RISK NEUTRAL AND REAL WORLD DENSITIES ON FTSE 100 DURING THE BREXIT REFERENDUM PERIOD

Athens University of Economics and Business

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Mystriotis Alexandros

Abstract

Risk Neutral and Real world densities implied by option prices, provide us useful information about the future prices of an asset. In this paper, we apply a mixture of two lognormals (MLN), a pure jump diffusion model (JDM) and a curve fitting spline method for flexible densities (SPL) in order to obtain RNDs for the FTSE 100 index during the Brexit referendum period. We use a power utility function to transform MLN into Real World Density (RWD). These transformations are performed by maximizing the log likelihood of the observed index prices and according to the likelihood ratio test we cannot reject the hypothesis that the representative investor earns no risk premium. We conclude that as the Brexit referendum approaches, RNDs become more and more negatively skewed and more and more leptokurtic. We also find that after the Brexit referendum announcement, the relative risk aversion of the representative investor shifts significantly from the period before.

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Introduction

One of the hottest issues European Union is facing recently, is the potential exit of the United Kingdom, widely known as “Brexit”. This could bear consequences of great volume not only for UK itself, but also for EU as well as for the global financial markets and nobody is in the position of estimating accurately this impact a priori. On February 2016, Prime Minister David Cameron announced the EU referendum date, after having secured a deal on Britain’s membership of EU, which was strongly criticized by Brexit campaigners. The referendum campaign kicks off on April 15 with events and rallies across all over the country. From May 27 civil servants were obliged not to provide information for the referendum able to sway and influence the public. This period was called “Purdah” and continued till the final run up of the referendum. On Thursday June 23, the polling day arrives and British people cast their vote in the historic EU referendum. Finally, the Brexit victory came true as 51,9% of British people voted for leave EU, against 48,1% who voted for stay. This result was the advent of the world’s most complex divorce. Speaking in parliament, Mr Cameron advocated: “This will be the most complex and most important task that the British civil service has undertaken in decades.’’ The impact on financial markets after the results was enormous. Immediately, sterling dropped more than 10 per cent against the dollar and UK stocks had the worst fall since the financial crisis, as markets incorporated the decision to leave EU. The pound continued to collapse the subsequent days, hitting its 30 years lowest levels against the dollar, despite the attempts made for reassurance. European banks were also caught in the crosshairs of Brexit, as investors started to fear for the collapse of an already struggling Eurozone economy. As a result, this high uncertainty was depicted in the markets making them highly volatile during that period. Consequently, investors lowered both their expectations and their willingness to bear risk. In this paper, we will use risk neutral densities as a window for grasping the market expectations of this extreme political event.

In comparison with time series of asset prices, option prices are regarded to be more informative and do better on gauging the market sentiment. Therefore, the risk neutral density derived from option prices is in the position of reflecting better both the representative investor's beliefs about future evolutions of the underlying assets and also, his estimation for the probability distribution of the underlying asset on the options expiration date. Moreover, the real world density transformed from risk neutral density deeply sheds light on the investor's attitudes towards risk. This paper tends to explore how the investors' expectations are reflected in risk neutral densities and how they alter as the leave or stay referendum approaches. In addition, by transforming risk neutral into real world densities we examine the risk preferences of the investors and their shift during this extraordinary period. We compare different methods for extracting risk neutral densities and we also examine the relationship between risk neutral and real world densities.

The following section includes a brief literature review, with studies on implied volatilities, scholars about methods of extracting risk neutral densities and their applications, as well as studies on deriving the real world distribution and the implied risk aversion. Section three gives a description for the methodologies applied in this paper in order to estimate both risk neutral and risk adjusted densities and also the risk aversion for option prices. In section four, we analyze the data we have collected in order to accomplish our research. The dataset includes European options written on FTSE 100, FTSE 100 index data and riskless rates. In section five, the empirical results of our findings about risk neutral densities are interpreted and discussed by comparing the different methods for exacting risk neutral densities and simultaneously we connect these findings with the period we are examining. In section six, we pay attention on the empirical results of real world densities as well as on the interpretable risk aversion parameters. It commences with the explanation of the parameters, goes on with the likelihood ratio tests and finishes with the comparison between risk neutral and real world densities. Finally, in the last section we take everything into consideration and we provide a conclusion of our findings.

Section II Literature Review

2.1 Implied volatility

Option prices, due to their forward looking character and to the fact that there exists a sufficient range of strike prices corresponding to them, they are able to serve as a rich source of information for making estimations about the market perceptions of the underlying asset price in the option's expiration date. Initially, investors exploited the process of obtaining option prices from the Black and Scholes model inversely, in order to retrieve the implied volatilities of options (Jackwerth, 2004). There has been an extensive discussion about the ability of implied volatility to predict the forthcoming volatility of the underlying asset. (Anagnou et al, 2002; Perignon & Villa, 2002). Latane & Rendleman (1976), Trippi (1977), and Beckers (1981) shed light on the implied volatilities on European stock options. Later, greater emphasis was given on European index options. A wide majority of studies, including Day & Lewis (1988, 1990), Lamoureux & La Strapes (1991), Harvey & Whaley (1992), Canina & Figlewski (1993), Fleming (1993), and Christiansen & Prabhala (1998) examined S&P 100 index. Some other studies, like Park & Sears (1985) and Feinstein (1989) examined the options on S&P 500 index futures. Moreover, some other scholars payed attention on options on different financial assets like German benchmark bond (Neuhasu, 1995) and currency futures (Jorion, 1995). These researches concluded contradictory results. For S&P 100 stock index market (Canina & Figlewski, ibid) as well as German government bond market (Neuhasus, ibid), the implied volatility seemed to make inaccurate estimations about future realized volatility. However, Christensen & Prabhala and Jorion noted the predictive power of implied volatility for both S&P 100 index as well as currency futures. These findings were enforced more recently by Georgios Chalamandaris & Andrianos Tsekrekos

(2009) who found profitable delta hedged positions on OTC currency options based on the predictable dynamics in implied volatility surfaces. On the other hand, a vast majority of studies concluded the existence of an upward bias in the implied volatility. (Anagnou et al, ibid; Jackweth, ibid). Additionally, another drawback of the implied volatility is the fact that it violates the assumption of the Black and Scholes model for constant variance. (Perignon & Villa, 2002; Jackwerth, 2004). Given the sets of the strike prices, market implied volatilities usually present a skewed structure, widely known as volatility smile or volatility smirk. This effect is supported by scholars that focus on stock index options, for example Rubinstein (1994) for the US stock index, in Tompkins (2001) for the Japanese, and in Pena (1999) for the Spanish index. Furthermore, Rubinstein (1985) noted slightly U shaped volatility smiles for options written on common US stocks. In the mean, Mayhew (1995), Toft & Prucyk (1997) and Dennis & Mayhew (2002) noted the downward volatility smiles for common stock options, less steep than the index smiles. For options written on the interest rate caps market, Jarrow et al (2003) noted a downward volatility smile. Because of this bias as well as of the smile nature of the implied volatility, investors have to apply more informative models in order to estimate future asset prices.

2.2 Risk neutral density

Recently, attention has focused on extracting the whole distribution of the underlying asset price, rather than to obtain the implied volatility. The distribution derived from option prices is the risk neutral probability distribution (RND) (Perignon & Villa, 2002). In a risk neutral world, due to arbitrage pricing theory, the present value of a security equals to the present value of its expected payoffs discounted by a riskless rate, and therefore RNDs can be obtained by taking advantage of the observed option prices (Taylor 2005; Monteiro et al, 2008). The first one who advocated that a set of European option prices can be exploited in order to extract RNDs was Ross (1976). According to Breeden and Litzenberger (1978) and to Banz and Miller (1978), the RND equals to the second derivative of the price of the option with respect to its strike price. Due to the fact that the expected payoffs of options depend on the future outcome of the underlying asset, the implied RND has a forward looking nature, and consists as a forecast of the probability distribution of the underlying asset (Monteiro et al, ibid). There follows a respective summary about the methods of extracting RNDs and their applications.

2.2.1 Methods to extract risk neutral densities

A wide variety of methods in order to obtain risk neutral densities have been developed. In a general way, we can divide them into two major fields: The parametric and nonparametric methods.

- Parametric Methods

As far as they are concerned, the RND is a part of a general distribution category. The market option prices are calibrated to the assumed distribution in order to estimate the unknown parameters (Anagnou et al, 2002). This family of RNDs includes mixture methods, expansion methods, generalized distribution methods as well as stochastic process methods.

1. <u>Mixture methods</u>

Ritchey (1990) was the first one to propose a mixture of two lognormal densities (MLN). It is nothing more than a weighted average of lognormal distributions. Mixtures of lognormals is a very widespread method, applied by many scholars such as, Soderlind & Svenson (1997), Bahra (1997) and Coutant (2001) for interest rates. Campa et al (1998) and Jondeau & Rockinger (2000) used it for exchange rates. Gemmil & Saflekos (2000), Bliss & Panigirtzoglou (2002), Anagnou et al (2002) and Liu et al (2007) used for equity indices. Additionally, Melick & Thomas (1997) applied a seven parameter mixture of three lognormals in order to estimate the crude oil future prices during the Gulf War. Mixture methods are preferable by policy makers in many industrialized nations because of their being easy to implement (Taylor 2005). Furthermore, they are considered as flexible densities which means that they can have a wide variety of shapes. Nevertheless, the more flexible they are, the more the number of parameters increases (Jackwerth 2004).

2. Expansion methods

Their foundations are theoretically strong and are related to expansions of Taylor series for more simple functions (Jackwerth 2004). These methods start with a simple probability distribution and then, there are incorporated correction terms (Jackwerth, ibid). Jarrow & Rudd (1982) applied the edgeworth expansion method. A Gram Charlier expansion approximation was applied by Corrado & Su (1996) in order to take a picture of the implied RNDs. Similarly, Abadir & Rockinger (1997) developed an adjustment to the normal distribution based on a Kummer's function. Moreover, Madan & Milne (1994) and Abken (1996) applied the Hermite polynomial expansion method for the S&P 500 index. Their path followed also Jondeau & Rockinger (2001) for the French franc against Deutse franc, as well as by Coutant (2001) for French interest rates. A common problem that arises when applying expansion methods is that the constraints of the density may not be guaranteed due to the added correction terms. Hence, the extracted RND has to be checked for being strictly positive and for integrating to one. (Jackwerth, ibid).

3. Generalized distribution methods

For extracting the distribution of future prices of an asset in a general form, the parameter vector should include the moments up to fourth order (mean, variance, skewness, kyrtosis) (Jackwerth , 2004; Taylor, 2005). The generalized beta distribution of kind two, known as GB2 was noted by Bookstaber & McDonald (1987, 1991). This kind of density was used in a scholar by Anagnou et al (2002) in order to get a picture of the S&P 500 index as well as the sterling rates. Moreover, Aparicio & Hodge (1998) and also Liu et al (2007) applied the GB2 density in order to estimate respectively the S&P 500 spot index as well as the futures written on it. In addition, Sherrick et al (1992, 1996), in order to estimate RNDs for soybean futures, they a applied a three parameter Burr distribution. In comparison with lognormals, generalized densities are considered to be more flexible, as they can be either positively or negatively skewed. In addition, since there exists a large family of these methods, the GB2 density can be replaced by a density of the same family (Bookstaber & Mcdonald, ibid). However, its parameters are not interpretable (Taylor, ibid).

4. Stochastic process methods

These methods in order to be applied, the stochastic process of the price of the underlying asset has to be fully specified. A realistic specification incorporates the stochastic volatility. Hull & White (1987), Chesney & Scott (1989), Melino & Turnbull (1990), and Ball & Roma (1994), assumed that the volatilities followed a diffusion process in their stochastic volatility models. Further assumptions noted the correlation between the returns of the underlying asset and the volatilities, and have to be implemented in order for the models to be more reliable. (Jondeau & Rockinger, 2000). Additionally, Heston (1993) assumed a different stochastic process for the volatilities. In order to obtain closed form solutions for option prices, he applied a different numerical approach (Jondeau & Rockinger, 2000). On the contrary, Jorion (1989) and Taylor (1994) focused on the price jumps. Therefore, the stochastic process of the price of the underlying asset is regarded as a log normal jump diffusion in its Bernulli version. This stochastic process could be assumed as the sum of a GBM plus a poisson jump process (Jondeau & Rockinger, ibid). Merton (1976), Cox & Ross (1976) and Bates (1991, 1996) in their scholars noted the pricing closed closed form solution for the jump diffusion. The assumptions of jump diffusion models were made more simple by Malz (1996). As far as he is concerned, there can be mostly one size with constant volume over the life of the option.

- Non parametric methods

These methods, contrary to the parametric ones, they do not assume a distribution family, neither any parametric model to calibrate the data. As far as they are concerned, RNDs are obtained only from the observed option data (Monteneiro et al, 2008). They are considered far more flexible for depicting the data, as they bear much fewer assumptions (Monteneiro et al, ibid). Hence, it is better to implement non parametric methods when one is not in the position of figuring out the stochastic process generating the time series of the data or, when this process drifts over time. Nevertheless, a drawback of these models is that a vast number of variables has to be estimated (Jackwerth, 2004). Furthermore, the densities obtained from these methods are often violated in the form of positivity, integration to one and smoothness (Jackwerth, ibid). Next, we present the four sub groups of nonparametric methods.

1. Maximum entropy methods

The goal of these methods is to maximize the volume of the missing information of the RNDs that fit the date, and is achieved by maximizing the cross entropy (Jackwerth, 2004). This methodology was first implemented by Rubinstein (1994) with lognormal prior, by Buchen & Kelly (1996) assuming uniform and lognormal priors, by Stutzer (1996) with historical distribution priors and also by Rockinger & Jondeau with normal, t as well as generalized distribution priors. An advantage of this kind of methods is the fact that they regard very few assumptions and put no constraints on the dataset (Taylor, 2005). However, large negative values might serve as denominators for these methods because they use the logarithm of very small probabilities.

