traditional MLE which use only the one component of joint MLE (underling returns in terms of volatility) fitting. In addition under joint MLE , the risk neutral parameterization that was used by mapping the physical parameters inside the Joint MLE.

We identify an average volatility risk premium of -3.01% using a joint MLE model with asymmetry equity and volatility risk premium and -2.8% when separated MLE procedure is implied. This indicates the importance of letting the volatility risk premium to be free parameter (incorporated in the model) to better capture the behavior of investors. When estimating the variance response function we found an approximated 250 k-day ahead horizon of forecast for variance with volatility risk premium. In addition if compare the risk neutral innovation distribution with the physical one , clearly we can observe the risk neutral one have variance more negative skewness and higher kurtosis.

In terms of out-of-sample option pricing exercise the symmetric model has the best performance for short term maturities (in term of days to expiration) less than one month and deep out of the money calls with moneyness less than 0.79. The most general model outperforms BS sym and asymmetric models for all maturities (in term of days to expiration) and moneyness higher than 0.79.

An interesting direction for futures studies is the estimation and option valuation using high frequency returns and implied volatility return data to improve further the short term performance of the models. An alternative direction would be to include jumps into the underlying return dynamics.

9 Aappendix

9.1 Volatility Indexes Around the World

Table 2: Volatility Indexes Around the World

Country	Exchange	Index	Underlying	Maturity	Launch Date	Method
US	Chicago Board Options Exchange (CBOE)	VIX	S&P 500	1 month	Sep 2003 (old index renamed VXO, 1993-)	Demeterfi, Derman, Kamal, and Zou (99) - Goldman Sachs (VIX methodology)
US	CBOE	VXV	S&P 500	3 months	Nov 2007	VIX
US	CBOE	VXO	S&P 100	1 month	1993	Whaley (1993)
US	CBOE	VXD	DJIA	1 month	Mar 2005	VIX
US	CBOE	VXN	Nasdaq 100	1 month		VIX
US	CBOE	VXAZN, VXAPL, VXGS, VXGOG, VXIBM	Stocks - Amazon, Apple, Goldman Sachs, Google, IBM	1 month	Jan 2011	VIX
US	CBOE	EVZ, GVZ, OVX, VXEEM, VXSLV, VXFXI, VXGDX, VXEWZ, VXXLE	ETFs - EuroCurrency, gold, crude oil, emerging markets, silver, China, gold miners, Brazil, energy sector	1 month	2008	VIX
US	CBOE	ICJ, JCJ, KCJ	S&P 500	As of May 2011, KCJ - Jan 2012, ICJ - Jan 2013, JCJ - Jan 2014, the tickers are to be recycled as they expire	Jul 2009	Skintzi and Refenes (2005)
Australia	Australian Securities Exchange	S&P/ASX 200 VIX (ASX code: XVI)	S&P/ASX 200 (XJO)	1 month	Sep 2010	VIX
Belgium	Euronext	VBEL	BEL 20	1 month	Sep 2007	VIX
Canada	TMX	S&P/TSX 60 VIX (VIXC)	S&P/TSX 60	1 month	Oct 2010	VIX
Europe	Eurex	VSTOXX	Euro STOXX 50	30, 60, 90,..., 360 days	Apr 20, 2005 (30 days); May 31, 2010 (60-360 days)	VIX
France	Euronext	VCAC	CAC 40	1 month	Sep 2007	VIX
Germany	Deutsche Borse	VDAX-NEW	DAX	1 month	Apr 2005 (previously VDAX, Dec 1994)	VIX
Hong Kong	Hong Kong Futures Exchange	VHSI	HSI	1 month	Feb 2011	VIX
India	National Stock Exchange of India	India VIX	NIFTY	1 month	Jul 2010	VIX
Japan	CSFI, Univ. of Osaka	CSFI - VXJ	Nikkei 225	1 month	Jul 2010	VIX
Mexico	Mexican Derivatives Exchange	VIMEX	Mexican Stock Exchange Price and Quotation Index (IPC)	3 months	Apr 2006	Fleming and Whaley (1995)
Netherlands	Euronext	VAEX	AEX	1 month	Sep 2007	VIX
South Africa	Johannesburg Stock Exchange	New SAVI	FTSE/JSE Top40	3 months	2010 (previously SAVI, 2007-)	VIX
South Korea	Korea Exchange	V-KOSPI	KOSPI200	1 month	Apr 2009	VIX
Switzerland	Six Swiss Exchange	VSMI	SMI	1 month	Apr 2005	VIX
UK	Euronext	VFTSE	FTSE 100	1 month	Jun 2008	VIX

Table 2 Volatility Indexes Around the World

9.2 Semi closed Heston-Nandi GARCH Call option pricing function

```
function OptionPrice=HestonNandi_giannis(w,a,b,g_star,r,S,h,K,T)

OptionPrice=.5*S+(exp(-r*T)/pi)*quad(@Integrand1,eps,100000)-K*exp(-r*T)...
    *(.5+(1/pi)*quad(@Integrand2,eps,100000));

    % function Integrand1 and Integrand2 return the values inside the
    % first and the second integrals

    function f1=Integrand1(phi)
        f1=real((K.^(-1i*phi).*charac_fun(1i*phi+1))./(1i*phi));
    end

    function f2=Integrand2(phi)
        f2=real((K.^(-1i*phi).*charac_fun(1i*phi))./(1i*phi));
    end

    % function that returns the value for the characteristic function
    function f=charac_fun(phi)
        phi=phi';    % the input has to be a row vector

        % recursion for calculating A(t,T,Phi)=A_ and B(t,T,Phi)=B_
        A(:,T-1)=phi.*r;
        B(:,T-1)=-.5.*phi+.5*phi.^2;
        for i=2:T-1
              A(:,T-i)=A(:,T-i+1)+phi.*r+B(:,T-i+1).*w-.5*log(1-2*a.*B(:,T-i+1));
              B(:,T-i)=phi.*((1.3580e-05-.5)+g_star)-.5*g_star^2+b.*B(:,T-i+1)+.5...
                  *(phi-g_star).^2./(1-2.*a.*B(:,T-i+1));
        end
         % A(t;T,phi)
        A_=A(:,1)+phi.*r+B(:,1).*w-.5*log(1-2.*a.*B(:,1));
         % B(t;T,phi)
        B_=phi.*(-.5+g_star)-.5*g_star^2+b.*B(:,1)+.5*...
            (phi-g_star).^2./(1-2.*a.*B(:,1)); % B(t;T,phi)

        f=S.^phi.*exp(A_+B_.*h);
        f=f'; % the output is a row vector

    end

end
```

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