2. Kernel regression methods

Data of option prices are used to calibrate either the call option price formula, or the implied volatility function, without any assumption about the shape of a regression function (Jackwerth, 2004; Taylor, 2005). Rookley (1997) in his scholar applied a bivariate kernel method in logmoneyness and time to maturity. In the aftermath, Pritsker (1998) implemented the same kernel method for options written on interest rates. Their path also was also followed by Ait Sahalia & Lo (1998, 2000) who implemented kernel regression methods in order to obtain RNDs for S&P 500 index. These methods appear to have two disadvantages. Firstly, they are considered to be data intensive and secondly, in the very case that data have gaps, it might be impossible to smooth RNDs. (Jackwerth, ibid).

3. Implied binomial trees

They assume a priori that the RND for all possible knots is established using binomial trees (Monteiro et al, 2008). According to Cox Rox and Rubinstein, the binomial tree consists as a discretization of the Black and Scholes model. A generalization of the already existing binomial trees was mentioned by Jackwerth (1997) as an expanded form of the model of Rubinstein. On the contrary, a different approach was noted by Derman & Kani (1994), who used the stepping forward method in order to construct the tree, which could be characterized as numerically instable. Later, Chriss (1996), and Barle & Cakici (1998) implemented some methods in order to make this method more stable. A similar approach which also suffers from instabilities is the Dupire tree.

4. Curve fitting methods

Shimko (1993) was the precursor of the implied volatility function method. He mentioned that firstly option prices have to be converted into implied volatilities with the Black and Scholes formula in order to interpolate and smooth the implied volatility curve. Subsequently, implied volatilities which lie on the smooth curve have to be extrapolated into option prices in order to obtain the RNDs (Figlewski, 2009). This method was applied by Malz (1997) who converted the observed market option and exercise prices into sets of deltas as well as implied volatilities, which seems to have flexible RND tails. Apart from the method of Shimko, which is nothing more than a simple quadratic polynomial of strike prices, Campa et al (1998) implemented a cubic spline polynomial for interpolating the implied volatility. Later on, Bliss & Panigirtzoglou (2002, 2004) took advantage of this method but in a different way. Instead of using a volatility/exercise price space, they introduced a volatility/delta space. It seems to be more flexible than the quadratic function at the cost of having more parameters to be estimated (Taylor, 2005). The drawback of these methods is firstly that the first two derivatives should be continuous, and also that sometimes negative probabilities may arise.

- <u>Comparison of different methods</u>

A wide range of scholars has focused on comparing different methods of obtaining RNDs (Bahra, 1997; Jondeau & Rockinger, 2000; Coutant et al, 2001). Andersson & Lomakka (2003) examined the stability of different methods, while Markose & Alentorn (2005) occupied with the pricing errors that arise. They did not come to a similar conclusion. For Bahra and Jondeau & Rockinger mixtures of lognormals appear to be more preferable. As far as Coutant is concerned, hermite polynomials seem to outperform the mixtures and maximum entropy methods. On the other hand, Bliss and Panigirtzoglou (2000) and Campa (1998) find the curve fitting methods better than the parametric ones. As a result, taking the available literature into consideration, we cannot conclude that there exists a unique method or a family of methods which is always better than the others. Hence, we are in the position of advocating that all methods mentioned yield sensible densities.

2.2.2 Applications of risk neutral density

There is a wide variety of researches which seek to evaluate the estimated implied RNDs apart from just applying a methodology. To begin with, Longstaff (1995) and Rosenberg (1998) implemented RNDs methods in order to price derivatives. According to Perignon & Villa (2002), implied RNDs serve as appropriate tools for pricing exotic derivatives with complex payoffs. Secondly, RNDs are very common tools in risk management (Perignon & Villa, ibid). Jackwerth & Rubinstein (1996), Ait Sahalia & Lo (2000) and Berkowitz (2001) implement implied RNDs from option prices in order to estimate the value at risk for extreme losses. More recently, Navatte & Villa (2000), Han (2008) and Conrad (2008) in their scholars shed light on the higher moments of RNDs in the optimal portfolio choice and also in asset pricing. Moreover, a strong majority of researches exploits RNDs as a medium in order to grasp market expectations about very important economic and political events. One category of the most popular events examined by RNDs are the stock market crashes.

(Bates, 1991; Gemmil, 1996; Malz, 1996; Jackwerth & Rubinstein, ibid; Melick & Thomas, 1997; Jemil & Saflekos, 1999; Bahra et al, 2001). In Bate's point of view, risk neutral densities did not predict the market crash of 1987, and this was in accordance with the findings of Gemmil on FTSE 100 index. In addition, Shiratsuka (2001) who investigated RNDs for the Japanese and Korean markets, concluded that RNDs seem to be reactive than predictive. Another family of studies emphasizes on information deriving from news. Bahra (1997) as well as McManus (1999) approved the predictive power of RNDs' in the alterations in Government's interest rate policies. These findings were also enforced by Campa et al (1999) with respect to the international finance. More specifically, his scholar illustrated that the investors' expectations in the currency market can be modeled by RNDs. Furthermore, Jondeau & Rockinger (2000) in their research occupied with the French legislative election in 1997 and to their point of view, RNDs did well on grasping the market anticipation about the election results. On the other hand, due to Gemmill & Saflekos (2000) RNDs could not depict market expectations before the general elections in the UK. Last but not in the least, RNDs can serve as an innovative way to estimate the implied risk aversion of the representative investor (Ait Sahalia & Lo, ibid; Jackwerth, 2000; Rosenberg & Engle, 2002; Bliss & Panigirtzoglou, 2004).

2.3 Real world density (RWD)

In the recent years, a hot issue concerning many scholars is the relationship between risk neutral and real world densities. In a theoretical basis, RND is similar to RWD if and only if the investors did not seek extra return for taking riskier investments in other words, if they were risk neutral. Hence, we can assume that the only difference between RND and RWD stems from the representative investor's risk aversion (Anagnou et al, 2002). Due to Jackwerth's point of view, the following equation must hold:

Risk neutral density = real world density * risk aversion adjustment.

One majority of researches payed attention on the methodology of converting RNDs into RWDs, while another one focuses on the estimation and interpretation of the implied risk aversion. Ait Sahalia & Lo (1998, 2000) and Rosenberg & Engle (2002) mentioned the risk adjustment with a stochastic discount factor or with a pricing kernel after having combined the modern asset pricing theory with present as well as future consumption. Assuming that the moments of the returns do not change significantly over time, or else that the returns are stationary, Ait Sahalia & Lo (ibid) came up with a risk aversion measure in a dynamic exchange economy. Similarly, Rosenberg & Engle (ibid) figured out the "empirical pricing kernel" and via this, they modeled a dynamic risk aversion function. Under the assumption that the investors' risk preferences do not shift by the passage of time, his marginal utility of terminal consumption is proportional with the stochastic discount factor (Jackwerth, 2000). Nevertheless, these functions suffered from some anomalies. A more appropriate idea in order to extract RWDs was the power utility function, which was developed in the later studies of Bakshi & Kapadia (2003) as well as of Liu et al (2007). Bliss & Panigirtzoglou (2004) under the assumption that the representative investor is rational, they implemented not only power utility function, but also an exponential one in order to estimate the risk aversion for FTSE 100 and S&P 500 options. Bunn (1984) and Fackler & King (1990) did the same thing but in a different way. Apart from utility functions, they applied a recalibration method, which can be used to any set of risk neutral densities. By this method, RNDs are converted directly into RWDs via the cumulative distribution of the Beta function. Finally, Liu et al ibid derived RWDs from mixtures of lognormals and GB2 RNDs by using both power utility function and calibration method.

Section III Methodology

3.1 Risk neutral density estimation methods

As far as Breeden and Litzenberger (1978) are concerned, the unique risk neutral density f(x) for every possible value x of the price of the underlying asset St can be extracted from European style call options C(X) if and only if there are observed contracts for all exercise prices X and under the assumption of the absence of arbitrage opportunities. Then, the RND f(x) equals

f(x) = $e^{rt}\frac{\partial^2 c}{\partial X^2}$ and

$$c(X) = e^{-rt}\int_X^{\infty}(x-X)f(x)dx, \quad (1)$$

where r stands for the continuous risk free rate and t is the time to the option's maturity and Q represents the risk neutral measure

$$c(X)=e^{-rt}E^Q[\max(\mathrm{St}-\mathrm{X},0)]. \quad (2)$$

In this paper, we will discuss further two methods of the parametric family, the mixtures of two lognormals and the jump diffusion model.

❖ Mixtures of lognormal method (MLN)

According to the majority of the previous studies, the prices of financial assets S, follow a geometric brownian motion process (GBM),

$$dS = \mu S dt + \sigma S dz \tag{3}$$

Assuming a risk neutral world, we replace μ with r-q and according to Liu et al (2007) the RND of the underlying asset's price at the expiration date of the contract could be defined as a weighted combination of two lognormal densities f_{LN}

$$f_{MLN}\,(x|\theta) = w f_{LN}(x|S1, \sigma 1, T) + (1-w) f_{LN}(x|S2, \sigma 2, T) \tag{4}$$

Where,

$$f_{LN}(x|S, \sigma, T) = \frac{1}{x\sigma\sqrt{2\pi T}} \exp(-\frac{1}{2}\left[\frac{\log(x) - (\log(S) - 0.5\sigma^2 T)}{\sigma\sqrt{T}}\right]^2) \tag{5}$$

The parameter vector equals to $\theta = [S1, S2, \sigma 1, \sigma 2, w]$, where $0 \le w \le 1$ because w represents a probability. In order for the density to be characterized as risk neutral, the following constraint should hold: $wS1 + (1-w)S2 = S$, where S is the current price of the underlying asset. The MLN considers the returns' non stationarity and unlike lognormal polynomials, its tails are always positive. It assumes that the returns are normally distributed and its parameters shift by the passage of time because the financial risk fluctuates. (Ritchey 1990). In comparison with a single lognormal (SLN), the MLN can take flexible shapes which depict better the market expectations against the rigid shape of the SLN due to the five parameters against the one of SLN. Hence, an MLN considers the price of a European call option to be a mix of Black and Scholes prices and thus, the theoretical price of a European call option is

$$c(X|\theta, r, t) = wCbs(S1, X, \sigma 1, r, t) + (1-w)Cbs(S2, X, \sigma 2, r, t) \tag{6}$$

where Cbs denotes the call price of the Black and Scholes formula.

❖ Jump diffusion model (JDM)

At the advent of extreme events, the Black and Scholes model may misprice the observed option data, due to the huge price variations of the assets (Bedoui & Hamdi, 2010). There stands a majority of scholars who take into consideration that the stochastic process of the assets is a lognormal jump diffusion process, rather than a GBM (Jorion, 1989; Taylor, 1994). This process, assumes that the price follows a sum of GBM and a jump diffusion component. According to Bedoui and Hamdi, this method can illustrate the effects of excess skewness and kurtosis. It is then defined

$$dS = \mu S dt + S\sigma dz + kSdp \quad (7)$$

with p representing the poisson probability. λ is the average jump rate and k in its absolute value represents the volume of the jump. It is a random variable and its sign determines if the jump is positive or negative (Jondeau & Rockinger, 2000). Under the assumption of a risk neutral world, the stochastic process of the price is

$$dS = \left(r - q - \lambda E(k)\right) S dt + \sigma S dz + kSdp \quad (8)$$

Considering that during the life of the option there will occur at least one jump of constant size we define the components of the Black and Scholes model as

no jump: $$d1 = \frac{\ln\left(\frac{S}{X}\right) + (r - q - \lambda k - 0.5\sigma^2)T}{\sigma\sqrt{T}} + \sigma\sqrt{T} \quad (9)$$

$$d2 = d1 - \sigma\sqrt{T} \quad (10)$$

Jump: $$d1' = \frac{\ln\left(\frac{S}{X}\right) + \ln(1+k) + (r - q - \lambda k - 0.5\sigma^2)T}{\sigma\sqrt{T}} + \sigma\sqrt{T} \quad (11)$$

$$d2' = d1' - \sigma\sqrt{T}$$

(12)

In accordance with the Bernulli version of the jump diffusion, like the MLN the theoretical price of a European call option is a weighted sum of two call Black and Scholes call prices

$$C(X) = (1 - \lambda T)Cbs(S, T, X, r, q + \lambda k, \sigma) + (\lambda T)Cbs(S(1 + k), T, X, r, q + \lambda k, \sigma)$$

(13)

with $1 - \lambda T$ representing the no jump probability in the option's horizon and λk the volume of the jump in its absolute form. Furthermore, it can be considered that the risk neutral density of this model is a mixture of two densities and then it is defined as:

$$fq(x) = (1 - \lambda T)\frac{1}{x\sigma\sqrt{2\pi T}}\exp[-0.5(d2(X))^2] + \lambda t\frac{1}{x\sigma\sqrt{2\pi T}}\exp[-0.5(d2'(X))^2] \quad (14)$$

with parameter vector $\theta = [\sigma, \lambda, k]$ under the constraint $0 \leq \lambda \leq \frac{1}{T}$

- **Spline Method (SPL)**

In the case that the option's implied volatility is not constant, but a dynamic function, the true density is not a lognormal distribution. Consequently, in order to determine the RND shape, there should be specified a function that fits the implied volatility. We consider $\sigma(X) = \sigma(X|\theta)$ with θ representing the parameter vector that should be estimated. In this paper, we apply the method proposed by Andersen and Wagener (2002). In order to specify the implied volatility function, this method considers a spline of fourth order polynomials in a σ/delta space under the condition that it is three times differentiable at the knot points. Under this case, the second order derivative of this function is a differentiable spline of parabolic functions, which gives a much more flexible shape in the RND than the Shimko's method which is a simple quadratic function in a volatility/Strike price space. Compared with cubic formulations proposed by Bliss and Panigirtzoglou (2002) and Liu et al (2007) which are not three times differentiable, it solves the problem of zig zag shapes, or “kinks” which appear in these methods. Furthermore, it is more computationally feasible than cubic splines because they assume one parameter for every knot point making the size of the parameter vector large. However, like all curve fitting methods, there may arise negative probabilities (Bliss & Panigirtzoglou,ibid). The implied volatility function we apply is

$$\sigma(\delta|\theta) = \alpha_0 + \alpha_1\delta + \alpha_2\delta^2 + \alpha_3\delta^3 + \alpha_4\delta^4 + \sum_{i=1}^{s} \alpha_{4+i}\,(\delta - ki)^4$$

(15)

Where s represents the number of knot points, δ is the delta implied by the Black and Scholes model and θ is the parameter vector. First, we convert observed option prices into implies volatilities in order to calculate deltas. We choose one knot point in our implementation with the value of 0.5 $k1 = 0.5$ because it is enough to provide reasonable fit while preserving the flexible shape and so, our parameter vector consists of six free parameters $\theta = [\alpha_0 \dots \alpha_6]$. Afterwards, implied volatilities are extrapolated into call option prices and hence, we can have option prices for all strikes. This allows us to calculate the RND directly from (1). The second order derivative of the call price with respect to the strike price can be approximated numerically from the following expression:

$$\frac{\partial^2 c}{\partial X^2} = \frac{C(X+\Delta X)-2C(X)+C(X-\Delta X)}{\Delta X^2}$$

(16) where, ΔX represents a small change in the strike price so we define it 0.5 equal to the tick size of FTSE 100.

3.2 Risk transformation

The role of risk aversion is fundamental in economics and finance and has to be taken into consideration. A task of great importance is to assess the agent’s behavioral patterns under risky situations. Risk neutral densities assume a utopic world where agents are indifferent about taking extra risk or no, which is irrational. Assuming that the representative investor abhors risk and has rational expectations, a transformation from a risk neutral density to a real world density should be made in order to take more informative results. The method we implement for this transformation is based on the economic theory and assumes a power utility function for the investor's risk preferences. In order to extract the real world density (RWD), the risk neutral density has to be multiplied by the agent's marginal utility function u(x) (Ait-Sahalia & Lo, 2000; Jackwerth 2000), and therefore, the following relationship holds for the real world density

$$fp(X) = \frac{fQ(X)/u\prime(x)}{\int_0^{\infty} fQ(Y)/u\prime(y)dy}$$

(17)

The power utility function we implement u(x) seems to be the most widespread for real world transformations (Bliss & Panigirtzoglou, 2004; Liu et al, 2007). Assuming this function, the relative risk aversion is constant and proportional to the CRRA parameter γ. The function has the following form

$$u(X) = \frac{X^{1-\gamma}}{1-\gamma} \quad \text{if} \quad \gamma \neq 1 \quad \text{and} \quad u(X) = \log(X) \quad \text{if} \quad \gamma = 1$$

(18)

As a consequence, the RWD is given by

$$fp(X) = \frac{X^{\gamma} fQ(X)}{\int_0^{\infty} Y^{\gamma} fQ(Y)dy}$$

(19)

If the risk aversion parameter γ is positive it indicates that the agent is risk averse and seeks risk premium for taking risky investments. If it is zero, the risk neutral assumption is the correct one, and if it is negative, the investor is risk seeker (Taylor, 2005).

- **Mixtures of lognormals transformation**

Due to the fact that there are no closed formed solutions for the transformations of the jump diffusion model and for the spline, we transform the mixtures of lognormals method. We assume that the RWD is given by (19) and that the γ parameter is constant over an annual period. Obviously, if the RND is a mixture of lognormals, the RWD will also be a mixture of lognormals (Liu et al, 2007) and therefore the following relationship holds

$$fp(X|\theta,\gamma) = w^{*}fq(X|S1^{*},\sigma 1,T) + (1-w^{*})fq(X|S2^{*},\sigma 2,T)$$

(20)

and the real world parameters are

$$\theta^{*} = [S1^{*}, S2^{*}, w^{*}, \sigma 1, \sigma 2]$$

(21)

$$Si^{*} = Si\exp(\gamma\sigma i^{2}T) \quad for\ i = 1,2$$

(22)

$$\frac{1}{w^{*}} = 1 + \frac{1-w}{w} \left(\frac{S2}{S1}\right)^{\gamma} \exp[0.5\ (\gamma^{2}-\gamma)(\sigma 2^{2} - \sigma 1^{2})T]$$

(23)

3.3 Estimation of RND and the transformation parameters

At a first step we estimate the RND vector of parameters θ. For the mixtures of lognormals and for the jump diffusion model, we derive it by minimizing the root of the average squared differences between the observed prices of call options and the theoretical ones. Hence,

$$G(Theta) = min\sqrt{\frac{\sum_{i=1}^{N}(Cmkt(Xi) - C(Xi|theta)^{2}}{N}}$$

(24)

For the estimation of the parameter vector of the spline we use the method proposed by Andersen and Wagener (2002). The function which has to be minimized has a more complex form.

The spline parameters are obtained by minimizing numerically the average squared error between the observed implied volatilities σi's and the calibrated implied volatilities $\hat{\sigma}(\delta i)$ given by (15) with a smoothing penalty

$$G(Theta) = \min \sum_{i=1}^{N} [\sigma i - \hat{\sigma}(\delta i)]^2 + \lambda \int_0^1 \kappa(\delta)^2 d\delta$$

(25)

Where

$$\kappa(\delta) = \frac{|\hat{\sigma}''(\delta i)|}{[1+\hat{\sigma}'(\delta i)^2]^{3/2}}$$

(26)

The parameter λ represents the weighted penalty between goodness of fit and smoothness. Like Bliss & Panigirtzoglou we choose it to be 0.0011 because for this value the RND has the optimal relationship between goodness of fit and flexibility. For lower values of λ, densities are not smooth and for higher values, they lack in goodness of fit. Commonly, κ is chosen to be simply the second derivative of the spline function, which penalizes significantly the curvature at steep segments. This problem is solved by the application of (26), the denominator of which considers the steepness of the curve.

At a second step, we estimate the relative risk aversion parameter γ. For this procedure, we apply the maximum likelihood estimation method, which is commonly considered enough consistent for optimization problems (Liu et al, 2007). We define S_{Ti} the prices of the underlying at the options' expiration dates and we also define t_i^* the expiration dates. The log likelihood function which has to be maximized has the following form

$$LogL(S_{Ti}|\gamma) = \sum_{i=1}^{N} \log(fp, i(S_{Ti}|\gamma, \theta^*))$$

(27)

Section IV Data analysis

In our study, we collect from Bloomberg the daily closing prices for European style call and put options which are written on FTSE 100 spot index and traded at LIFFE, with the contract symbol "ESX". Each contract is valued at 10 pounds per index point and is quoted on index points. These options expire on every third Friday of the expiration month and the trade ceases at 10:15 (London time). The delivery months are the serial months out to two years. The exercise prices are found on intervals between 25, 50, 100 or 200 index points with respect to the lifetime of the expiration month and the minimum tick size for them is 0.5. The daily cash settlement for these contracts is determined by taking into consideration the FTSE 100 average index level in a sample of 15 consecutive seconds in an interval between 16:20 and 16:30 (London time basis). In the last trading day it is determined between 10:10 and 10:30. The buyer of such a contract is obliged to pay the premium in full the business day after the transaction and the minimum trade is 500 contracts. These options are chosen because the underlying index is considered as the "UK market portfolio".

According to the put call parity, all European put options can be converted into calls and vice versa. This relationship holds if and only if there are no arbitrage opportunities. However, in the real market this may be violated because of the mispricing of many options. Hence, in our study in order avoid this problem and get more informative results we use only call options.

In order to shed light on the market expectations for the Brexit referendum day, we choose options which expire the nearest dates before and after the referendum date. The referendum date was on 23 June 2016 and the nearest expiration dates before and after were 17 June of 2016 and 15 July of 2016. Options of the first expiration date started to trade on 23 June of 2014, exactly two years before while the contracts of the second expiration date on 7 August of 2015. We estimate RNDs and RWDs for every consecutive month or two weeks for each period which finally gives us 41 overlapping distributions, 27 for the first period and 14 for the second.

In order to secure that our results are informative and make sense, the collected data pass from some exclusion criteria. Firstly, we exclude options for which the following relationship does not hold: $C > S - Ke^{-rt}$ We know for the financial theory that the value of a European option is the sum of its intrinsic and time value, so the lower bound is the intrinsic value and options that trade less than this are considered mispriced. Also, we exclude options with values less than the tick size because they are considered illiquid and account for only a tiny proportion of our sample. Finally, we eliminate options which do not trade for more than four consecutive days, because they are also assumed as illiquid. After the implementation of the filtering criteria, our data sample shrinks approximately per 50%. The following table gives information about the observations per day for each period. By t1, we define the options that expire on 17/6/2016, one week before the referendum and by t2, we define the options with expiration on 15/72016, approximately one month after the referendum. It refers to the call option data after the filtration we applied. As we see, in 2014 we have observations only for t1 options. On 2014 for example, there are traded on average t1 calls in intervals of 22 strike prices, which implies that for this year RNDs and RWDs are derived by taking into consideration 22 observations average. It is interesting to notice that for all years except 2014, out of the money options are more than the in the money ones. It is also very interesting that from 2015 to 2016, which is the referendum year, the in the money options traded for both periods decline, while increases significantly the number of out of the money call options. For t1 they increase by 35% (31 over 23 contracts) and for t2, they increase by 75% (28 over 16 contracts). This means that there is a galloping demand for out of the money call options in the referendum year. At a first glance, this seems irrational as a buyer of an OTM call option has optimistic expectations about the underlying asset. A feasible explanation for this increase is that they may serve as tools for speculative strategies. For example, investors may use them as protective calls while they open short positions against the FTSE 100 index before the referendum, or others who want to speculate on the volatility of the index might use them for applying straddles or butterfly spreads.

Table 1 Descriptive statistics of observed call options per day							
		Total number of calls		ITM		OTM	
		t 1	t2	t1	t2	t 1	t2
2014	Max	49	-	24	-	24	-
	Average	22	-	8	-	8	-
2015	Max	69	58	26	23	52	43
	Average	37	30	13	11	23	16
2016	Max	68	60	15	20	59	52
	Average	37	35	9	10	31	28

Table 1 shows the summary statistics of the observed call options for both expiration dates as mentioned by t1 and t2. They are generated by the observations of 533 days in total and are expressed in annual basis. There are included the data that passed the filtering criteria.

Furthermore, we collected from Bloomberg the adjusted closing prices for the FTSE 100 index which correspond to the issue as well as to the expiration dates of the options and they are used in order to obtain the RNDs. We did not take into account dividend yields for the index because they have little or no effect on the results.

As a risk free rate, we consider the 6month short sterling London interbank offered rate (LIBOR) on UK pound, because it is considered globally as a benchmark for approximating the riskless return and the majority of transactions are based on it. It is also collect it from Bloomberg and then is transformed with regards to the continuous compound. We observe that for our sample period it ranges from 0.55% to 0.75%, which indicates that it bears tiny or no effect on our results.

Section V Empirical Results of RNDs

After having applied the methodology discussed earlier, we have extracted RNDs and RWDs every one month or two weeks for each of the expiration days we mentioned. In total we have 41 overlapping densities, 27 of which representing the estimations on one week before the referendum (t1) and 14 of which the estimations on 22 days after the referendum(t2). For the MLN and JDM, the parameters are estimated by minimizing the root mean squared errors of the call prices defined by (24) and for the SPL parameters we minimized the equation (25). In this section, we are going to discuss the options at the money implied volatility in comparison with the index returns. Then, we present the pricing errors of the methods we applied. Furthermore, the moments of the RNDs are discussed. In addition, we show the RND estimations for two specific dates respectively and also the evolution of the RNDs by the passage of time.

5.1 Implied volatility

At a first step, before presenting the RNDs, we should get a first glance at the forward looking nature of the options. As far as Taylor, Yadav and Zhang (2010) are concerned, when the prediction horizon coincides with the expiration of the options, the at the money implied volatility is the best prediction for the future volatility of the underlying asset compared with volatility forecasts based on historical data and generated by ARCH models. However, they advocate that when the forecast horizon is one day ahead, volatility predictions from historical data outperform those based on the implied volatility. The following graphs could show some verification on these results. Figure 1 depicts the daily returns of FTSE 100. The blue area refers to the period before the brexit referendum announcement starting from January of 2014 and ending on 19 February of 2016, which is the date of referendum announcement by Kameron. The red area refers to the period from the announcement to July of 2016, a month after the referendum. Figure 2 shows the time evolution of the at the money implied volatility for the options sets for the two expiration dates as mentioned earlier. The shaded area of figure 2 illustrates the period after the referendum announcement. The samples of the two figures correspond chronologically. From figure 1, we can grasp that in the period after the announcement, the index is more volatile than before.

More specifically, in the first period, the annualized volatility was 16% while in the second one it increased by 4 percentage points (20%). This is a first sign of the investors' anxiety for the referendum results and the volume of the impact it will carry. On 23 of June, the referendum date, the index had a positive return of 1,12%. However, the day after, when the referendum results were released, the index fell by -3,2% and the fall continued in the consecutive days. This upward trend of the index volatility is explained by the proportional upward trend of the at the money implied volatility for both expiration dates. The inclining trend starts by the referendum announcement when we observe a high for both expiration dates (14,6% for t1 and 15,4% for t2) and goes on till the expiration. We can observe that the ATM implied volatility of the options which expire before the referendum is higher than of those which expire after. This inverses after the announcement. Contracts that expire after the referendum have significantly higher ATM implied volatility which peaks at the referendum date (26,9%). This hints that investors seem to be more anxious about the after referendum period and gives a first impression that observed option prices carry useful information about the economic sentiment.

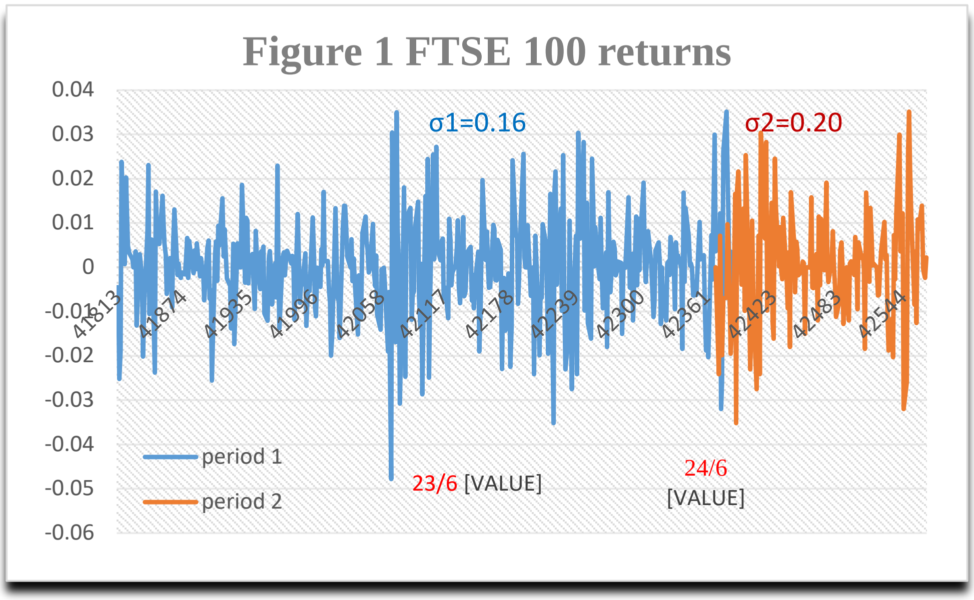


Figure 1 shows the daily returns of FTSE 100. The blue area refers to the period from June 2014 to 16 February 2016 and the orange area refers to the period from 16 February to 14 July. By σ1 and σ2 are defined the realized annualized standard deviations of the returns that correspond to each period.

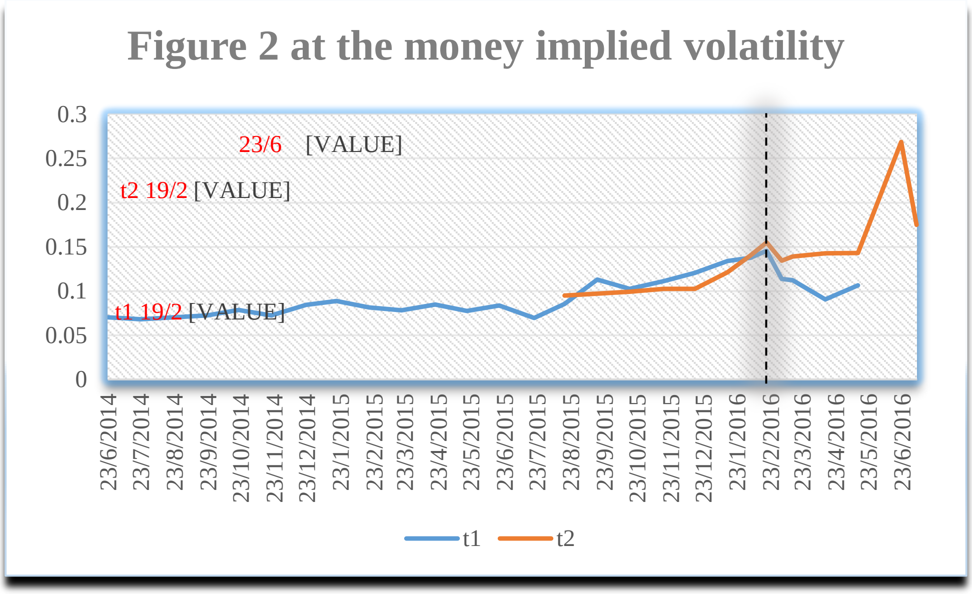


Figure 2 shows the at the money implied volatility every month for the two sets of options as described and mentioned earlier by t1 and t2. The shaded area separates the two periods before and after The announcement. The value marked is the value on the referendum date.

5.2 Pricing errors of the three methods

In this section, there are examined the descriptive statistics of the average errors of each method. Table 2 gives an overall summary of the root average squared error of all methods for options of both expiration dates (t1, t2), and is derived by the procedure discussed earlier. Tables 3, 4, 5 summarize the error of each method for options of both expiration dates (t1, t2) respectively in an annual basis. (Further details are given below each table). We examine the descriptive statistics of the average error, because they provide us useful information on whether we can price options accurately by using the density function we assumed. First of all, we can stress that overall, the MLN has the lowest error compared to the other methods for both sets of options. (For t1: 0.72<0,99<2,26 and for t2: 1,62<2,06<2,57). Furthermore, we observe that the spline method performs the worst in fitting the data, as it shows the largest pricing error. This coincides with the findings of Liu et al

(2007), who advocated that spline methods are inferior to the classic parametric ones in pricing accurately the observed data. In our case, a possible explanation for this could be the fact that there are observed less in the money options than out of the money (table 1) which can make the implied volatility function less consistent. However, an interesting point to stand is that the spline's error is the less volatile with respect to the options that expire after the referendum compared to the other methods (1,39<3,05<2,66). The maximum error of the spline for this set of options is 4,88 significantly lower than the 12,05 of JDM and 10,45 of MLN. Consequently, despite the highest average error, the spline method should be taken seriously into account. In addition, some useful information we can grasp is that for all methods, the average error is higher and more volatile for the options that expire after the referendum. This is driven by the year of 2016, the referendum year. More specifically, for the MLN the average error in 2016 for t2 options is 2,34, much higher than 0,32 in 2015. For the JDM, it is even higher (2,98 in 2016 over 0,41 in 2015). For the spline method, we do not mention such an increase between these two consecutive years. (2,64 in 2016 over 2,4 in 2015). A feasible explanation for this could be that due to the higher uncertainty in the referendum period, many (t2) options might trade mispriced. On the other hand, for the contracts that expire before the referendum (t1), they are priced more accurately in 2016 than in the years before by all methods. However, the differences in the average error between the years are less significant compared with those of the second set of options (t2) for all methods.

Table 2	MLN		JDM		Spline	
G(theta)	t1	t2	t1	t2	t1	t2
Max	1,90	10,45	3,52	12,05	6,31	4,88
Min	0,18	0,13	0,25	0,13	0,38	0,76
Mean	0,72	1,62	0,99	2,06	2,26	2,57
Std	0,52	2,66	0,83	3,05	1,54	1,39
25% percentile	0,34	0,34	0,43	0,44	1,13	1,15
50% percentile	0,44	0,79	0,65	1,26	1,97	2,60
75% percentile	1,25	1,58	1,37	2,39	2,90	3,44

Each pricing error is the root of the average squared difference between the observed price and the price implied by each RND. Each summary statistic is derived from a set of 27 values of errors for options that expire on t1 and from a set of 14 values for options that expire on t2.

Table 3	2014		2015		2016	
G(theta) MLN	t1	t2	t1	t2	t1	t2
Max	1,90		1,51	0,73	0,77	10,45
Min	0,33		0,20	0,13	0,18	0,57
Mean	1,20		0,64	0,32	0,43	2,34
Std	0,60		0,48	0,26	0,18	3,14
25% percentile	0,44		0,33	0,14	0,30	0,77
50% percentile	1,31		0,40	0,16	0,40	1,15
75% percentile	1,64		1,07	0,57	0,53	2,41

Each pricing error is the root of the average squared difference between the observed price and the price implied by MLN. Each summary statistic is derived from a set of 7,10,10 values of errors which correspond to 2014, 2015 and 2016 for options that expire on t1 and from a set of 6 and 8 values of errors which correspond to 2015 and 2016 for options that expire on t2.

Table 4	2014		2015		2016	
G(theta) JDM	t1	t2	t1	t2	t1	t2
Max	2,80		3,52	1,09	0,77	12,05
Min	0,70		0,25	0,13	0,33	0,86
Mean	1,71		0,87	0,41	0,53	2,98
Std	0,75		0,88	0,42	0,15	3,52
25% percentile	0,85		0,39	0,13	0,37	1,17
50% percentile	0,36		0,48	0,14	0,54	1,46
75% percentile	2,20		1,10	0,82	0,63	3,20

Each pricing error is the root of the average squared difference between the observed price and the price implied by JDM. Each summary statistic is derived from a set of 7,10,10 values of errors which correspond to 2014, 2015 and 2016 for options that expire on t1 and from a set of 6 and 8 values of errors which correspond to 2015 and 2016 for options that expire on t2.

Table 5	2014		2015		2016	
G(theta) SPL	t1	t2	t1	t2	t1	t2
Max	5,85		6,31	4,31	2,08	4,88
Min	2,73		0,89	0,76	0,38	0,92
Mean	3,74		2,13	2,40	1,15	2,67
Std	1,12		1,53	1,49	0,66	1,42
25% percentile	2,90		1,12	0,89	0,42	1,30
50% percentile	3,51		1,58	2,93	1,29	2,58
75% percentile	4,56		2,70	3,64	1,70	3,93

Each pricing error is the root of the average squared difference between the observed price and the price implied by SPL. Each summary statistic is derived from a set of 7,10,10 values of errors which correspond to 2014, 2015 and 2016 for options that expire on t1 and from a set of 6 and 8 values of errors which correspond to 2015 and 2016 for options that expire on t2.

5.3 Moments of the RNDs methods

In this section, we elaborate on the first four moments of the densities extracted by the RNDs methods. Table 6 summarizes the mean, standard deviation, skewness and kurtosis of the derived densities for both sets of options (t1, t2). Table 7 gives a summary of the average first four moments of the RNDs for both sets of options respectively for the period before and after the referendum announcement. Figures 4,5,6,7 depict the time evolution of the skewness and kurtosis implied by the RNDs for both sets of options. Further details are given below each table and figure. First of all, in order to make sense, the mean and standard deviation are divided by the closing index price on the expiration date for each set of options. Furthermore, as far as standard deviation is concerned, due to the fact that the densities are overlapping with fixed expiration date for each set of options, it shrinks in proportion with the time to maturity. In other words, the RND standard deviation and percentile range are functions of the time to maturity. (Birru & Figlewski, 2009). In order to be a comparable measure, it is multiplied by the "squared root of T rule" ($\sqrt{\frac{365}{calendar\ days\ to\ option\ expiration}}$) (Birru & Figlewski, ibid). To begin with, it is observed that the mean of the first moment of the densities of all methods is very similar and close to 1. This hints that the estimated values for the index on the expiration date implied by the RNDs, are close to the actual closing index prices (on each expiration date). Secondly, we are in the position of grasping that for the densities implied by the first set of options which expire before the referendum, the first moment is greater than the unit (on average) for all methods. This stresses, that the estimated value is greater than the actual value of the index or in other words, that the expectations are kind of optimistic. Nevertheless, this fraction is below unit for the densities of all RND methods for options that expire after the referendum, which on its turn is a first sign of the pessimism about the after referendum period. It is also interesting to notice that for all methods and for both sets of options, the average estimation is higher in the period before the referendum announcement and declines in the period after. Furthermore, as far as the second moment is concerned, the three methods show similar results. Overall, the RNDs estimated by the contracts which expire after the referendum (t2) are slightly more volatile than those which stem from options with expiration before the referendum (t1). Investigating each period separately (before and after the announcement), it is observed that the second moment of the RNDs implied by options expiring on t2 rises significantly in the period after the announcement. The same is also observed in the RNDs by contracts that expire on t1, but the differences are slight. Taking this into account, we can assume that investors and market makers discount the uncertainty that brings the referendum announcement and make volatile expectations. In addition, cumulative the MLN RNDs on t1 are positively skewed against MLN RNDs on t2 which are negatively skewed (0,08 vs -0,28). JDM RNDs are negatively skewed on both expiration dates (-0,27 -0,29) while SPL RNDs are positively skewed on both t1 and t2. (0,58 0,05). However, examining each period separate, we get a different view. With respect to the expiration date t1, we can stress that MLN and SPL RNDs become negatively skewed after the referendum announcement, while JDM RNDs are negatively skewed in both periods and become slightly more left tailed after the announcement. RNDs on t2, similarly change sign by the advent of the announcement and become more negatively skewed in comparison with RNDs on t1. Negative skewness by definition means greater probability for a substantial negative return, because more observations are in the left tail of the distribution than in the right. Normal distribution assumes zero skewness, or else symmetric densities. This shift in the skewness between the two periods sheds light on the negative sentiment about the referendum. Moreover, as for kurtosis, we mention that cumulative for all RNDs and for both expirations, it is near

3 which assumes the normal distribution except for spline RNDs which have leptokurtic properties (3,60 for t1 and 4,21 for t2). However, making the same distinction between the two periods, it is observed that for all RND methods, densities on t2 become significantly more leptokurtic after the referendum announcement. Values of kurtosis greater than 3 (assumed by normal distribution), imply heavy tails, or else probabilities for extreme scenarios. Leptokurtic properties are another sign of the negative economic climate. Another point to stand, is the fact that kurtosis estimations are more volatile than those of the three first moments.

Table6		MLN		JDM		SPL	
Moments summary		t1	t2	t1	t2	t1	t2
mean/St*							
	Max	1,19	0,99	1,16	0,98	1,17	1,01
	Min	0,97	0,89	0,97	0,87	0,97	0,87
	Mean	1,04	0,93	1,05	0,93	1,06	0,93
	Std	0,06	0,03	0,05	0,03	0,06	0,03
Std*/St*							
	Max	0,18	0,43	0,25	0,35	0,17	0,39
	Min	0,09	0,11	0,11	0,12	0,09	0,11
	Mean	0,13	0,17	0,16	0,17	0,12	0,16
	Std	0,02	0,08	0,03	0,06	0,02	0,07
Skewness							
	Max	0,62	0,50	0,05	0,47	1,57	1,04
	Min	-0,56	-2,22	-1,79	-1,95	-1,74	-2,29
	Mean	0,08	-0,28	-0,27	-0,29	0,58	0,05
	Std	0,30	0,89	0,37	0,76	0,83	1,03
Kurtosis							
	Max	3,33	9,37	7,81	7,90	6,80	11,57
	Min	2,17	2,17	1,83	2,25	2,36	2,70
	Mean	2,66	3,48	2,53	3,42	3,60	4,21
	Std	0,30	2,16	1,10	1,74	1,23	2,55

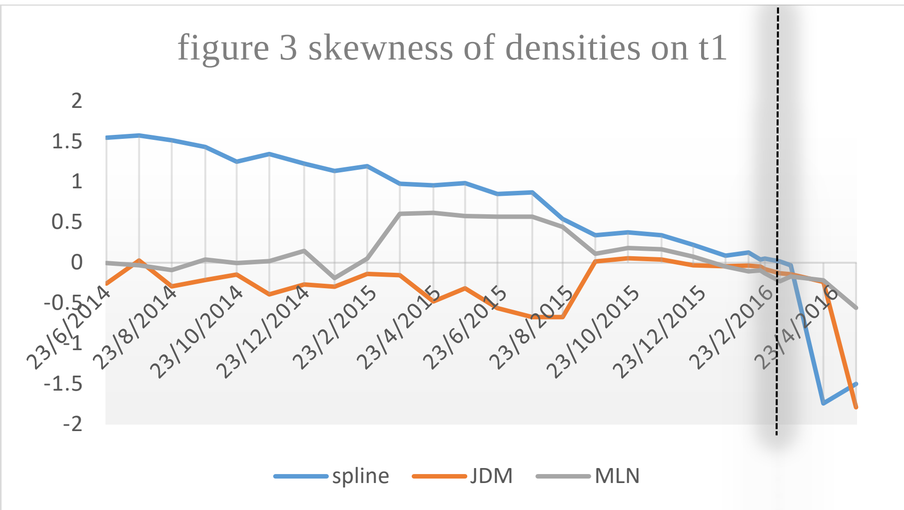
Each summary statistic is derived from the first four moments of 27 densities for RNDs estimations on t1 and of 14 RNDs estimations on t2

Table 7 Average moments divided by period		MLN t1	t2	JDM t1	t2	SPL t1	t2
mean/St*	Period 1	1,10	0,94	1,06	0,93	1,10	0,94
	Period 2	1,01	0,92	1,01	0,92	1,01	0,92
Std*/St*	Period 1	0,13	0,14	0,17	0,14	0,11	0,12
	Period 2	0,14	0,19	0,15	0,20	0,15	0,19
Skewness	Period 1	0,20	0,33	-0,24	0,18	0,94	0,75
	Period 2	-0,20	-0,72	-0,35	-0,72	-0,43	-0,48
Kurtosis	Period 1	2,72	2,85	2,32	2,84	3,57	3,66
	Period 2	2,53	4,19	3,11	4,03	3,67	4,88

Period 1 refers to a time sample from 23 June of 2014 to 18 February of 2016 and period 2 refers to a sample from 19 February 2016 to 15 July 2016. Overall, with respect to options expiring on t1, we have 20 RNDs for period 1 and 7 RNDs for period 2. With respect to options expiring on t2, for period 1 we have 6 RNDs and 8 RNDs for period2.

In order to get an even clearer view on the expectations of the investors before and after the referendum announcement, we elaborate more on the third and fourth moment of the implied RNDs for both expiration dates. In figures 3 and 4, we can see the time evolution of skewness and kurtosis of RNDs estimations on t1. The shaded areas separate the periods before and after the referendum announcement. Taking into consideration firstly the period before the announcement, we can mention that Spline RNDs in 2014 were positively skewed (between 1 and 1,5). This indicates that they were estimating higher probabilities for positive returns. In the middle of 2015, they remain right tailed but with a downward trend. MLN RNDs are symmetric at most in period 1, except for the first middle of 2015 when they turned to be positively skewed. JDM RNDs seem to be slightly negatively skewed in 2014 and even more left tailed in the first middle of 2015, but in the second middle of 2015 they turn out to be symmetric. It bears great significance the fact that in February of 2016, exactly after the referendum announcement, the skewness coefficient implied by all RNDs starts to plummet and this downward trend continuous till the expiration of the contracts. We can observe that in April for instance, two months before the referendum, JDM and SPL RNDs are negatively skewed approximately by -2, while MLN RNDs by -1. This reflects all the anxiety in the British market for this extreme political event. The proportional is observed also in the RND estimations on t2 (about

22 days after the referendum). As figure 5 illustrates, the declining trend in skewness commences by the referendum announcement and finally, on March JDM and MLN RNDs become negatively skewed, while SPL RNDs start to have left tails two months later on May. This decline continuous proportionally with the passage of time and is in a greater magnitude compared to RNDs implied by contracts that expire before the referendum. As we can see, in June RNDs have skewness coefficient less than -2. This hints, that the uncertainty is even higher in the distance between June and July (after referendum period), which coincides with the expiration of the second set of options we use. Finally, taking a glance at kurtosis coefficient, we can stress that in RNDs on t1, in the period before February 2016 it is around 3, as it is assumed by the normal distribution. Nonetheless, after February, there starts an inclining trend in the kurtosis coefficient of JDM and SPL RNDs, which reaches an apex in May, with a value greater than 7. This upward trend is slightly observed in MLN RNDs with respect to the first expiration date (t1). Similarly, the same is noticed in the RNDs estimations on t2 but in an even greater climax. The kurtosis coefficient starts to increase in May 2016, after the announcement, and this increase continuous in a galloping rate as time to expiration approaches. The highest value is observed on SPL RNDs by the start of July 2016, after the referendum (around 12 for SPL, 10 for MLN and 8 for JDM). This indicates, that the negative economic sentiment does not cease by the referendum, but continuous because markets cannot discount accurately the potential consequences that may arise from an exit from the EU.



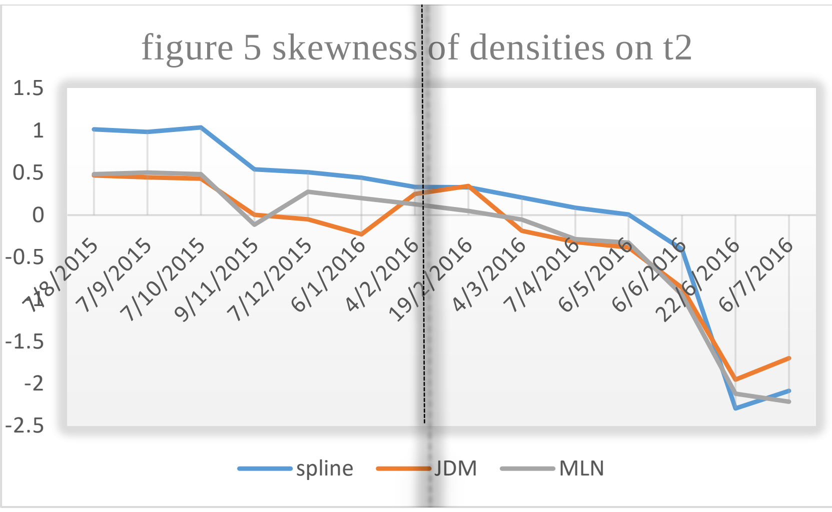


figure 5 skewness of densities on t2
1.5
1
0.5
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-0.5
-1
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-2
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7/8/2015
7/9/2015
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spline
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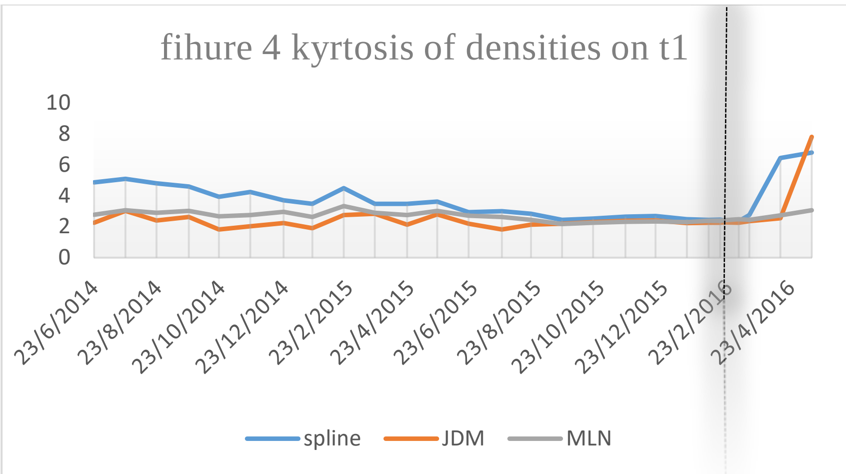
fihure 4 kyrtosis of densities on t1
10
8
6
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2
0
23/6/2014
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spline
JDM
MLN

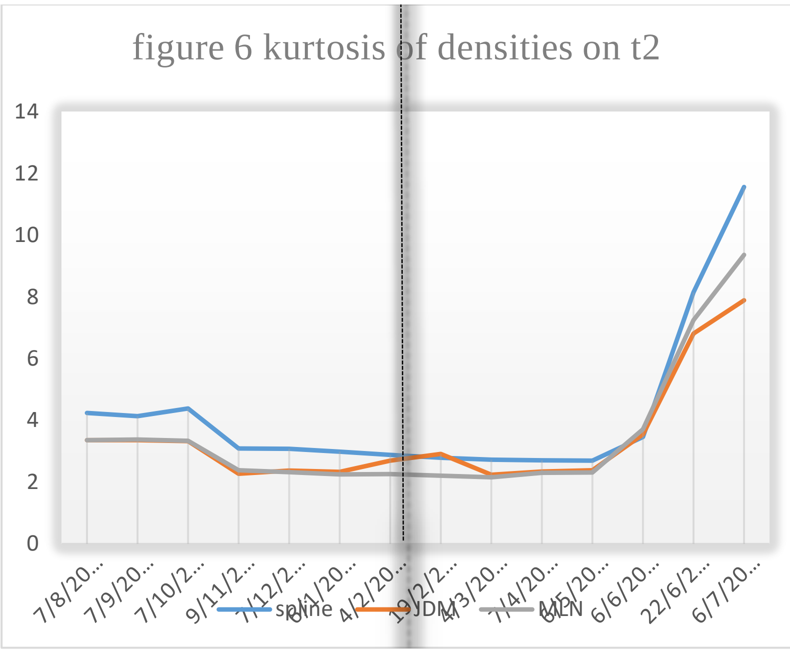


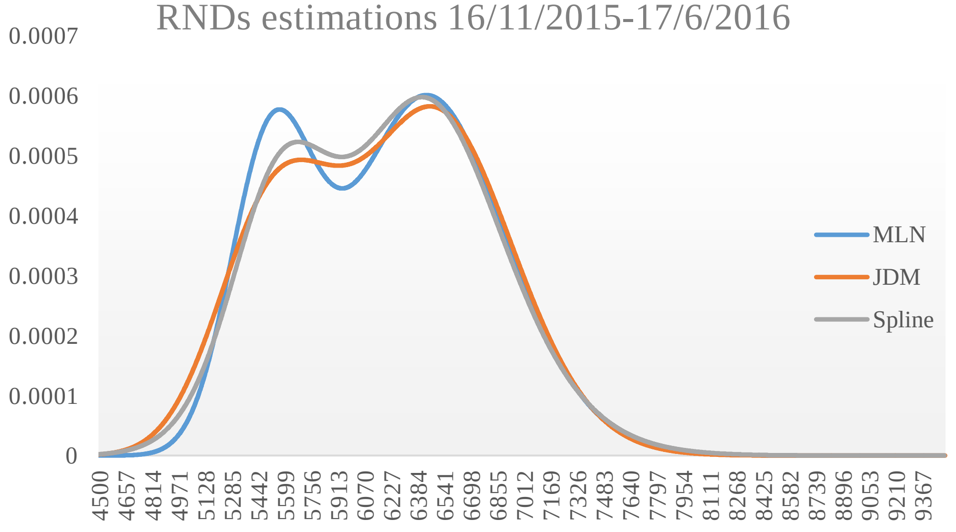
figure 6 kurtosis of densities on t2
14
12
10
8
6
4
2
0
spline
JDM
MLN

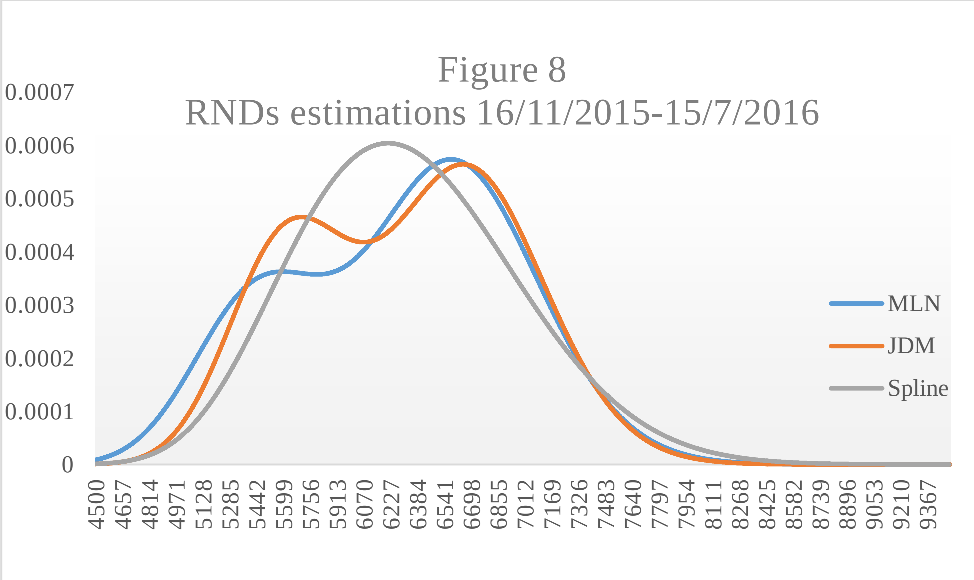
5.4.1 RNDs on November 11, 2015 (before referendum announcement)

In this section, we elaborate on the RNDs estimations at 11 November of 2015, about four months before the referendum was announced, with respect to both expiration dates t1 and t2. Figure 7 illustrates the estimations on 17 June 2016 (t1). We can observe that there are slight differences between the three RND densities. All of them are high peaked and in the peaks we can notice the flexible shapes they take. The right tails are almost identical. The left tails show some slight differences but not significant ones. Furthermore, JDM and SPL appear to be kind of smoother than MLN. As far as the first moment is concerned, it is quite similar between the three methods. All of them estimate the closing index price at the expiration date to be more than 6100 index points, which is an optimistic scenario, as it finally closed at 6021. They are slightly positively skewed, with the MLN showing the highest value (0,17) which means that they are almost symmetric. Moreover, the kurtosis is below 3 which hints that values near the mean are more probable to arise than tail values. Taking these into account, we are inclined to believe that before the referendum announcement, the markets didn't expect any negative shock. We are in the position of mentioning the same also by taking a glance at the estimations on 15 of July 2016 (figure 8). There are some differences in the shapes of the RNDs but not significant ones. These differences lie on the peaks and on the left tails, while the right tails are almost identical. It is also observed that the Spline RND is smoother than MLN and JDM which take more flexible shapes. Their mean is around 6300 index points, lower than the closing price at the expiration date 6669,5, which could be characterized at a first step kind of pessimistic. As far as standard deviation is concerned, they seem to be more volatile than those on figure 7, because the time to expiration is more and as we mentioned, standard deviation is a function of the time to maturity. However, their skewness is around zero. SPL is slightly positively skewed (0,54), JDM symmetric and MLN slightly negatively skewed which implies that they do not estimate probabilities for substantial negative returns. Moreover, they do not appear leptokurtic properties as their kurtosis coefficient is around 3. Consequently, we can advocate that in this date RNDs estimations do not seem to discount any adverse scenarios.

Table8 RNDs moments		MLN	JDM	SPL
t1	Mean	6184,17	6175,09	6146,38
	Std	597,19	616,37	605,48
	Skewness	0,17	0,04	0,34
	Kurtosis	2,33	2,38	2,65
t2	Mean	6279,29	6307,96	6295,16
	Std	699,74	655,55	645,92
	Skewness	-0,12	0,00	0,54
	Kurtosis	2,39	2,28	3,09

Figure 7
RNDs estimations 16/11/2015-17/6/2016



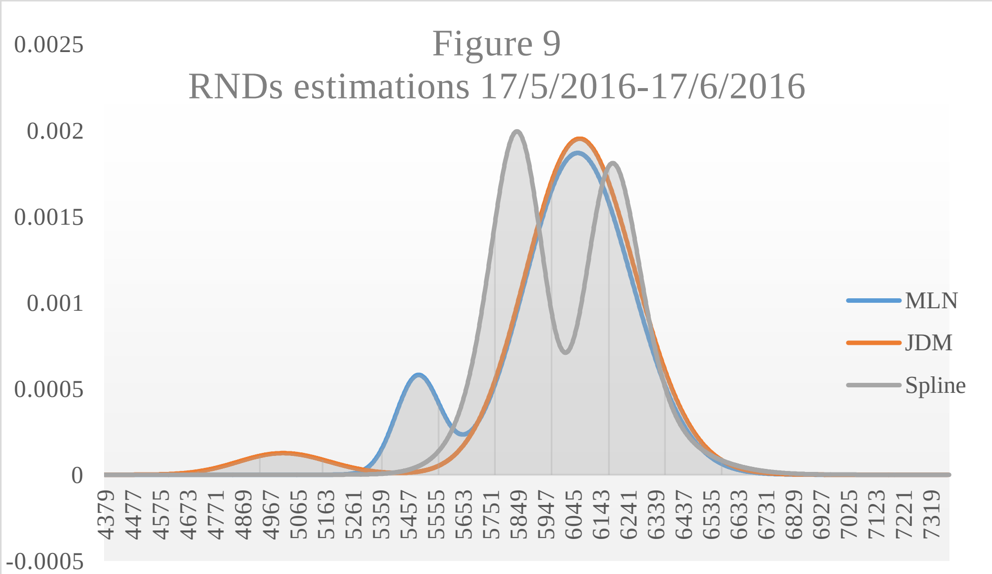


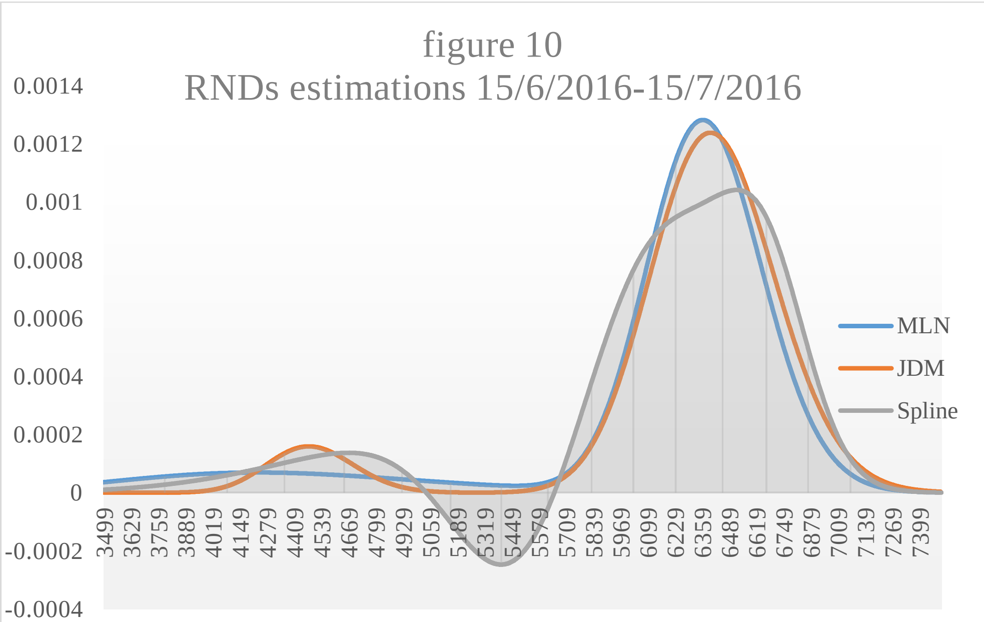
5.4.2 RNDs on the period after referendum announcement

In this section, we elaborate on the RNDs estimations after the announcement of the referendum. Our goal is to compare the estimations on 17/6/2016 (t1) and on 15/7/2016 (t2). In order for the comparison to be feasible, we choose the issue date of the estimation to be exactly one month before each expiration date t1 and 12. The purpose of this is that the options of each expiration date have exactly the same number of days to expiration. First of all, on the one hand, we describe the RNDs estimations on t1. The expected value of the Index is around 6130 points for all methods with slight differences. This value is near the closing price at t1(6029), but slightly higher about 2%. As for the second moment, it ranges from 12% to 16% with JDM estimating the highest value (16%) and SPL the lowest (12%). As far as the skewness coefficient is concerned, all methods are negatively skewed, which illustrates the negative sentiment about the forthcoming referendum. The JDM RNDs are the most negative skewed (-1,79), the MLN the less (-0,56) and the SPL in the middle (-1,5). Taking the kurtosis coefficient into account, we observe that only the JDM has leptokurtic properties with a value of 7,81, much higher than 3. MLN and SPL have kurtosis value around 3. We can attribute this difference between the RNDs to the nature of the JDM to catch the shocks either positive or negative. Figure 9 shows the three RNDs on 17/6/2016 (one week before the referendum). MLN and JDM have similar shapes. They are both left tailed, but the JDM has much heavier left tail than MLN which hints the higher probability it estimates for extreme low values for the index. Its left tail ranges from 4500 to 5200 index points, which is an extreme adverse scenario while MLN's left tail ranges from 5200 to 5600 index points. The SPL RND looks different. It takes a flexible shape, negatively skewed, but without significant tails. On the other hand, we take into consideration the RNDs on 15/7/2016 (after the referendum). We can notice that the expected index value is around 6200 with the MLN RND estimating the lowest price (6135). Compared to the closing price on 15/7/2016 which was 6669,5, the RNDs estimations are significantly lower around 8%. Taking this into consideration, we could grasp that the option market on this date was discounting a significantly negative return for the FTSE 100 which actually did not occur. According to the second moment, The RNDs on t2 estimate 2 times higher standard deviation than the RNDs on t1 in both absolute and "annualized" value or else, the uncertainty in the after referendum period is two times higher than the before referendum period due to the option market. Examining the skewness coefficient, RNDs on t2 are significantly negatively skewed, more than those on t1. It ranges from -1,95 (JDM) to -2,29 (SPL) and indicates that all of them negatively asymmetric. We can also notice an excess kurtosis ranging from (7,26) MLN to 8,16 (JDM) greater than 3 and also greater than the values of RNDs on t1. Figure 10 gives a picture of these RNDs. Firstly, we can notice that the spline method estimates some negative probabilities. It is irrational, but is a common problem of curve fitting methods and is mentioned in the literature (Bliss & Panigirtzoglou 2002, Andersen & Wagener 2002, Liu et al 2007). It arises especially in high uncertainty periods when either the number of in the money and out of the money options is unbalanced or when many options are mispriced and hence there are observed some extreme values of implied volatilities. Nevertheless, the spline densities integrate to one and thus the moments implied make sense. We can also mention from figure 10 that the right tails of the three densities are quite similar and thin. The interesting part lies on the left tails. All of the RNDs appear to have long and heavy left tails ranging from 5500 to 3400 index points. The probability mass they contain is far higher than the corresponding one of those on t1. To sum up, both RNDs on t1 and on t2 show the investors' anxiety and uncertainty about this extreme political event. Furthermore, this anxiety is much greater for the time period about a month after the referendum compared to the one about a week before the referendum.

Table 9	MLN	JDM	SPL

Moments				
17/5/2016-17/6/2016	Mean	6129,91	6146,04	6138,50
	Mean/St*	1,02	1,02	1,02
	Std	256,80	301,28	229,97
	Std*/St*	0,20	0,16	0,2
	Skewness	-0,56	-1,79	-1,50
	Kurtosis	3,07	7,81	3
15/6/2016-15/7/2016	Mean	6135,65	6264,96	6260,56
	Mean/St*	0,92	0,92	0,94
	Std	734,55	604,89	666,11
	Std*/St*	0,43	0,35	0,39
	Skewness	-2,12	-1,95	-2,29
	Kurtosis	7,26	8	8,16





5.5 Time evolution of the RNDs estimations

The purpose of this section is to show how the RNDs estimations of all methods mentioned and for both expiration dates t1 and t2 were shifting by the passage of time. Figures 11, 12 and 13 illustrate the RNDs estimations of MLN, Spline and JDM starting from 23 June of 2014 (when options which expire on t1 started to trade) and ending to 17 May 2016 with respect to 17/6/2016, the first expiration date (t1). The referendum announcement occurred when these contacts had 0,5 years to maturity. Figures 14, 15 and 16 illustrate the RNDs estimations of MLN, Spline and JDM starting from 7 August of 2015 (when options which expire on t2 started to trade) and ending to 7 July 2016 with respect to 15/7/2016, the second expiration date (t2). The referendum announcement occurred when these contacts had 0,6 years to maturity. First of all, we observe that in all figures the densities in front are the wider ones and shrink by the time. This wideness is proportional to the time to expiration. The longer the time to maturity, the higher the probability of large price fluctuations. As the expiration approaches, densities become high peaked and narrower which reflects the less time for huge price fluctuations. The interesting point to stand is that all RNDs for both maturities start with positive skewness. As far as they are concerned, the probabilities are higher for positive returns. This indicates a positive economic sentiment in the years before 2016. As time passes by, we could observe that RNDs, apart from shrinking which stems from the shrink of the time to maturity, their peaks shift from the right to the left driving to lower expectations for the index value. Also, the heavier probability mass starts to concentrate to the left tails. The less the time to maturity, the closer the referendum date and therefore, the thicker the left tails. Contrary to the shrink of the density which is a function of the expiration time, negative skewness and leptokurtic properties are attributed to the shift in the economic sentiment. According to the leverage effect theory, negative shocks lead to higher volatility than equally positive shocks and therefore, there is a negative correlation between volatility and returns. In other words, by the advent of negative shocks the volatility increases which makes the index a riskier investment. Hence, agents seek higher expected return in order to invest and consequently, the prices drop. This theory complies with the case we are examining. Finally, we can notice that all RNDs take flexible shapes.

Figure 11

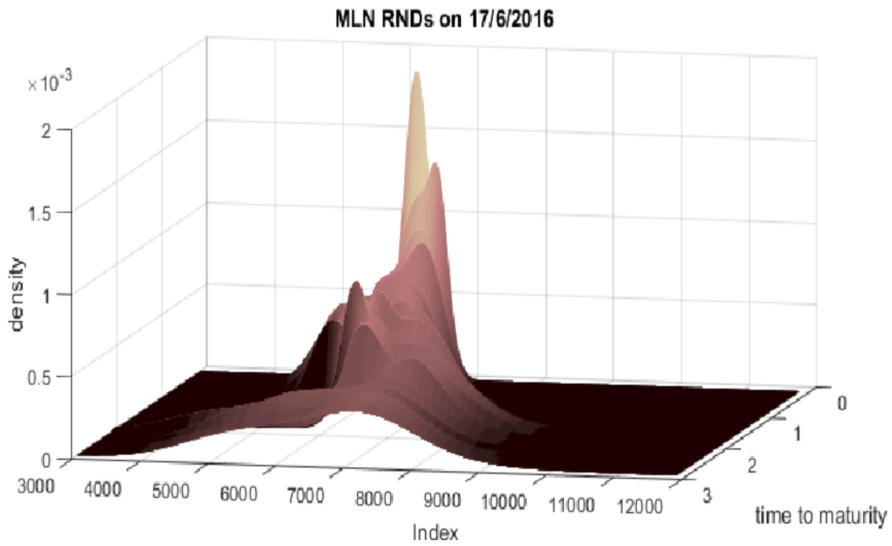


Figure 11 shows 27 MLN overlapping RNDs estimations from 23/6/2014 to 17/5/2016 with respect to the first expiration date (17/6/2016). The referendum announcement occurred when the options had 0,5 years to maturity.

Figure 12

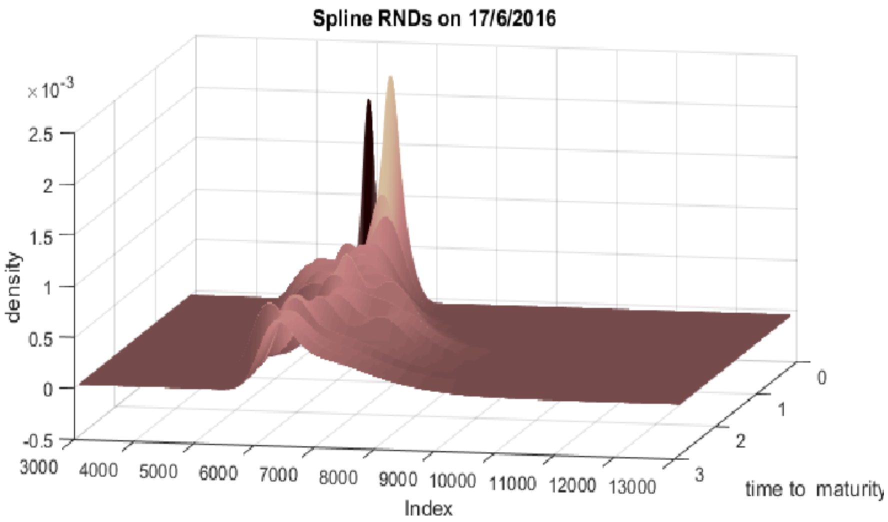


Figure 12 shows 27 Spline overlapping RNDs estimations from 23/6/2014 to 17/5/2016 with respect to the first expiration date (17/6/2016). The referendum announcement occurred when the options had 0,5 years to maturity.

Figure 13

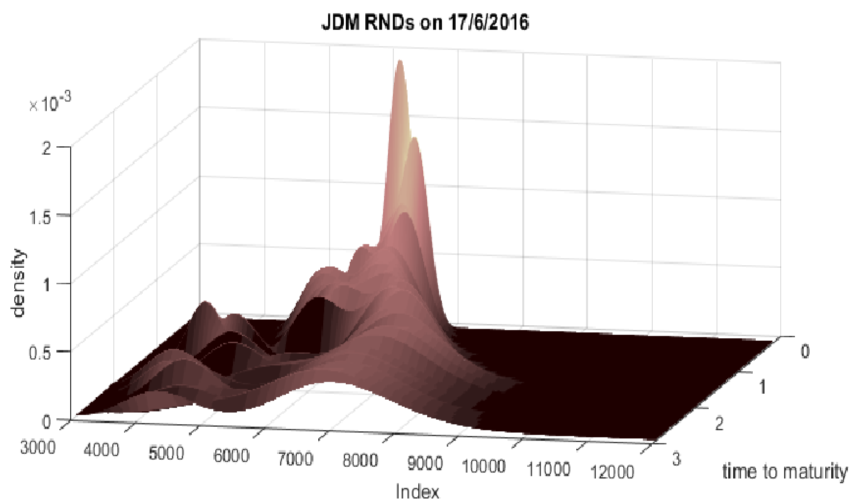


Figure 13 shows 27 JDM overlapping RNDs estimations from 23/6/2014 to 17/5/2016 with respect to the first expiration date (17/6/2016). The referendum announcement occurred when the options had 0,5 years to maturity.

Figure 14

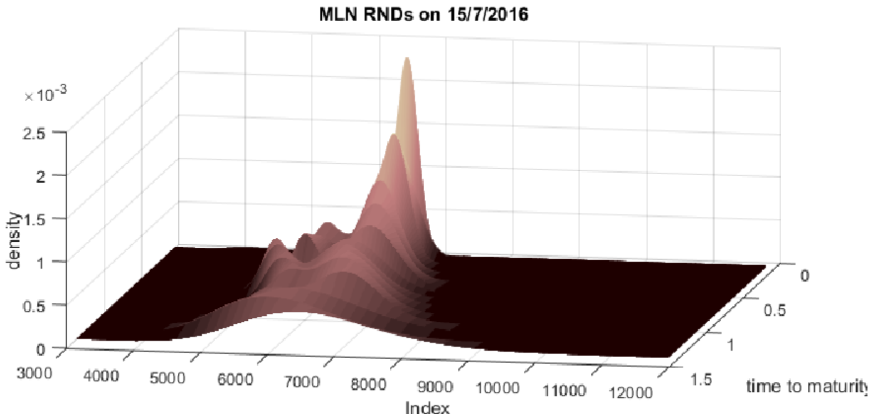


Figure 14 shows 14 MLN overlapping RNDs estimations from 7/8/2015 to 5/7/2016 with respect to the second expiration date (15/7/2016). The referendum announcement occurred when the options had 0,6 years to maturity.

Figure 15

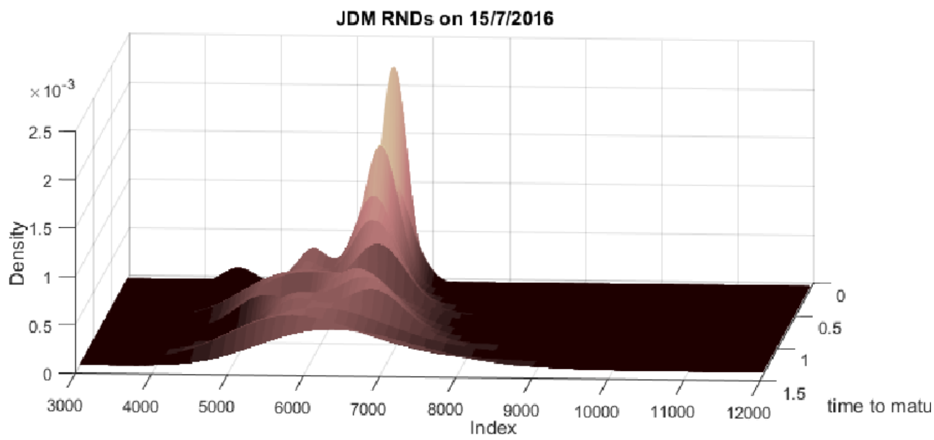


Figure 15 shows 14 JDM overlapping RNDs estimations from 7/8/2015 to 5/7/2016 with respect to the second expiration date (15/7/2016). The referendum announcement occurred when the options had 0,6 years to maturity.

Figure 16

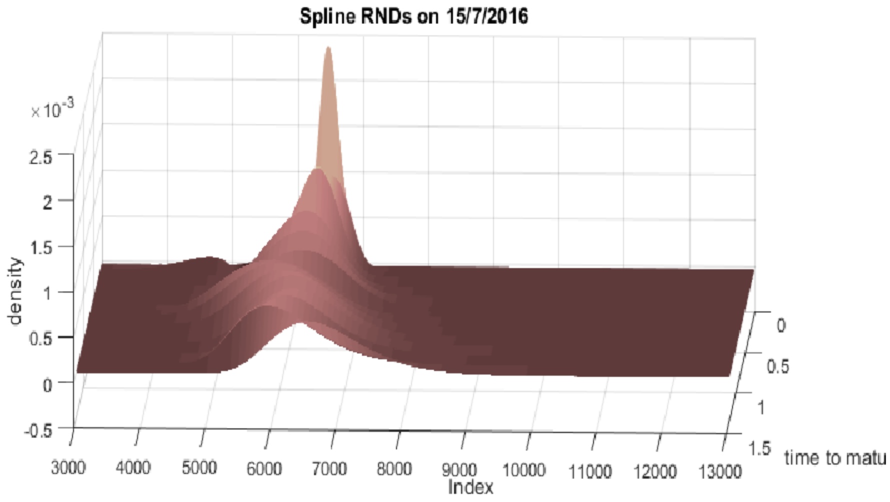


Figure 16 shows 14 Spline overlapping RNDs estimations from 7/8/2015 to 5/7/2016 with respect to the second expiration date (15/7/2016). The referendum announcement occurred when the options had 0,6 years to maturity.

Section VI Empirical Results of Real World Densities (RWDs)

As we mentioned before, RNDs assume that investors do not seek to earn excess returns for taking risk, which is utopic. Therefore, taking this into consideration, we transform the mixtures of two lognormals RNDs into real world densities (RWDs), with respect to the investors' risk preferences. We transform only the MLN RNDs due to the fact that there are no closed form transformations for the JDM and the spline method we apply in the available literature. For each MLN distribution, we calculate the log likelihood at the index levels St* when options expire (t1, t2). We assume that the investors' risk preferences are described by the power utility function and also that the relative risk aversion (γ) changes annually. Then we estimate γ by maximizing the log likelihood function (equation 27). We exclude the RNDs during 2014 from this process, because we only have observations for options expiring on t1 and thus, it may cause bias to the results.

6.1 Estimation of the risk parameters

Assuming the MLN RND, our maximum log likelihood γ estimations are 0,16 and 9,03 for 2015 and 2016 equivalently. The time framework during which we select to change γ is subjective, but the majority of previous studies do select to change it annually. However, we are aware that the overlapping nature of our data may cause some bias in the predictive power of the extracted RWDs. In our study we have 2 fixed expiration dates and thus, the forecast horizon differs among each issue date we extracted RNDs. Most previous studies who followed the same procedure for risk transformation used non-overlapping densities with constant forecast horizons. However, their goal was to make forecast evaluations between RNDs and RWDs (Ait Sahalia & Lo 2000, Bliss & Panigirtzoglou 2004, Liu et al 2007). In this paper, we do not seek to compare the forecast ability of densities. On the other hand, we seek to shed some light on the risk preferences of the investors especially after the referendum announcement. In order to test the statistical significance of the γ parameter, we implement the likelihood ratio test. In 2015, the log likelihood increases very slightly after the risk adjustment from -74,24 to -74,21. The likelihood ratio test statistic of the null hypothesis of the zero risk parameter is given by the increase in the log likelihood times 2. This figure is compared to the test statistic's asymptotic distribution $X^2(1)$ with 1 degree of freedom for the significance levels of 5%, 1% and even 0,5%. We observe that for 2015, the value of the statistic is 0,07 less than the critical values for every significance level. Thus, we cannot reject the null hypothesis of zero risk aversion. Nevertheless, considering 2016 which coincides with the referendum announcement, the γ parameter has the value of 9,03. The likelihood ratio statistic is 8,06 greater than the critical values for every level of significance, even for 0,5%, which indicates that we do reject the null hypothesis of no risk aversion. First of all, risk aversion near zero seems irrational. Taylor 2005 and Jackwerth 2002 elaborate on zero and negative risk aversions. To their point of view, negative or zero risk aversion parameters arise in post crisis periods due to the options' being mispriced. According to them, risk averse investors have the tendency to liquidate their positions. As a matter of fact, risk aversion decreases among the existing investors. Therefore, zero risk aversion implies that agents are pessimistic rather than risk averse. However, in our case 2015 was not a post- crash period for the UK. On the other hand, Bliss & Panigirtzoglou in 2004 proved that risk aversion is strongly dependent with the forecast horizon of the density. As far as they are concerned, the longer the investment horizon the less the risk aversion. Agents who take long horizon investments have alternatives to overcome market shocks. For example, they may smooth their consumption and simultaneously, they can increase their no investment income by working harder. On the contrary, short-term investors do not appear to be very flexible with this and thus, they are more risk averse. For FTSE 100 they found that for the investment horizon of 4 months (the maximum they examined), the risk aversion had the less value (1,9). In our research, in 2015 the average investment horizon is more than 1 year. Consequently, we could attribute the insignificant risk aversion of 2015 to the very long time horizon of the options expiration. The average investment horizon in 2016 is less than half a year. We cannot be sure if this huge increase in the risk parameter between the two years stems from the decrease in the investment horizon or from the referendum announcement. In fact, the representative agent in 2016 became far more risk averse than the previous year (more than 9 times) which hints that more risk premium is demanded for investmenting in the market portfolio. The most feasible explanation we can give, is that it is an outcome of the combination of both facts we mentioned.

Table 10	γ	ML1	ML0	2(L2-L1)	Chi @5%/1%/0,5%
2015	0,16	-74,21	-74,24	0,07	3,84/6,63/7,88
2016	9,03	-52,90	-56,93	8,06	

6.2 Comparison of moments between Risk Neutral and Real World Densities

In this section, we focus on the comparison of moments between MLN RNDs and RWDs in the year 2016, because only for this year we found a statistical significant γ parameter. For the year 2015, RNDs and RWDs are almost identical as γ is very close to zero and statistical not significant. Table 10 summarizes the moments of MLN RNDs and RWDs during 2016 for both expiration dates t1 and t2. At a first step, we are in the position of observing that the mean of the RWDs is higher than the corresponding one of the RNDs for densities on both t1 and t2. (1,04>1,01 for t1 and

0,97>0,92 for t2). Yet, with respect to densities on t2 the fraction between the mean and the index price at the expiration date is still less than 1, which indicates that the RWDs similar to RNDs underestimate the index price after the referendum. Furthermore, considering the “annualized” standard deviation, RWDs seem to be less volatile than RNDs. For densities on t1 the difference is very slight while for those on t2 it drops from 19% to 16%. Moreover, after the risk adjustment, the densities become more negatively skewed and more leptokurtic with respect to both expiration dates, because the kurtosis coefficient increases. We could stress from this that under the assumption that investors are risk averse, greater probabilities are estimated for significant negative returns and for scenarios far away from the mean. It is also significant that after the adjustment, the moments become less volatile.

Table10 2016 Moments summary		MLN q		MLN p	
		t1	t2	t1	t2
mean/St*					
	Max	1,06	0,96	1,07	0,99
	Min	0,97	0,89	1,03	0,96
	Mean	1,01	0,92	1,04	0,97
	Std	0,03	0,02	0,04	0,01
Std*/St*					
	Max	0,17	0,43	0,16	0,24
	Min	0,11	0,12	0,11	0,13
	Mean	0,14	0,19	0,13	0,16
	Std	0,02	0,10	0,02	0,04
Skewness					
	Max	-0,05	0,20	-0,27	-0,14
	Min	-0,56	-2,22	-0,47	-1,87
	Mean	-0,20	-0,62	-0,41	-0,74
	Std	0,16	0,94	0,06	0,60
Kurtosis					
	Max	3,07	9,37	3,44	12,14
	Min	2,30	2,17	3,03	3,10

Mean	2,53	3,76	3,19	5,42
Std	0,25	2,68	0,12	3,46

MLN q refers to the risk neutral MLN while MLN p refers to the risk adjusted MLN. In 2016 we have 8 RNDs and RWDs on t1 and 9 RNDs and RWDs on t2.

The following figures depict the change in the third and fourth moment of RNDs and RWDs during 2016. First of all, taking into consideration the densities with forecast horizon before the referendum, we can grasp RWDs are more negative skewed for the most time. While in the RNDs the declining trend in skewness starts by the referendum announcement and goes on till the expiration, in the RWDs we cannot mention such a downwards trend. On the contrary, after April RWDs become less left tailed and finally on May, RNDs appear to be more negative skewed than RWDs. Something similar is observed on densities estimations on t2. RWDs are more left tailed until June. In July, this inverses as RNDs seem to have more negative skewness coefficient. Taking a glance at the kurtosis figure, with respect to both expiration dates the RWDs are more leptokurtic than RNDs for the whole year. More specifically, this difference shrinks among RWDs and RNDs estimations on t1, while it widens if we examine those on t2. Figure 20 shows that RWDs kurtosis peaks in June, some days before the referendum. This might be regarded as a sign that the risk adjustment drives to more pessimistic estimations.

figure 17 skewness of densities on t1

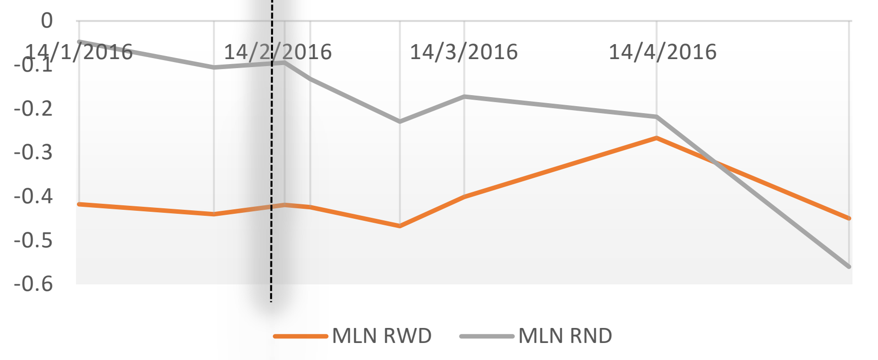


Figure 17 shows the time evolution of the skewness of the 8 RNDs and RWDs in 2016 from January 14 to May 17 with respect to 17/June/2016 (t1).

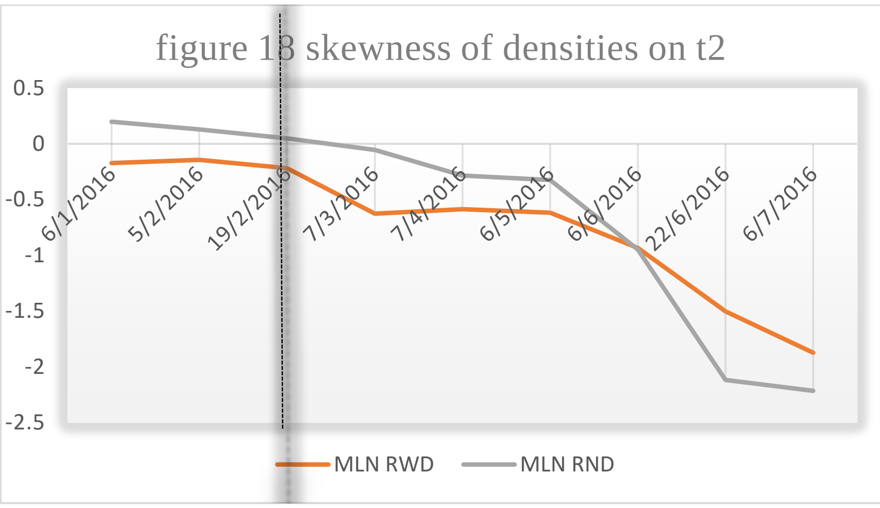


Figure 18 shows the time evolution of the skewness of the 9 RNDs and RWDs in 2016 from January 6 to July 7, with respect to 15/July/2016 (t2).

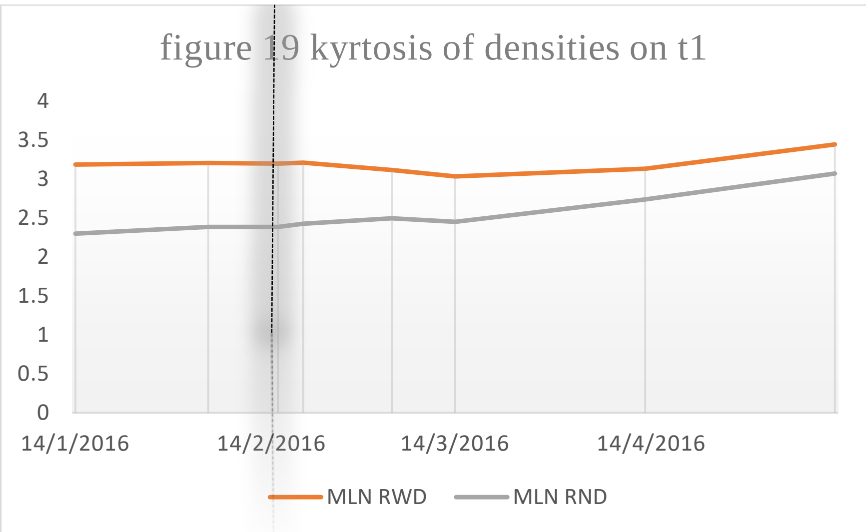


Figure 19 shows the time evolution of the kurtosis of the 8 RNDs and RWDs in 2016 from January 14 to May 17 with respect to 17/June/2016 (t1).

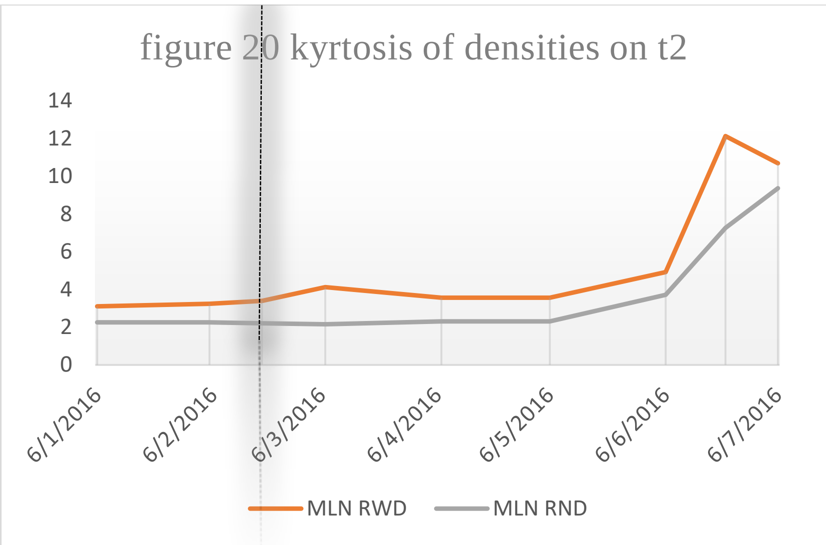


Figure 20 shows the time evolution of the kurtosis of the 9 RNDs and RWDs in 2016 from January 6 to July 7, with respect to 15/July/2016 (t2).

6.3 RNDs versus RWDs on specific dates

In this section, we examine the RNDs and RWDs estimations on t1 (17/6/2016) and on t2 (15/7/2016) exactly one month before. To begin with, taking into account the first date, the RWD appears to have significantly higher mean with the value of 6450 vs 6129, which is much greater than the closing price at the expiration date. According to standard deviation, the RWD is slightly less volatile than the RND, in both absolute and annualized terms. It is also slightly less negative skewed and slightly more leptokurtic. Figure 21 illustrates the RND and RWD on the date we mentioned. We can observe that the RWD has a higher peak and is more right centered which indicates the higher estimation. The left tail looks smoother and the RWD appears to be more leptokurtic. Taking a glance at the second date, we can also stress that the RWD has a higher estimation but still less than the actual closing price on July 15. It is interesting to notice that it is by half less volatile than the RND in both absolute and annualized terms. We might stress that the higher mean, closer to the actual index price and the lower volatility could indicate more accurate estimation. Moreover, the RWD in this date looks less negative skewed, but yet more leptokurtic. These properties are depicted in figure 22. We can observe approximately the same as in figure 21. The RWD has a higher peak, which has shifted to the right compared to the RND. The right tail is almost similar, while the left one looks thinner in the graph. The higher kurtosis coefficient could be attributed to the leptokurtic properties, for example the higher peak and the lower volatility in combination with the long left tail.

Table 11 Moments of RNDs and RWDs		MLN q	MLN p
17/5/2016-17/6/2016	Mean	6129,91	6450,00
	Mean/St*	1,02	1,07
	Std	256,80	240
	Std*/St*	0,14	0,13
	Skewness	-0,56	-0,45
	Kurtosis	3,07	3,44
15/6/2016-15/7/2016	Mean	6135,65	6459,51
	Mean/St*	0,92	0,97
	Std	734,55	365,96
	Std*/St*	0,43	0,24
	Skewness	-2,12	-1,50
	Kurtosis	7,26	12,14

Table 11 summarizes the moments of MLN RNDs and MLN RWDs for the estimations on 17/6/2016 and on 15/7/2016 (t1, t2) exactly 1 month before. The summary statistics are derived from the moments of 8 densities on t1 and of 9 densities on t2.

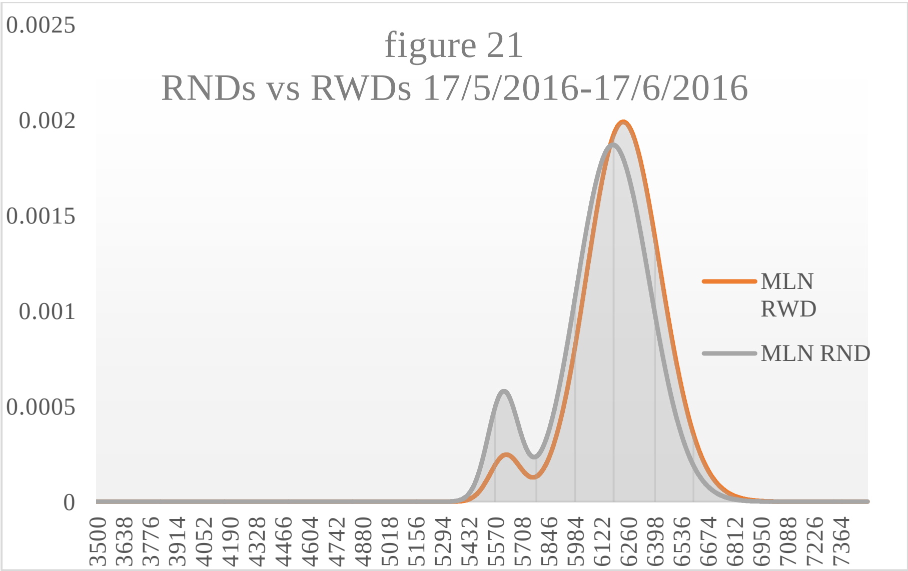


figure 21
RNDs vs RWDs 17/5/2016-17/6/2016
0.0025
0.002
0.0015
0.001
0.0005
0
MLN RWD
MLN RND
3500
3638
3776
3914
4052
4190
4328
4466
4604
4742
4880
5018
5156
5294
5432
5570
5708
5846
5984
6122
6260
6398
6536
6674
6812
6950
7088
7226
7364

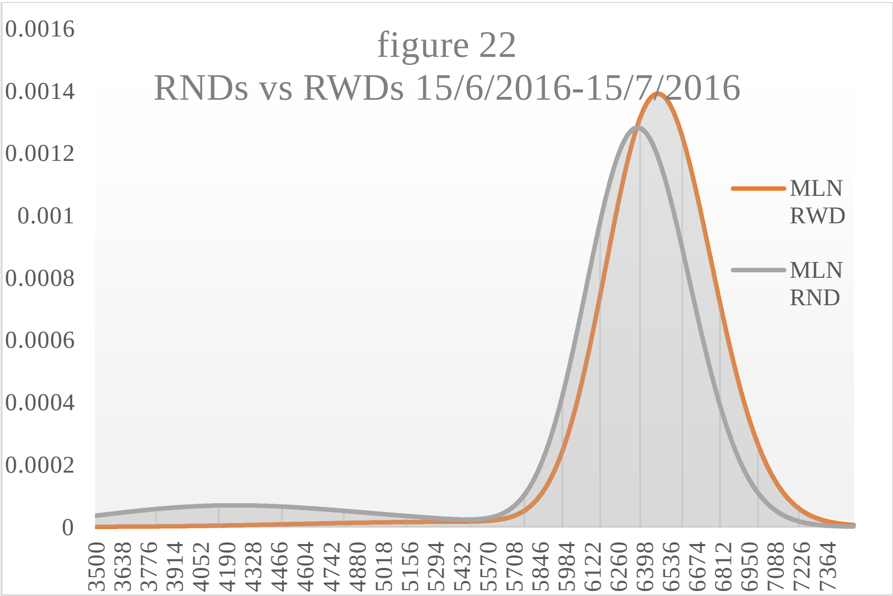


figure 22
RNDs vs RWDs 15/6/2016-15/7/2016
0.0016
0.0014
0.0012
0.001
0.0008
0.0006
0.0004
0.0002
0
MLN RWD
MLN RND
3500
3638
3776
3914
4052
4190
4328
4466
4604
4742
4880
5018
5156
5294
5432
5570
5708
5846
5984
6122
6260
6398
6536
6674
6812
6950
7088
7226
7364

Section VII Conclusion and Further research

In this paper we estimated MLN, JDM and Spline RNDs for the FTSE 100 index implied by options that expire on June 17, 2016 (before the referendum) and on July 15, 2016 (after the referendum). We found that densities derived from option prices are very informative and reflect the market anxiety about such an extreme political event, the consequences of which could not be discounted accurately a priori. This stems from the forward looking nature of options contracts. We conclude that after the referendum announcement, the densities become more volatile, negative skewed and leptokurtic which hints the uncertainty and pessimism agents have about the final outcome of the referendum. Our findings also show, that RNDs on July 15, are even more volatile, left tailed and leptokurtic than those on June 17. This indicates that the anxiety is even greater for the after Referendum period. Furthermore, we transformed the MLN RNDs into MLN RWDs by maximizing the likelihood function with respect to the closing prices at the expiration dates. We found that there is no significant risk aversion in the year 2015, while in 2016 the relative risk aversion parameter has the value of 9 and is statistical significant in every significance level. On its turn, this illustrates that after the referendum announcement, agents became more risk averse and demanded higher risk premium for investing in the index than before. However, as we mentioned it is difficult to distinguish between pessimism and risk aversion. RWDs seem to have higher mean, to be less volatile, more negative skewed and more leptokurtic than RNDs.

The main disadvantage of the RNDs and RWDs estimations methods we described is that they can be implemented only to European options. In spite of the fact that for most equity indices, only European style options are available, this is not the case for stocks or currency. American options are more widely traded, despite being over the counter derivatives. Thus, it might be more practical to extract RNDs and RWDs from American options because they might be more informative. According to Yisong, 2010 there exists a twostep method to obtain RNDs from American style options. Firstly, American options are converted into European. After the transformation, the RNDs moments are approximated. Very few researchers have tried to follow this procedure. Furthermore, an even more ambitious researcher could analyze the higher order moments of the RNDs. For example, the fifth order moment indicates the asymmetry in the tails, which can be also very informative.

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